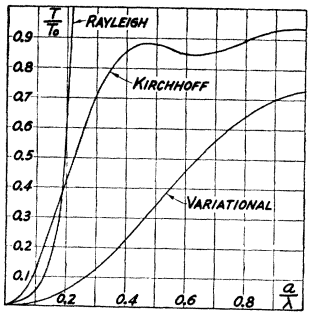

FIG. 3. Transmission cross section of circular aperture of radius a.

where $E_i(\lambda)$ and $E_\sigma(\lambda)$ are the "vector" transforms of the transverse components (parallel to screen) of the incident (at $z=0$) and aperture electric fields, respectively, in the sense of reference 4, the angular brackets with subscript Av denote the complex conjugate, S_λ denotes integration over the entire transform (λ) spectrum, and $Y(\lambda)$ represents the admittance for the eigenmode at a point λ in this spectrum and is defined by

$$Y^{(1)}(\lambda)=[1-(\lambda/k)^2]^{-\frac{1}{2}}, \tag{3a}$$

$$Y^{(2)}(\lambda)=[1-(\lambda/k)^2]^{\frac{1}{2}}, \tag{3b}$$

where the superscripts 1 and 2 denote the TM and TE modes, respectively. k is the wave number (2π/wavelength), and the paths of integration in the λ-spectrum are indented under and over the branch points $-k$ and $+k$, respectively. Equation (2) is variational in the sense that both its real and imaginary parts are stationary with respect to variations of the aperture field about the true solution to the aforementioned integral equation. In the case where only a single class of modes (TM or TE) is excited both the real (G_σ) and imaginary (B_σ) parts of Y_σ are positive definite forms and are therefore absolute minima, while in the more general case only G_σ is an absolute minimum. It is of further interest to remark that *if the approximation $E_\sigma=E_i$ is made in Eq. (2)*, the ratio of the transmission coefficient predicted by Kirchhoff optics to that predicted by geometric optics is given by G_σ/Y_i.

As examples, the problems of diffraction through a slit (magnetic vector parallel to slit) and a circular aperture were considered. The incident field was used to obtain the first variational approximations for the cases of normal and 45° oblique incidence of a plane wave on the slit, and the results are compared with the Kirchhoff, Rayleigh,[5] and exact results[6] in Figs. 1 and 2. In the case of normal incidence of a plane wave on the circular aperture of radius a the field in the variational approximation was adjusted so as to reduce to the known, exact results in the limits of both large wave-length[7] and small wave-length (geometrical optics). The results are plotted in Fig. 3.

An alternative approach, more suitable for the case where the obstacle (τ) is of finite area, is to formulate the problem in terms of the current in the obstacle. The analysis is quite analogous to the foregoing. In the case of a complementary interchange of σ and τ and $\mathbf{E}_i$ and $-\mathbf{H}_i$ this formulation is related to the foregoing by Babinet's principle.[8] Thus, Figs. 1 and 2 give the reflection cross section for a plane wave with the electric vector parallel to a flat strip, and Fig. 3 gives the reflection cross section for a plane wave normally incident on a circular disk.

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[2] H. Levine has informed the writer (personal correspondence) that he intends to carry out a similar investigation for the vector problem.
[3] John W. Miles, J. Acous. Soc. Am. (to be published).
[4] John W. Miles, Phys. Rev. **74**, 1531 (1948).
[5] Lord Rayleigh, Phil. Mag. **43**, 259–272 (1897).
[6] P. M. Morse and P. J. Rubenstein, Phys. Rev. **54**, 859–898 (1938).
[7] Lord Rayleigh, Phil. Mag. **44**, 28–52 (1897); H. Bethe, Phys. Rev. **66**, 163–182 (1944).
[8] E. T. Copson, Proc. Roy. Soc. London **A186**, 100–118 (1946).

Hf^{175}, a New Radioactive Isotope of Hafnium

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A STUDY has been made of hafnium activities produced by bomardment of lutecium with 19-Mev deuterons and 10-Mev protons from the 60-inch Crocker Laboratory cyclotron.

The lutecium used for bombardment was Johnson and Matthey's "Specpure" oxide, prepared by Professor Marsh of Oxford University and purified in this laboratory by ion-exchange resin column separation. Spectroscopic analysis[1] showed absence of yttrium and rare earth contamination. The oxide, with sodium silicate as a binder, was bombarded on a platinum interceptor target placed in the cyclotron beam. After bombardment, the target material was dissolved in boiling, concentrated nitric acid and the insoluble silica residue discarded. After addition of a few milligrams of hafnium carrier, the hot solution was adjusted to $3N$ in nitric acid and the lutecium precipitated by addition of hydrofluoric acid. After centrifuging, the filtrate containing hafnium was scavenged repeatedly by addition of lanthanum carrier solution in order to remove any rare earth contamination. Hold-back carriers for the various radioactivities likely to be formed from the target materials, copper, sodium silicate, etc., were added and the solution adjusted to $3N$ in both nitric and hydrofluoric acids. Excess barium nitrate solution was then added to precipitate barium hafnium fluoride. The washed precipitate was dissolved in hot $8N$ nitric acid saturated with boric acid, and the hafnium hydroxide recovered by addition of ammonium hydroxide. The hydroxide was dissolved in nitric acid, and barium hafnium fluoride reprecipitated. The precipitation of the barium metal fluoride from strongly acid solutions is specific for hafnium and zirconium.

The lutecium and hafnium were finally converted to the oxides for weighing and estimation of chemical yields.

The chemically separated hafnium has been found to contain a single radioactivity, emitting electrons and γ-radiations, which decays with a half-life of 70 ± 2 days. No evidence of zirconium contamination, or shorter-lived

activities, was seen. Standard 3-mg/cm² mica window counters filled with argon-alcohol mixture were used in the measurements. An aluminum absorption curve of an "infinitely thin" hafnium sample, mounted on thin mica, is shown in Fig. 1; the soft quantum radiation background was determined after removal of electrons by a 100-mg/cm² beryllium absorber. The lead absorption curve (Fig. 2) was measured using an unshielded counter. The radiations consist of electrons range 82±2 mg/cm² (300 kev); electromagnetic radiation of half-thicknesses 14.5±0.3-mg/cm² aluminum (8.2 kev), 100±5-mg/cm² lead (55 kev), 2.2±0.1-g/cm² lead (350 kev), and 13.3±0.2-g/cm² lead (1.5 Mev). The energies of the two soft components agree well with those to be expected for lutecium L and K x-radiation.

From the measurements, the intensity ratios of the various radiations were estimated. Corrections were made for absorption of electrons in the counter window, air gap, etc., and counting efficiencies for the 8.2-kev, 55-kev, 350-kev, and 1.5-Mev γ-rays were taken as 5 percent, 0.5 percent, 0.5 percent, and 1.5 percent, respectively. A fluorescence yield of 0.5 for L and 0.8 for K x-radiation was assumed. The fully corrected ratios obtained are:

$$e^-: L \text{ x-rays}: K \text{ x-rays}: 350\text{-kev}\gamma: 1.5\text{-Mev}\gamma = 0.1:0.1:1:0.2:0.05.$$

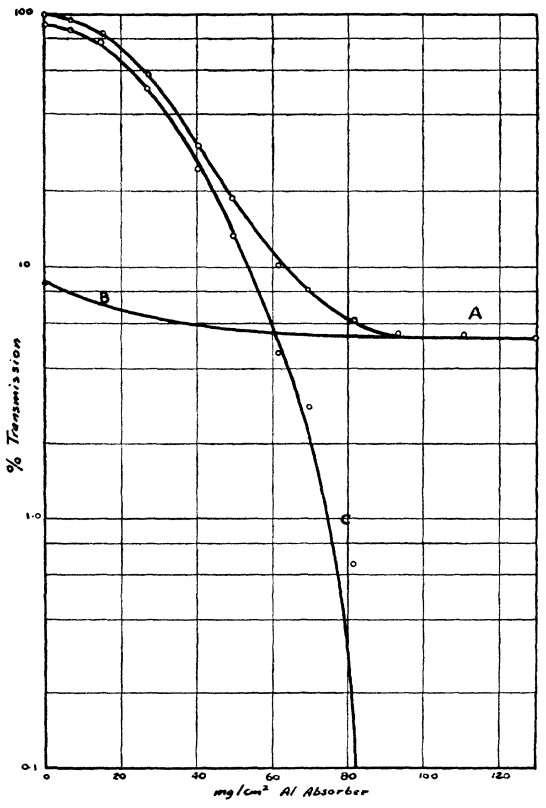

FIG. 1. Aluminum absorption of Hf^{175} radiations: Curve A is the gross absorption curve; curve B, L x-rays, K x-rays, and γ-radiation; curve C, electrons.

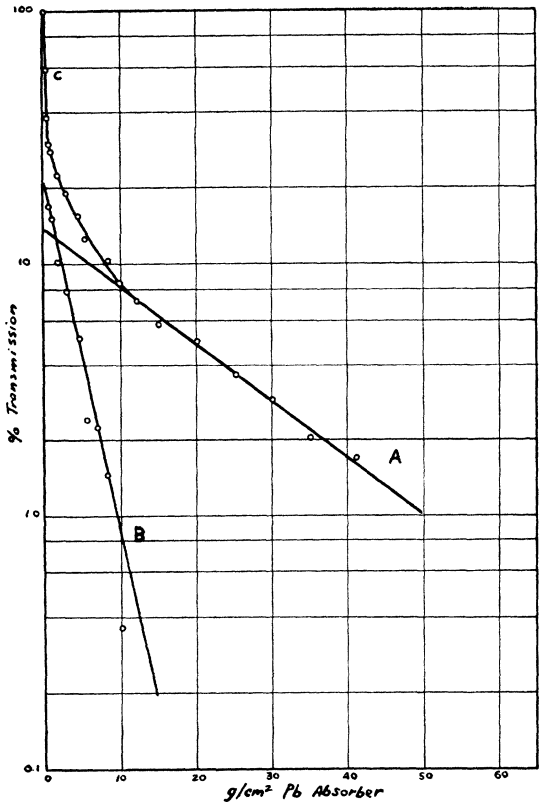

FIG. 2. Lead absorption of Hf^{175} electromagnetic radiations: Curve A represents the hard γ-radiation; Curve B, the resolved unconverted part of the 350-kev γ-ray; Curve C shows the presence of K x-rays.

While it is realized that such ratios may be in error by factors of two or more, more accurate information being difficult, if not impossible, to obtain by the simple counting techniques used, it seems justifiable to attempt to estimate the radiations arising from one disintegration, in order to allow calculation of cross sections of reactions producing the isotope. The isotope thus probably decays by electron capture, mainly to the ground state, and also to two or more excited or metastable levels. The transition from the lower excited state is accompanied by conversion, the coefficient for which is 0.4. The measured energies of the conversion electrons agree with K shell conversion. It is assumed that about 0.1 of the observed K x-rays arise from conversion, and, accordingly, one disintegration by orbital electron capture is represented by 0.9 K quanta corrected for counting efficiency, etc. Using this value, the measured chemical yields, and bombardment data from the cyclotron instruments, the cross sections for the formation of the isotope were calculated. For 10-Mev protons and 19-Mev deuterons on natural lutecium, the cross sections are, respectively, 3×10^{-2} barn and 5×10^{-2} barn. These values are reasonable for the p,n and $d,2n$ reactions, and agree with other measurements for similar reactions in this region.

The new isotope is allocated to mass 175.

We are greatly indebted to Dr. J. G. Hamilton, Mr. T. Putnam, Mr. B. Rossi, and the crew of the 60-inch

Crocker Laboratory cyclotron for their cooperation in making bombardments; we also wish to thank Professors G. T. Seaborg, I. Perlman, and B. B. Cunningham for their continued interest and advice.

This work was carried out under the auspices of the United States Atomic Energy Commision.

[1] We are indebted to Mr. J. G. Conway of this laboratory for spectroscopic analyses.

The Penetration of μ-Meson Decay Electrons and Their Bremsstrahlung Radiation

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MEASUREMENTS of the penetration of the charged particles from the 2.2-μsec. meson decay using the arrangement of counters and delayed coincidence circuits previously described[1] have been extended with absorbers of lead and aluminum, in addition to carbon. The results confirm our previous conclusion that at least a substantial number of particles have an energy >25 Mev. Although detailed analysis of the absorption curves does not seem justified, a soft and a hard component are evident. In lead, for example, the "soft" component is essentially absorbed by about 20 g/cm², and the "hard" one is easily detected after 38 g/cm², where its intensity (~1 count/day) is about ten times the casual rate.

While our results will be published in full later, we report here an investigation of the "hard component." First, the arrangement of Fig. 1 was used to determine whether or not the "hard component" is composed of penetrating charged particles. Mesons stopped in 8.5 g/cm² of graphite are detected by the anticoincidence (*AB–C*). If a decay particle traverses trays *C*, *D*, or *E*, between 1 and 6 μsec. after a meson stops, a delayed coincidence is registered which we designate as $(C)_{del}$, $(D)_{del}$, or $(E)_{del}$. Ordinary coincidence mixers detect the events* $(CD)_{del}$, $(CE)_{del}$, while delayed coincidences of greater complexity are observed by recording all delayed pulses with a ten-pen recorder.

By this technique we have observed that in a run of 103 hr. with 16.4 g/cm² of lead between both *C* and *D*, and *D* and *E*, there were 9 events of the type $(CDE–B)_{del}$, and 10 of the type $(CE–D–B)_{del}$. We expect in the same time about 8 casual $(CDE–B)_{del}$ events, but only about 0.5 casual $(CE–D–B)_{del}$ events. The experiment when repeated using a similar arrangement with 13.6 g/cm² of graphite between *C* and *D* (but lead still between *D* and *E*), gave similar results.** This demonstrates the production of a neutral radiation in the lead or carbon absorbers and indicates that most or all of the "hard component" consists, in fact, of photons from bremsstrahlung by the particles of the "soft component."

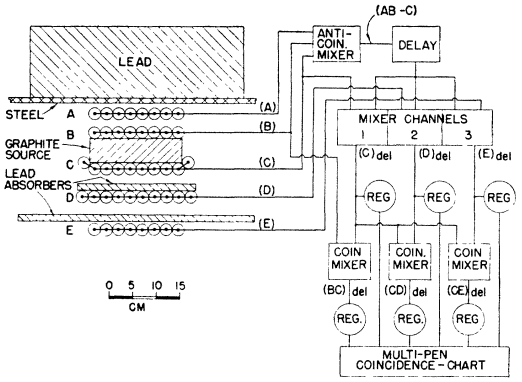

FIG. 1. Experimental arrangement for the detection of bremsstrahlung. Length of counters = 40 cm.

To test further this interpretation, we have compared the intensities of production of the "hard component" in elements of different *Z*, *viz.* carbon and lead. The counter arrangement is shown in Fig. 2, and the circuits are as in Fig. 1. A constant thickness of lead (8.2 g/cm²) is kept above tray *E*, while an 8.2-g/cm² lead absorber and an 11.0-g/cm² graphite absorber are alternated in the two arrangements: (I), *C*–graphite–*D*–lead, and (II), *C*–lead–*D*–graphite, as shown. The rates $(CE–D–B)_{del}$, taken as a measure of the intensity of the "hard component," are given in Table I. The ratio (2.8±0.9) of the number of $(CE–D–B)_{del}$ counts in Case II to that in Case I is consistent with the ratio 2 to 3 to be expected*** if the effect is due to radiation from electrons. In Table I we have also given the $(CD–B)_{del}$ rates, which show that the "soft component" is, in fact, more strongly absorbed by 8.2-g/cm² lead than by 11.0-g/cm² carbon.

Our conclusion that the "hard component" is bremsstrahlung radiation produced by the charged particles of the "soft component" has the following consequences:

(1) The charged particles are indeed electrons, for if they were heavier (2 m_e or more) it would be difficult to account quantitatively for the absolute intensity of the bremsstrahlung radiation, and for the ratio of the intensities of such radiation produced in lead and carbon.

(2) Comparison of measured with calculated intensities of bremsstrahlung indicates that the *average* electron energy is >25 Mev;

TABLE I. Results obtained with the two arrangements of Fig. 2 (corrected for casual events).

Absorber between trays *C* and *D*	Absorber between trays *D* and *E*	Hours of observation	$(CE–D–B)_{del}$/hr.	$(CDE–B)_{del}$/hr.	$(CD–B)_{del}$/hr.
None	None	47	0.11 ±0.05	1.8 ±0.2	5.7 ±0.4
I.—Carbon (11.0 g/cm²)	Lead (8.2+8.2 g/cm²)	392	0.046±0.013	0.02±0.02	1.50±0.07
II.—Lead (8.2 g/cm²)	Carbon (11.0 g/cm²) +lead (8.2 g/cm²)	392	0.13±0.02	0.01±0.02	1.02±0.06