of 4.5 Mc/sec., the field then being about 1000 gauss. As the modulating field was 10 gauss, the accuracy of the thallium measurements was not duplicated. The result obtained for the ratio of resonant frequencies was

$$\nu_{F^{19}}/\nu_{H^1}=0.9407\pm 0.0003.$$

A subsequent series of measurements was made at a higher frequency, in the neighborhood of 19 Mc/sec. One sample coil was used. The field was kept constant and the frequency varied so that first one resonance was obtained and then the other. Slow field drifts, not exceeding one gauss, which sometimes occurred during the interval between measurements, were corrected. The resonances were displayed on the oscilloscope and located by reducing the modulating field until it was about 0.015 percent of the applied field, and keeping them centered on the trace. Four independent determinations were made, the results being given in Table I. A probable error of 0.01 percent is considered to include possible errors. The result is then

$$\nu_{F^{19}}/\nu_{H^1}=0.94077\pm 0.0001.$$

Taking into account the diamagnetic correction gives for the moment, in nuclear magnetons,

$$\mu(F^{19})=2.626\pm 0.001,$$

most of the error again arising from that in the proton moment. The value is in close agreement with the molecular-beam value.

ACKNOWLEDGMENT

The author is indebted to Professor F. Bitter for guiding the course of this research.

PHYSICAL REVIEW VOLUME 75, NUMBER 4 FEBRUARY 15, 1949

Magnetic Moments of Sunspots*

JAMES W. BROXON
Department of Physics, University of Colorado, Boulder, Colorado
(Received September 30, 1948)

Simple approximate representations of the magnetic fields of sunspots by dipoles or ring currents are considered, and expressions deduced for the magnetic moment of a sunspot in terms of its area and its maximum central magnetic field strength. These are extended to include sunspot groups, and a function (SS-MM) is defined as the arithmetical sum of the equivalent moments of all visible groups. Statistical treatment by Chree's method of superposed epochs provides some indication of a 27-day interval between recurrences of fluctuations in SS-MM.

IN an earlier paper[1] the writer provided an estimate of the magnetic moment of a sunspot. This was incidental to the discussion of the statistically determined recurrence of fluctuations in cosmic-ray intensity at 27- or 28-day intervals, and their relations to areas of sunspots and other variables. Because a new function has been devised which is based upon the earlier estimate of sunspot magnetic moment and which appears to be related to variations in cosmic-ray intensity, it is deemed worth while to provide further details regarding the manner in which that estimate was made.

How best to approach the representation of the magnetic properties of sunspots is not obvious. So far as the writer is aware, the only magnetic data regularly available in the literature are the values of H for individual sunspot groups determined at the Mt. Wilson Observatory by observation of the separation of the components of the Zeeman triplet λ6173.553 (Fe) in the second-order spectrum of their 75-foot spectrograph and published in the *Publications of the Astronomical Society of the Pacific.* Only one value of H is given for each group. This is the

* Presented at the meeting of the American Physical Society at Chicago, November 26–27, 1948.
[1] J. W. Broxon, Phys. Rev. **62**, 508 (1942).

maximum value of H ever observed at any point in that particular group while it remains visible. The unit employed is 100 gauss (or oersteds) which appears to serve as a practical limit of resolution since only integral values are listed. Because spot groups are likely to consist of anything from a single spot to several spots, sometimes with a single and sometimes with dual polarity and because the sizes and shapes as well as numbers and distributions of the spots in a group and their associated fields all vary with time, it appears to be impossible (with the limited data available) to devise any simple function to represent entirely satisfactorily the magnetic properties of a group. The complexity of the situation needs to be borne in mind when contemplating the adequacy of any function designed to represent even crudely the magnetic properties of a sunspot group.

In the case of a single sunspot of single polarity which may be regarded as approximately circular, something may be deduced regarding its magnetic state in terms of its size and the maximum magnetic field associated with it. On a statistical basis certain general statements may be made regarding the distribution of the field over the area of the sunspot. According to Chapman,[2] Nicholson has found the inclination of the magnetic field from the axis of a sunspot of radius a to vary with the distance r from its center in approximate accordance with the relation

$$\theta=(\pi/2)(r/a).$$

As was correctly surmised by Chapman, Dr. Seth B. Nicholson of Mt. Wilson observatory also informed the writer that H, the intensity of the field, may be regarded as varying over the area of the sunspot according to the relation

$$H=H_c(1-r^2/a^2),$$

where H_c is the intensity at the center.

With such a specification of the distribution of field over the area of a sunspot and with H_c and a given, one may, for instance, compute the total flux leaving or entering it and from this obtain the strength of the magnetic pole, assuming that the sunspot represents one pole of a pair. Chapman[2] has done this and in the case of the great bipolar group M.W. 6725, multiplied the pole strength computed for one of the two principal spots of the group by the distance between them to obtain the dipole moment of the pair, $5\times10^{16}\Gamma$ km^3 or 5×10^{31} e.m.u. He provided a close approximation to the appropriate distribution of H over the surfaces of the spots by imagining this to be due to a solenoidal distribution of electric current in a curved solenoid or vortex imbedded in the sun with its ends coinciding with the two spots. He found the appropriate current density in the interior of the solenoid to be

$$j=-2(H_c/\pi a)(r/a)(1-r^2/a^2),$$

where H_c is the magnetic intensity at the center of the sunspot, a its radius, and r the distance from the axis of the solenoid. The maximum value of j is given as 0.77 $H_c/\pi a$ at $r=a/\sqrt{3}=0.577a$.

One difficulty with the representation of sunspot moments in the manner outlined above is that some sunspot groups (about 9 percent[3]) are unipolar and some are irregular and there appears often to be no way to locate elsewhere the path of the flux associated with a particular sunspot. This was the chief consideration which led the writer in 1942 to attempt to represent the field of a unipolar spot approximately as that of a magnetic dipole with its axis coincident with that of the sunspot. As pointed out in the earlier paper,[1] if we imagine the dipole to be located along the solar radius through the center of a sunspot of radius a at a distance $a/\sqrt{2}$ nearer the center of the sun, then it would provide magnetic fields in the appropriate directions at the center and at the edge of the sunspot. To provide the field intensity H_c at the center of the sunspot, the moment of the dipole must be

$$M_a=\sqrt{2}a^3H_c/8=0.177a^3H_c.$$

The magnitude of its field would not vary over the area of the sunspot in close agreement with the empirical formula provided by Nicholson. Curve C in Fig. 1 shows how the intensity of the

[2] S. Chapman, Mon. Not. R. Astr. Soc. **103**, 117 (1943); see also Terr. Magn. Atmos. Elect. **49**, 37 (1944). K. O. Kiepenheuer (Zeits. f. Ap. **15**, 53 (1938)) has estimated that a sunspot has a magnetic moment $\geqq10^{30}$ oersted cm^3, but does not make clear his method of evaluation.

[3] R. S. Richardson, Astrophys. J. **107**, 78 (1948).

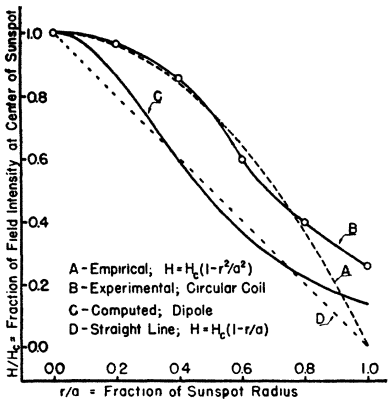

FIG. 1. Contemplated variations of H in a sunspot with distance from its center.

magnetic field produced by the dipole would vary with distance from the center of the sunspot in the plane of the sunspot. Except rather near the center and at the edge, it is seen to be much more closely represented by the linear relation

$$H = H_c(1 - r/a),$$

curve D.

Before proposing this type of representation, the writer had a student (Mr. R. C. Allen) investigate experimentally the magnetic field in the neighborhood of a simple circular coil. With a current-bearing coil of mean radius 5.66 inches, he found that in a plane parallel to the coil and at a distance of 5.66 inches from it, the magnetic field was perpendicular to the axis at a distance of 10 inches from the axis. Hence this experiment indicates that if the magnetic field of a sunspot of radius a were supposed to be due to a current I flowing in a circle of radius $0.566a$ coaxial with the sunspot, and with its plane $0.566a$ beneath that of the sunspot, then this would also yield magnetic fields in the correct directions at the center and edge of the sunspot. To produce the field H_c at the center of the sunspot, we must have a current

$$I = \sqrt{2}RH_c/\pi \text{ e.m.u.}$$

where $R = 0.566a$ is the radius of the circuit. (In this connection it is of interest to recall that Chapman found the maximum current density in his curved solenoid to occur at $r = 0.577a$.) The magnetic moment of the magnetic shell equivalent to the circular current according to Ampere's theorem would be

$$M_c = \pi R^2 I = \sqrt{2}R^3 H_c = (0.566)^3\sqrt{2}a^3 H_c = 0.256a^3 H_c = 1.45 M_d.$$

Moreover, Mr. Allen's experimental observations of the magnetic field due to the circular current showed that the magnitude of H due to the current I located as specified would vary over the surface of the sunspot in accordance with curve B in Fig. 1. Curve B lies quite close to curve A which represents Nicholson's empirical relation, except near the edge of the sunspot where it is presumed that the empirical relation provides only a fair approximation. It is perhaps doubtful whether the accuracy of the empirical expressions for the magnitude and direction of H over the area of a sunspot would justify any closer approximations to either of these. Because it was considered that M_c provided quite a satisfactory representation of the magnetic moment to be associated with the sunspot and because M_d differed from this by only 45 percent while one can scarcely hope for much better than the proper order of magnitude under the circumstances, the simpler representation by the dipole was proposed in the earlier paper.

For the large unipolar sunspot group M.W. 6618 having a maximum area $A = 1939$ (millionths of the Sun's visible hemisphere) and hence 44,000 km equivalent radius, and a maximum $H_c = 36$ (hundred gauss), the maximum dipole moment was given as $M_d = 5.2 \times 10^{31}$ e.m.u. Corresponding to this we have $M_c = 7.5 \times 10^{31}$ e.m.u. representing a ring current of 4.0×10^{13} amperes with a radius of 25,000 km located 25,000 km closer to the center of the sun. It is perhaps of some interest to note that Chapman[2] found for the total flow of current across a radial half-plane through the axis of his curved solenoid $10^6 H_c/2\pi$ amperes per km length. For the bipolar pair he considered, with a pole separation of 14° or 170,000 km and with $H_c = 3900$ gauss, one obtains a total current of 1.7×10^{14} amperes if he supposes the solenoid to be semicircular, or 1.1×10^{14} amperes if he supposes it to extend almost straight from the one sunspot to the other.

If one desires to consider the magnetic properties of even a well-behaved sunspot throughout its history, he is confronted with further difficulties. A sunspot is likely to begin as a rather small spot (its magnetic field can sometimes be detected before the spot is visible) which grows for several days and then decreases, perhaps breaking up into several parts meanwhile. While the sunspot varies in size, its central magnetic field varies somewhat correspondingly, but apparently not according to a well-defined law. Since we have available only one value of H for each group, namely, the maximum value ever observed for the group, it appears likely that this corresponds to the value of H at the center of the largest sunspot in the group when that sunspot is largest, though it appears that this is not necessarily the case. In order to make any estimate regarding the magnetic state of a group at any time other than that for which H is given, it appears necessary to make further rather bold assumptions which may be justified only partially on a statistical basis. Some information is available as a guide for such assumptions. On p. 55 of *Publications of the Astronomical Society of the Pacific* **39** (1927) we read:

> Up to a certain area, the field-strength is almost directly proportional to the diameter of the spot, with its maximum intensity near the middle of the umbra, decreasing to zero just beyond the outer edge of the penumbra. But when the spot is split into several members, the maximum field-strength in any one of them is much below that shown by an unbroken spot.

Dr. Nicholson has informed the writer that the phrase *up to a certain area* is important and that for greater areas the central field intensity is likely to be more nearly proportional to the square root of the diameter or the fourth root of the area.

If we make the simplifying assumption that the central field intensity in any sunspot is proportional to its radius and hence to the square root of its area, we have $H_c = H_m(A/A_m)^{\frac{1}{2}}$ where A is its area, A_m its maximum area, and H_m the maximum field intensity ever associated with it. Using the dipole representation, we might then assign to it the magnetic moment $M_d = \sqrt{2}a^3H_c/8 = \sqrt{2}/(8(\pi^3)^{\frac{1}{2}})(A^2H_m/(A_m)^{\frac{1}{2}}) = 0.0317A^2H_m/(A_m)^{\frac{1}{2}}$, or $M_c = 1.45M_d$ for the ring-current representation. This provides an expression presumably somewhat representative of the magnetic moment of the sunspot throughout its life. If H_m is expressed in gauss and A and A_m in cm^2, then M_d or M_c is expressed in e.m.u.

In order to obtain a function capable of evaluation and hence statistical investigation, further assumptions were made. Since only the H_m for a sunspot *group* is available, the entire area for the group was treated as if it were the area of a single circular sunspot, irrespective of the polarity of individual spots of the group. While this may appear to be an extraordinary procedure, an example indicates that it may not be so bad as it appears. If one applies this procedure to the great bipolar pair considered by Chapman,[2]

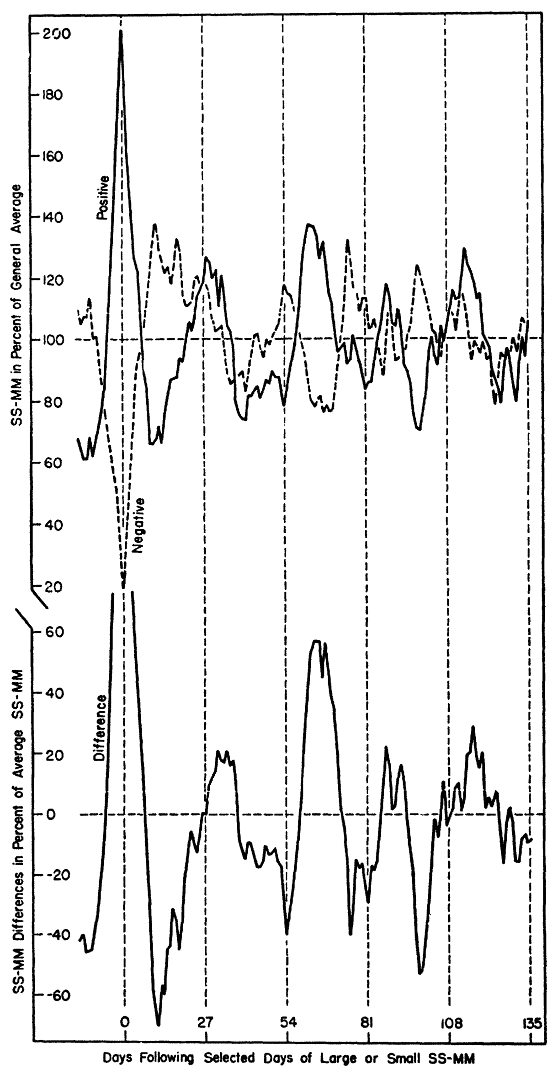

FIG. 2. Primary and subsequent subsidiary pulses in SS-MM.

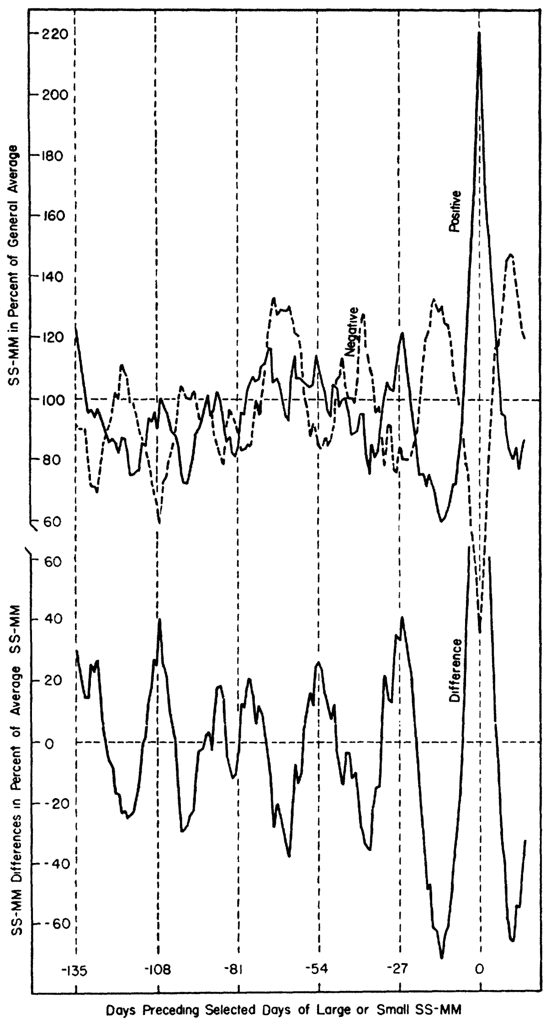

FIG. 3. Primary and preceding subsidiary pulses in SS-MM.

one obtains $M_d = 1.6\times10^{31}$ or $M_c = 2.3\times10^{31}$ e.m.u., instead of the 5×10^{31} obtained by him. Incidentally, as he mentioned, he might have obtained an estimate half as great by dividing the magnetic flux by 4π rather than 2π to obtain the pole strength. The important and rather surprising fact, however, is that these two very different methods of approach lead to magnitudes of the same order for this very large bipolar pair. The supposed orientations of magnetic axes are quite different, of course.

In order to obtain a single function somewhat representative of the state of magnetization of all the sunspots visible on the Sun, capable of evaluation, and not unwieldy, a further step was taken. The equivalent dipole (or ring-current) moment was computed as outlined above for each sunspot group for each day from daily values of their areas published in the *Monthly Weather Review*. (The writer corrected some obvious errors in tabulation.) These magnetic moments for all the sunspots visible on a particular day, irrespective of their locations, were then added together to provide a function which it was thought might prove to be of interest. This function, defined to be the arithmetic sum of the equivalent dipole or ring-current moments of all sunspot groups visible on a particular day, perhaps in some degree representative of all the sunspot magnetic moments on the nearer side of the Sun, will hereinafter be designated SS-MM. The possible usefulness of this function requires investigation, of course. It has been shown that terrestrial magnetic disturbances,[4] cosmic-ray intensity,[1] and frequency of small cosmic-ray bursts,[5] all bear some relation to the simple sum of the areas of the visible sunspots. Inasmuch as it provides some recognition of a physical property other than size, it occurred to the writer that a function such as the SS-MM might be found to be more closely related to such variables than is the total area. While application of Chree's method of analysis[4] showed recurrences of fluctuations of magnetic character, cosmic-ray intensity,[1] and burst frequency[6] at intervals of about 27 or 28 days, a casual investigation of sunspot areas by this method did not indicate such a recurrence interval.[7]

The SS-MM was subjected to Chree's superposed-epoch method of statistical analysis in the usual manner. Because the same period had been employed in the earlier work, the SS-MM was determined for each day with available data during the interval from May 25, 1938, to December 1, 1939, inclusive. For 13 of the days in this interval, data were not available; these were rather widely distributed, only two of them being consecutive.

Figures 2 and 3 show the results of the analysis. Figure 2 was obtained with zero-days se-

[4] C. Chree, Phil. Trans. Roy. Soc. **A212**, 75 (1913); **A213**, 245 (1914).
[5] J. W. Broxon, Phys. Rev. **72**, 1187 (1947).
[6] J. W. Broxon, Phys. Rev. **70**, 494 (1946).
[7] Note particularly Fig. 13 of reference 1.

lected from the first 15 months (June 1938–August 1939) and Fig. 3 with zero-days selected from the last 15 months (September 1938–November 1939). In each case the selected zero-days for the positive-pulse curve (marked Positive) were the five in each month with the largest values of SS-MM. For the negative-pulse curves (marked Negative) the selected zero-days were the five in each month with the smallest values of SS-MM. In each diagram the difference-pulse curve (marked Difference) was obtained by subtracting the ordinate for a particular day number on the negative curve from the ordinate for the same day number on the positive curve to provide the ordinate for that day number on the difference curve.

The positive and negative primary (zero-day) pulses in SS-MM are seen to represent variations of some 70 to 120 percent from the mean, with preceding and following secondary pulses representing variations of some 20 to 40 percent from the mean. The mean value of the SS-MM for all days in the period of investigation is 1.7×10^{31} or 2.5×10^{31} e.m.u. according to whether the dipole or ring-current representation is employed. This is only one-third as large as the value given above for the single great unipolar M.W. 6618 whose area attained its maximum on Sept. 23, 1939. Its maximum area also exceeded the mean value of the sum of the areas of all visible sunspots, but this excess amounted to less than 3 percent.

While the subsidiary pulses following the primary pulse in Fig. 2 are quite large, in general, there does not appear to be any good evidence of a constant period of recurrence. The four subsidiary positive pulses appear to be centered about day numbers 33, 65, 90, and 116. The three most definite subsidiary negative pulses appear to be centered about day numbers 34, 67, and 124. The four principal subsidiary pulses in the corresponding difference curve are rather well centered about day numbers 30, 65, 90, and 115. Among these it appears difficult to pick out any single period which would fit well into these groups. In each of the three curves the first two pulses follow at intervals of 30 to 35 days, while in the case of the difference curve, at any rate, the third and fourth pulses follow at intervals of 25 days. Comparison with Fig. 13 of reference 1 shows that the subsidiary pulses in SS-MM following the primary are generally larger and more clearly defined than are those in sunspot area. In the latter there is some evidence of a 34-day period for three pulses in the positive-pulse curve and for the first two pulses in the difference curve.

The subsidiary pulses preceding the primary in Fig. 3 provide rather good evidence for recurrence at intervals of 27 days. This is particularly true of the negative-pulse curve and is quite apparent in the difference curve. In the positive-pulse curve there is no evidence of a positive subsidiary pulse at day -81, and this produces a distortion in the third preceding pulse in the difference curve. In general, the positive subsidiary pulses preceding the primary are also rather smaller than the negative pulses. Taken together, however, this set of curves appears to provide rather good evidence of subsidiary pulses of some 12 to 20 percent amplitude preceding the primary pulses in SS-MM at intervals of about 27 days for four or five intervals. In the case of sunspot area no investigation of preceding pulses with zero-days selected on the basis of sunspot area was made, so we are unable to make a direct comparison. In the case of sunspot-area pulses obtained with days selected on the basis of cosmic-ray intensity, however, both preceding and following pulses combined to indicate a recurrence period of about 34 days in the fluctuations of total sunspot area.

Since there is some evidence of a 27-day interval between recurrences of fluctuations of the SS-MM, it appears that this function may be more closely associated than is the total sunspot area with other variables displaying this recurrence interval.

This research was supported in part by the Office of Naval Research.