pressure, and therefore have no bearing on the theory valid for a gas at low pressure. The results of these experiments indeed seem to indicate that at high pressure the curve showing P_0 as a function of q is markedly different from the type of curve expressed by Eq. (14), especially at the foot of the curve. This is not surprising since the processes of spark formation in the two cases—high and low pressure—are quite different.

Electron Velocity Distribution Function in High Frequency Alternating Fields Including Electronic Interactions

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The Boltzmann transport equation has been solved for the electronic velocity distribution function in a high frequency gas discharge. The distribution is examined as a function of the electron density. The conductivity is computed for two electron-molecule cross sections, one, in which the cross section is inversely proportional to the electron velocity and two, in which the cross section is independent of electron energy. The results show the extension of Margenau's distribution to high densities.

THIS paper is an extension of the work of Margenau[1] in which the Boltzmann Equation

$$(e/m)\mathbf{E}\cdot\nabla f = (\partial_e f/\partial t) \tag{1}$$

was solved for high frequency fields $\mathbf{E}_z = E \cos\omega t$ for the case of low current densities. In it we will apply the results of a former paper[2] (hereinafter referred to as I) in which the Coulomb interaction term $(\partial_e f/\partial t)_1$ of the Boltzmann Equation was derived. Following the notation in I, the distribution function $f(\mathbf{v})$, normalized to the electron density n, will be expanded in the form

$$f(\mathbf{v}) = f_0(v) + v_z(f_1(v) \cos\omega t + g_1(v) \sin\omega t). \tag{2}$$

The approximate collision term $(\partial_e f_0/\partial t)_1$ is given by

$$\left(\frac{\partial_e f_0}{\partial t}\right)_1 = \frac{10}{s^{\frac{1}{2}}} \frac{ds^{\frac{3}{2}}}{ds}(Af_0' + Bf_0), \tag{3}$$

in which $s = v^2$, $A = \mathcal{L}'\int_0^\infty f_0 ds$, $B = \mathcal{L}'f_0(0)$, $\mathcal{L}' = 16\pi^2/15 \cdot e^4/m^2 \ln\bar{\varepsilon}\rho/e^2$, $\rho = (\bar{\varepsilon}_1/12\pi ne^2)^{\frac{1}{2}}$, e and

m are the charge and mass of the electron; the average energy of electrons relative to electrons is $\bar{\varepsilon}$, that relative to ions is $\bar{\varepsilon}_1$.

We use the same electron-molecule and electron-ion collision term as in I, namely

$$\left(\frac{\partial_e f_0}{\partial t}\right)_M = \frac{\delta}{\beta s^{\frac{1}{2}}} \frac{d}{ds} \frac{s^2}{\lambda}(f_0' + \beta f_0), \tag{4}$$

where $\beta = m/2kT$, $\delta = 2m/M$, T is the gas temperature, and M is the molecular mass, so that our total $\partial_e f_0/\partial t$ for electrons, molecules, and ions is the sum of (3) and (4).

We will make the same simplifications as in I in which the Coulomb interaction was neglected in the momentum balance equation. We can then use Eq. (23) in the paper by Margenau and Hartman,[3]

$$\frac{\partial_e f_0}{\partial t} = \frac{\gamma}{6v^2} \frac{d}{dv}(v^2 f_1), \quad \frac{v}{\lambda}f_1 + \omega g_1 = -\frac{\gamma}{v}f_0'(v),$$

$$g_1 = (\omega\lambda/v)f_1. \tag{5}$$

[1] H. Margenau, Phys. Rev. **69**, 508 (1946).
[2] J. H. Cahn, Phys. Rev. **75**, 293 (1949).
[3] H. Margenau and L. M. Hartman, Phys. Rev. **73**, 309 (1948).

After eliminating g_1 and f_1 in (5), we have finally

$$\frac{\partial_e f_0}{\partial t} = -\frac{\gamma}{3s^{\frac{3}{2}}}\frac{d}{ds}\frac{2\gamma\lambda s^2}{s+(\omega\lambda)^2}f_0' = \frac{10}{s^{\frac{1}{2}}}\frac{ds^{\frac{3}{2}}}{ds}(Af_0'+Bf_0) + \frac{\delta}{\beta s^{\frac{1}{2}}}\frac{d}{ds}\frac{s^2}{\lambda}(f_0'+\beta f_0).$$ (6)

As pointed out in I, (6) must be integrable because of particle conservation so that we obtain on integrating (6), a simple first order equation whose integral in turn is

$$f_0 = N_0 \exp\left\{-\int_0^s \frac{s^{\frac{1}{2}}+10\lambda B/\delta}{[1+\frac{2}{3}(\beta/\delta)(\gamma\lambda)^2/(s+(\omega\lambda)^2)]s^{\frac{1}{2}}+10\lambda\beta A/\delta}\beta ds\right\},$$ (7)

where N_0 is the normalization constant. As a check on (7) if we let ω be zero, (7) reduces to

$$f_0 = N_0 \exp\left\{-\int_0^v \frac{(10B\lambda/\delta)\beta v+\beta v^2}{\frac{2}{3}[(\beta\gamma\lambda)^2/\delta]+(10\beta A\lambda/\delta)\beta v+\beta v^2}(2\beta v dv)\right\},$$ (8)

which agrees with (19) in I if γ^2 is replaced by $2\gamma_{dc}^2$ because the R.M.S. value of γ is comparable to γ_{dc}.

A further check on (7) is found in Eq. (15) of Margenau[4] which reads

$$f_0 = N_0 \exp\left\{-\int_0^s \frac{ds}{1/\beta+[\frac{2}{3}(\gamma\lambda)^2/\delta/s+(\omega\lambda)^2]}\right\}$$ (9)

in our notation. Equation (7) is seen to reduce to (9) when B and A, the interaction terms, are negligible.

As was done in I, it is instructive to examine (7) for two types of electron-molecule mean free paths in which (a) $\tau=\lambda/v=$constant, and (b) λ is independent of the energy.

For case (a) in which the time τ between collisions is a constant, (7) reduces to

$$f_0 = N_0 \exp\left\{-\frac{1+10B\tau/\delta}{1+[\frac{2}{3}\beta(\gamma\tau)^2/\delta/1+(\omega\tau)^2]+10\beta A\tau/\delta}\cdot\frac{mv^2}{2kT}\right\}.$$ (10)

Computing A and B from (10) we obtain

$$\beta A = B\left[1+\frac{\frac{2}{3}\beta(\gamma\tau)^2/\delta}{1+(\omega\tau)^2}\right], \quad B=N_0L',$$

which when replaced in (10) give us the Maxwellian distribution

$$f_0 = N_0 \exp[-mv^2/2k(T+T')],$$ (11)

where

$$\frac{3}{2}kT' = \frac{M(\gamma\tau)^2}{4(1+\omega^2\tau^2)},$$ (12)

and

$$N_0 = [m/2\pi k(T+T')]^{\frac{3}{2}}n.$$

Distribution (11) is found to be independent of the electron density and so should be identified with (9) for the same assumption of mean free path. This is easily seen to be the case. It should

be noted that at frequencies of the order of 10^{10} c.p.s., the excess electron temperature T' is independent of pressure for a large range of pressures, and is given by

$$\frac{3}{2}kT' = M/4(\gamma/\omega)^2.$$ (12')

We next compute the drift velocity $\bar{v}_z$ from which the current density is computed. We have,
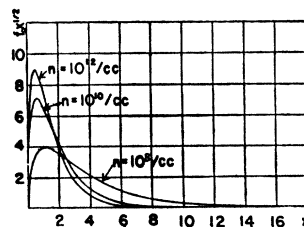

FIG. 1. Comparison of the electron distribution f_0 for three critical electron densities. The gas is assumed to have a pressure of 1 mm, atomic number of 20, temperature of 300°K. The electric field amplitude is assumed to be 1 volt/cm and the frequency, 10^{10} c.p.s. At electron densities of 10^8/cc or less, the excess electron temperature $T'=410°K$.

[4] See reference 1, p. 509.

TABLE I. Mean energies and velocities of electrons for three densities.

n/cc	$\bar{v}$	$\bar{\epsilon}$
10^8	$1.825 \ (10)^7$ cm/sec.	0.145 ev
10^{10}	1.347	0.055 ev
10^{12}	1.088	0.039 ev

from (2),

$$n\bar{v}_z = \frac{4\pi}{3} \int_0^\infty (f_1 \cos\omega t + g_1 \sin\omega t)v^4 dv. \quad (13)$$

Making use of (5), (13) becomes

$$n\bar{v}_z = -\frac{4\pi}{3} \int_0^\infty \frac{\gamma \tau v^3}{1+(\omega\tau)^2}$$

$$\times f_0'(\cos\omega t + \omega\tau \sin\omega t)dv. \quad (14)$$

When (11) is substituted in (14), $\bar{v}_z$ is found to be given by

$$\bar{v}_z = \gamma\tau(\cos\omega t + \omega\tau \sin\omega t)/1+(\omega\tau)^2.$$

The current density j is then given by

$$j = ne\bar{v}_z, \quad (15)$$

and the complex conductivity in turn by σ in the relation

$$j = \sigma\epsilon = \sigma E e^{i\omega t}. \quad (16)$$

We then obtain for the complex conductivity,

$$\sigma = \sigma_r - i\sigma_i$$

$$= ne^2\tau(1+i\omega\tau)/m(1+(\omega\tau)^2). \quad (17)$$

When $\omega \sim 10^{11}$, $\tau \sim 10^{-9}$, $\omega\tau \gg 1$ so that

$$\sigma = ine^2/m\omega.$$

This is the result obtained from ordinary dispersion theory when the damping is disregarded.[5] It should be pointed out that since (11) and (9) are identical for this choice of cross section, and hence independent of electron density, that (12) and (17) are special case of Margenau's results.[1]

In case (b) we consider the distribution (7) under the condition that the mean free path λ be independent of the electronic energy. Because the integrations must be dealt with numerically, and the frequency range for which this work is of interest is of the order of 10,000 megacycles, we choose pressures for which $\omega\lambda \gg v$ over most of the range of v and approximate (7) by

$$f_0 = N_0 \exp\left\{ -\int_0^x \frac{x^{\frac{1}{2}} + (10B\lambda/\delta)\beta^{\frac{1}{2}}}{\left[1 + \frac{2}{3}\frac{\beta}{\delta}\left(\frac{\gamma}{\omega}\right)^2\right]x^{\frac{1}{2}} + \frac{10\beta A\lambda}{\delta}\beta^{\frac{1}{2}}} dx \right\}, \quad (18)$$

where $x = mv^2/2kT$.

If we use the following notation,

$$b_1 = (10B\lambda/\delta)\beta^{\frac{1}{2}},$$

$$b_2 = 1 + \frac{2}{3}(\beta/\delta)(\gamma/\omega)^2,$$

$$b_3 = (10\beta A\lambda/\delta)\beta^{\frac{1}{2}},$$

we can write (18) as

$$f_0 = N_0 \exp\left\{ -\int_0^x \frac{x^{\frac{1}{2}} + b_1}{b_2 x^{\frac{1}{2}} + b_3} dx \right\}. \quad (19)$$

We find that the integrand of the indefinite integral may be written in a more meaningful fashion, i.e.,

$$\frac{x^{\frac{1}{2}} + b_1}{b_2 x^{\frac{1}{2}} + b_3} = \frac{B}{\beta A} + \frac{[1 - (B/\beta A)b_2]x^{\frac{1}{2}}}{b_3 + b_2 x^{\frac{1}{2}}}. \quad (20)$$

The significance of (20) is seen from the following. When the interaction terms b_1 and b_3 are neglected in (19), we have

$$f_0 = N_0 e^{-x/b_2},$$

so that

$$N_0 = n((\beta/\pi)b_2)^{\frac{1}{2}},$$

and

$$\beta A = b_2 B.$$

Thus, at low densities, the variable term in (20) vanishes with $1 - (B/\beta A)b_2$. The low density approximation to (17) is therefore the Maxwellian function

$$f_0 = N_0 \exp[-mv^2/2k(T+T')], \quad (21)$$

where the excess electron temperature $\frac{3}{2}kT' = M/4(\gamma/\omega)^2$, agreeing with the result in (12') for quite a different electron-molecule cross

[5] J. A. Stratton, Electromagnetic Theory (McGraw-Hill Book Company, Inc., New York, 1941), First Edition, p. 327.

section. This result is of course in accord with Margenau's Eqs. (23) and (24).[1]

At high densities, (19) reduces to the Maxwellian form

$$f_0 = N_0 \exp(-mv^2/2kT). \qquad (22)$$

The transition from low to high densities is shown in Fig. 1. The mean energies and velocities at the three densities exhibited in Fig. 1 are given in Table I.

We next compute the effect of electron density on the conductivity when λ is constant. After eliminating f_1 and g_1 from (13) using (5), and making the approximation $\omega\lambda \gg v$, we find that the drift velocity $\bar{v}_z$ is given by

$$\bar{v}_z = \gamma/\omega[(4\bar{v}/3\omega\lambda)\cos\omega t + \sin\omega t],$$

showing that the in phase current is very much smaller than the out of phase current at these frequencies and pressures. The complex conductivity σ defined in (16) becomes

$$\sigma = (ne^2/m\omega)[(4\bar{v}/3\omega\lambda) + i]$$
$$\doteq i(ne^2/m\omega).$$

This is identical with the result obtained for the constant τ case. The effect of varying density is here small, but of course can be readily estimated from the calculated values of $\bar{v}$ given in Table I.

In conclusion, it should be noted that whereas in the d.c. case, we noted a lowering of the entropy for increasing density, in the high frequency case, the drift velocity (in phase component) drops off with increasing density (see Table I) so that the same statement about entropy cannot be made. Since the out of phase component remains constant while the average velocity reduces for increasing density, the entropy is reduced to this extent.

Ferromagnetism at Very High Frequencies
II. Method of Measurement and Processes of Magnetization*, **

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A method for measuring the complex permeability, μ, of a ferromagnetic metal is described. The determination of both components of μ is accomplished by a simultaneous measurement of the changes in attenuation and phase velocity introduced into a conducting system by the ferromagnetism of one of its walls. An experimental technique involving pulsed magnetic fields is used. Values of μ for samples of magnetic iron at 200 and 975 Mc as a function of a polarizing magnetic field (which is parallel to the high frequency field) are compared to the static incremental permeabilities measured on the same sample. The interpretation of the experimental results indicates several characteristics of the magnetization in iron: the upper limit of domain dimensions in polarizing fields less than 500 oersteds is 10^{-4} cm; domains have different degrees of stability in the applied field; "weak-field domain" wall displacements are practically eliminated at the frequencies used; spin rotation and "strong-field domain" wall displacements are only slightly damped and contribute to the magnetization in weak as well as strong fields. Studies of hysteresis phenomena are discussed, and results on two types of permalloy presented.

I. INTRODUCTION

NUMEROUS investigations[1] have shown that two basic mechanisms are responsible for the changes in magnetization of a ferromagnetic substance placed in an external magnetic field. These mechanisms involve two different effects of the field on the configuration of the magnetically saturated regions, called domains,

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[1] R. Becker and W. Doering, *Ferromagnetismus* (Verlagsbuchhandlung Julius Springer, Berlin, 1939).