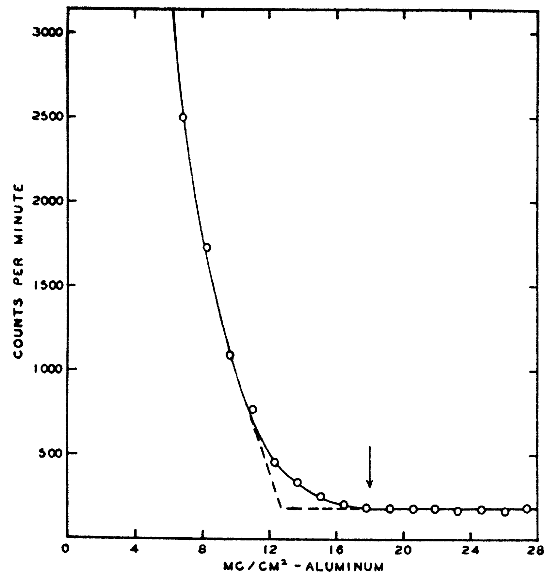


FIG. 3. Absorption in aluminum of the conversion electrons of *$Te^{125}$

can be concluded that the high energy beta-rays do not go directly to the 60-day tellurium level but are followed by gamma-rays which then lead to the tellurium isomer. The beta-gamma-coincidence rate, $0.42\times 10^{-3}$ coincidence per beta-ray, suggests that the beta-rays of the high energy group are followed by 0.31 Mev of "immediate" gamma-ray energy.

The conversion electrons of *$Te^{125}$ were absorbed in aluminum as shown in Fig. 3. The end point, shown by the arrow, occurs at 18 mg/cm², 120 kev, in exact agreement with the earlier measurement.[2] On the other hand, extrapolation of the curve as indicated by the broken line, gives an end point at 12.7 mg/cm² or 93 kev. Either value, together with the half-life, corresponds to a $2^5$-pole isomeric transition.

All measurements were completed within five days after the chemical separations. It should also be mentioned that a coincidence absorption experiment disclosed the presence of the 1.7-Mev gamma-ray of $Sb^{124}$ in the antimony fraction.

* Assisted by the joint program of the ONR and the AEC.
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[3] A. C. G. Mitchell, verbal communication. Spectrometric data of the Indiana group indicate the presence of soft spectra in $Sb^{125}$ in addition to the principal group at 0.704 Mev.

## On Radiative Corrections to Electron Scattering

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RADIATIVE corrections to the electromagnetic properties of the electron produce energy level displacements and modify electron scattering cross sections. Although the high accuracy of radiofrequency spectroscopy facilitates the measurement of energy level displacements, as in the Lamb-Retherford experiment[1] and the evidence on the anomalous magnetic moment of the electron,[2] nevertheless the correction to the cross section for scattering of an electron in a Coulomb field is not without interest, since it permits a comparison between theory and experiment in the relativistic region, as compared to the non-relativistic domain to which the energy level measurements pertain.

The radiative correction to the cross section for essentially elastic scattering of an electron by a Coulomb field has been computed with the form of quantum electrodynamics developed in several recent papers.[3] In addition to the emission and absorption of virtual quanta, we include the real emission of quanta with maximum energy $\Delta E$, which is small in comparison with $W=E-mc^2$, the initial kinetic energy of the electron. In other words, we treat only those inelastic events in which a small fraction of the original energy is radiated. The contribution of the remainder of the inelastic processes can be derived from the well-known bremsstrahlung cross section, and is not of principal interest. The result, expressed as a fractional decrease in the differential cross section for scattering through an angle $\vartheta$, is

$$\delta=2\alpha/\pi\left[\left(\log\frac{E}{\Delta E}-1\right)(K_0+K_1)+\tfrac{1}{2}K_0-K_1+\tfrac{1}{3}K_2-L+\tfrac{1}{2}\frac{(mc^2/E)^2}{1-\beta^2\sin^2\vartheta/2}K_0\right],\quad(1)$$

where

$$K_0=[\lambda/(1+\lambda^2)^{\frac{1}{2}}]\log[(1+\lambda^2)^{\frac{1}{2}}+\lambda],\quad \lambda=(p/mc)\sin(\vartheta/2),$$
$$K_1=[(1+\lambda^2)/\lambda^2]K_0-1,\quad(2)$$
$$K_2=[(1+\lambda^2)/\lambda^2]K_1-\tfrac{1}{3},$$

and

$$L=(\lambda^2+\tfrac{1}{2})\frac{(mc^2/E)^2}{\beta}\int_0^1\left[\frac{\log\frac{1}{2}(1-\beta x)}{1+\beta x}-\frac{\log\frac{1}{2}(1+\beta x)}{1-\beta x}\right]\frac{dv}{x}-\tfrac{1}{2}\frac{(mc^2/E)^2}{\beta}\left[\frac{\log\frac{1}{2}(1-\beta)}{1+\beta}-\frac{\log\frac{1}{2}(1+\beta)}{1-\beta}\right].\quad(3)$$

Here

$$x=[1-\sin^2(\vartheta/2)(1-v^2)]^{\frac{1}{2}}\quad(4)$$

and $\beta=pc/E$. Note that $\delta$ diverges logarithmically in the limit $\Delta E\rightarrow 0$. It is well known that this difficulty stems from the neglect of processes involving more than one low frequency quantum.[4] Actually the essentially elastic scattering cross section approaches zero as $\Delta E\rightarrow 0$; that is, it never happens that a scattering event is unaccompanied by the emission of quanta. This is described by replacing the radiative correction factor $1-\delta$ with $e^{-\delta}$, which has the proper limiting behavior. The further terms in the series expansion of $e^{-\delta}$ express the effects of higher order processes involving the multiple emission of soft quanta. However, for practical purposes such a refinement is unnecessary. The accuracy with which the energy of a particle can be measured is such that the limit $\Delta E\rightarrow 0$ cannot be realized, and $\delta$ will be small in comparison with unity under presently accessible circumstances.

In the non-relativistic limit, $\beta \ll 1$,

$$K_n = [1/(2n+1)]\beta^2 \sin^2(\vartheta/2),$$
$$L = [(4/3)(\log 2 - 1) - \tfrac{1}{5}]\beta^2 \sin^2(\vartheta/2), \quad (5)$$

and

$$\delta = (8\alpha/3\pi)\beta^2 \sin^2(\vartheta/2)[\log(mc^2/2\Delta E) + (19/30)], \quad (6)$$

which increases linearly with the kinetic energy of the particle. For a slowly moving particle, it is an elementary matter to include the additional scattering produced by the real emission of quanta with energies in the interval from $\Delta E$ to $W$. One thereby obtains the following fractional decrease in the differential cross section for scattering through an angle $\vartheta$, irrespective of the final energy:

$$\delta = (8\alpha/3\pi)\beta^2 \sin^2(\vartheta/2)[\log(mc^2/8W) + (19/30) + (\pi - \vartheta)\tan(\vartheta/2) + [\cos\vartheta/\cos^2(\vartheta/2)]\log\csc(\vartheta/2)]. \quad (7)$$

We may remark, parenthetically, that in the same non-relativistic approximation, the radiative correction to the energy of a particle moving in an external field with potential energy $V(r)$ is[5]

$$\begin{aligned}\delta E &= (\alpha/3\pi)[\log(mc^2/2\Delta w) + (31/120)](\hbar/mc)^2\langle\nabla^2 V\rangle + (\alpha/2\pi)(\hbar/2mc)\langle -i\beta\boldsymbol{\alpha}\cdot\nabla V\rangle \\ &= (\alpha/3\pi)(\hbar/mc)^2[(\log(mc^2/2\Delta W) + (19/30))\langle\nabla^2 V\rangle + \tfrac{3}{4}\langle\boldsymbol{\sigma}\cdot\mathbf{L}(1/r)(dV/dr)\rangle],\end{aligned} \quad (8)$$

where $\mathbf{L}$ is the orbital angular momentum operator in in units of $\hbar$, and $\Delta W$ is an average excitation energy of the system.[6] Applied to the relative displacement of the $2^2S_{\frac{1}{2}}$ and $2^2P_{\frac{1}{2}}$ levels of hydrogen, this formula yields 1051 mc/sec., to be compared with the experimental value[7] of $1062 \pm 5$ mc/sec.

The extreme relativistic limit of (1) is

$$\delta = (4\alpha/\pi)[(\log(E/\Delta E) - (13/12)) \times (\log(2E/mc^2)\sin(\vartheta/2) - \tfrac{1}{2}) + (17/72) + \phi(\vartheta)], \quad (9)$$

where

$$\phi(\vartheta) = \tfrac{1}{2}\sin(\vartheta/2)\int_{\cos(\vartheta/2)}^{1}\left[\frac{\log\frac{1}{2}(1+x)}{1-x} - \frac{\log\frac{1}{2}(1-x)}{1+x}\right] \times \frac{dx}{(x^2 - \cos^2(\vartheta/2))^{\frac{1}{2}}}. \quad (10)$$

The integral can be performed analytically for $\vartheta = \pi$, $\phi(\pi) = \pi^2/24$, but must be evaluated numerically for other angles. An approximation in excess, which has the correct asymptotic form at small angles, is provided by

$$\phi(\vartheta) \sim \frac{1 - \cos(\vartheta/2)}{[2\cos(\vartheta/2)(1 + \cos(\vartheta/2))]^{\frac{1}{2}}} \times \left[\log\frac{1}{2(1 - \cos(\vartheta/2))} + \frac{1 - \cos(\vartheta/2)}{2} + 1\right]. \quad (11)$$

This approximation is reasonably accurate even at $\vartheta = \frac{1}{2}\pi$, where the value yielded by (11) exceeds by only 8.6 percent the following result of a numerical calculation, $\phi(\pi/2) = 0.292$.

The asymptotic formula (9) is quite accurate for even moderate energies. Thus, with $\vartheta = \frac{1}{2}\pi$, $\Delta E = 10$ kev, and $W = 3.1$ Mev, which corresponds to $(E/mc^2)\sin(\vartheta/2) = 5$, the value of $\delta$ computed from (9) differs from the correct value, $\delta = 8.6\ 10^{-2}$, by only a few tenths of a percent. It is evident from this numerical result that radiative corrections to scattering cross sections can be quite appreciable. For the particular conditions chosen, $\Delta E$ can be materially increased (but still subject to $\Delta E \ll W$), without seriously impairing $\delta$. Thus, with $\Delta E = 40$ kev, $\delta = 6.3\ 10^{-2}$, while $\Delta E = 80$ kev yields $\delta = 5.1\ 10^{-2}$. As to the energy dependence of $\delta$, we remark that with a given accuracy in the determination of the energy, say $\Delta E/E = 0.04/3.6 = 1.1\ 10^{-2}$, an increase in the total energy by a factor of four produces an addition of $4.4\ 10^{-2}$ to $\delta$. Thus, for a kinetic energy of 14 Mev, $\delta = 11\ 10^{-2}$.

The variation of $\delta$ with angle, at moderate energies, cannot be studied with the asymptotic formula (9) alone, for at small angles the condition $(p/mc)^2\sin^2(\vartheta/2) \gg 1$, which underlies this formula, will not be maintained. It is evident from (1) that $\delta$ is proportional to $\sin^2(\vartheta/2)$ at angles such that $(p/mc)^2\sin^2\frac{1}{2}\theta \ll 1$. However, for $W = 3.1$ Mev, $\Delta E = 40$ kev, and $\vartheta = \pi/4$, which corresponds to $(p/mc)\sin(\vartheta/2) = 2.7$, the correct value of $\delta$, $4.2\ 10^{-2}$, exceeds that deduced from (9) by only 2 percent. For the same choice of $W$ and $\Delta E$, the value of $\delta$ associated with $\vartheta = 3\pi/4$ is $\delta = 7.2\ 10^{-2}$.

The verification of these predictions would provide valuable conformation for the relativistic aspects of the radiative corrections to the electromagnetic properties of the electron.

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## Measurements of Behavior and Mobility of Polyatomic Ions

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A TECHNIQUE capable of tracing continuously in time the motion of a body of ions in an electric field is desirable for the measurement of mobility and could indicate changes of mobility occurring due to changes in the nature of the moving charges (due to transfer of charge in ion-molecule collision, dissociation to smaller, or association to larger ionic masses, etc.).

The technique used by H. G. Stever[1] in obtaining a "recovery curve" for Geiger-Mueller tube discharges has been extended to give information concerning the actual progress of the sheath $r = r(t)$ by simultaneously performing three experiments: