Spin Dependence of Slow Neutron Scattering by Deuterons

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The cross section of deuterium gas at liquid air temperature for neutrons of average wave-length 5.43A is found to be 21.3×10^{-24} cm^2 per molecule. The cross section of deuterium atoms for neutrons of a few volts is found to be 3.44×10^{-24} cm^2 per atom. These two values are used to obtain information on the spin dependence of the neutron scattering by the deuteron. It is found that the two scattering lengths a_4 and a_2, corresponding to neutron spin parallel or anti-parallel to the deuteron spin, have the same sign but may differ in magnitude by as much as a factor 2.

I. INTRODUCTION

THE purpose of the present experiment is to obtain information on the spin dependence of the slow neutron scattering by deuterium. The scattering properties of slow neutrons are determined by the scattering lengths[1] for the various spin orientations. The spins of the neutron and the deuteron can take two relative orientations with resultant spins $\frac{1}{2}$ and $\frac{3}{2}$. To these correspond two scattering lengths, a_2 for the doublet state, $S=\frac{1}{2}$, and a_4 for the quartet state, $S=\frac{3}{2}$. If these two quantities and their signs were known, the scattering properties of the deuterium nucleus would be completely characterized.

The corresponding two quantities for hydrogen have been obtained by determining separately the cross sections of the ortho- and parahydrogen. The most recent of these measurements is due to Sutton *et al.*[2]

Since no liquid hydrogen was available for the present work and the ortho- and paradeuterium could not be separated, a somewhat different method was used. The cross section of the deuterium molecule at liquid air temperature was measured, using neutrons of very long wave-length in order to emphasize the interference phenomena. The ratio of this cross section to that of deuterium for epithermal neutrons was compared with a value of the same ratio calculated as a function of a_4/a_2. From the comparison one can assign limits to the possible values of a_4/a_2.

Unfortunately, a_4/a_2 turns out to be not very different from one—a value for which the sensitivity of the method is extremely poor. Consequently, a_4/a_2 could be determined only within rather wide limits.

II. DETERMINATION OF THE CROSS SECTION OF D_2 FOR FILTERED NEUTRONS

The deuterium gas was contained in a heavy-walled steel tube, the inside dimensions of which were 74.93 cm in length and 2.72 cm in diameter. In order to prevent condensation of water on the ends of the tube, when it was cooled to liquid air temperature, the ends of the tube were extended by false ends and the space in between was evacuated. The tube, together with the adjoining parts of the false ends, was immersed in liquid nitrogen. The temperature of the tube was measured by a thermocouple and was found to be quite constant at 77.5°K.

A beam of thermal neutrons from the thermal column of the Argonne heavy water pile was filtered through 40 cm of sintered BeO. This filter removes neutrons whose wave-length is less than 4.5A from the thermal beam.[1] The average wave-length of the neutron beam so obtained was determined in a separate experiment, which will be described below, and found to be 5.43A.

The neutron intensity transmitted by the deuterium was measured with a long BF_3 counter. The intensity was measured both with the tube empty and with the tube containing deuterium at a pressure of 406.9 cm Hg. Without deuterium the beam intensity was measured as 1672.1 ± 5 counts per minute with a background of 86 ± 2 c/min., corresponding to a net count of 1586 ± 6 c/min. With the deuterium in the tube the count was 781.9 ± 4 c/min. with a

[1] E. Fermi and L. Marshall, Phys. Rev. **71**, 666 (1947).
[2] Sutton, Hall, Anderson, Bridge, De Wire, Lavatelli, Long, Snyder, and Williams, Phys. Rev. **72**, 1147 (1947).

background of 74.8±2 c/min., corresponding to a net count of 707±5 c/min.

From these data the cross section of the deuterium molecule at 77.5°K for BeO filtered neutrons is found to be $(21.3\pm0.2)\times10^{-24}$ cm^2.

The average wave-length of the BeO filtered neutrons used in this experiment was measured by a subsidiary experiment. The cross section of B^{10} in the form of BF_3 was measured for monochromatic neutrons of known wave-length obtained by reflection from a (1, 1, 1) face of a LiF crystal, and for the filtered neutrons. Assuming a $1/v$ law for the boron absorption, the average wave-length of filtered neutrons is then calculated. The particular face (111) of LiF was chosen, because for it the correction caused by high orders in the range of wave-lengths used is negligible.

In order to measure transmissions of the order of 0.5 for a large range of wave-lengths, two interconnected transmission tubes filled with $B^{10}F_3$ were used. One was 3.569 cm long and the other was 17.556 cm. The short transmission tube was used to measure the cross section of the filtered neutrons, and the long tube was used to measure the cross sections of the monochromatic neutrons whose wave-length ranged from 0.8A to 1.3A. The average wave-length, after correcting for air absorption and for scattering, was found to be 5.43A.

III. DETERMINATION OF THE CROSS SECTION OF D_2 FOR EPITHERMAL NEUTRONS

The cross section of deuterium for epithermal neutrons was measured for resonance neutrons of indium and silver. A beam of neutrons coming from the surface of the pile and which contained therefore a large amount of epithermal neutrons was used. A cadmium foil was inserted into the beam to absorb the thermal neutrons.

When a resonance detector like silver or indium is used, the activity induced is due to some extent to very high energy neutrons. One expects that the cross section of deuterium is smaller for such neutrons than for neutrons of a few volts. In order to minimize the contribution of the high energy neutrons, each measurement of induced activity is taken with and without a thin absorber made of the same element as the detector, and the difference is used. In this way, only neutrons belonging to the low resonance bands are counted.

The deuterium was in the form of 99.74 percent pure D_2O, the contaminant being H_2O. If the cross section of oxygen were well known, it could be directly subtracted, since for epithermal neutrons interference phenomena are negligible. In order to minimize the error introduced by inaccuracies in the oxygen cross section, the following method to cancel it was used. In one measurement, the absorber in the neutron beam was n moles of D_2O and n moles of Be. The intensity transmitted through this absorber was compared with the intensity transmitted in a second measurement in which the absorber was n moles of BeO. The difference in these two transmitted intensities is due to the deuterium only.

Actually, the absorbers used in the two measurements contained the following quantities:

Absorber 1	moles/cm^2	Absorber 2	moles/cm^2
D_2O	0.2999	BeO	0.3016
H_2O	0.0008	O_2	0.00004
Be	0.3051	N_2	0.00018

If R is the ratio of the intensity transmitted by absorber 2 to that transmitted by absorber 1, one finds

$$\ln R = 0.6023[0.5998\sigma_D + 0.0035\sigma_{Be} - 0.0010\sigma_O + 0.0016\sigma_H - 0.00036\sigma_N].$$

The σ's represent the cross sections of the respective elements in units of 10^{-24} cm^2. All cross sections except σ_D appear with very small coefficients so that an error in their values is unimportant. The values which were used were $\sigma_{Be} = 6.1$, $\sigma_O = 4.1$, $\sigma_H = 21$, $\sigma_N = 10$.

The measurement with silver gave

$$\ln R = 1.293 \pm 0.023.$$

That with indium gave

$$\ln R = 1.247 \pm 0.021.$$

The average of these two, 1.27±0.02, was then used to calculate σ_D. One finds $\sigma_D = 3.44 \pm 0.06$.

IV. THEORY

The above experiments have determined the cross section of the deuterium molecule for neutrons of average wave-length 5.43A at 77.5°K ($\sigma(D_2)=21.3\times10^{-24}$ cm^2) and of the deuterium atom for neutrons of a few volts ($\sigma(D)=3.44\times10^{-24}$ cm^2).

These two cross sections can be expressed in terms of the scattering lengths a_2 and a_4 corresponding to antiparallel and parallel orientation of spins of the neutron and the deuteron, respectively.

$\sigma(D)$ is simply expressed by the formula

$$\sigma(D)=4\pi(\tfrac{2}{3}a_4^2+\tfrac{1}{3}a_2^2).$$

In this formula the factors $\frac{2}{3}$ and $\frac{1}{3}$ correspond to the probabilities of parallel and of antiparallel orientation. The calculation of the scattering cross section of the deuteron is more complicated and is similar to that made by Schwinger and Teller[3] for the scattering cross section of the hydrogen molecule. In the experiments the ortho-para ratio was unchanged from that at room temperature. On the other hand, the distribution of the rotational states was that for a temperature of 77.5°K. $\sigma(D_2)$ is an average of the values for the individual states corresponding to this distribution. The results of the calculation of the ratio $\sigma(D_2)/\sigma(D)$, not including the correction of the Doppler effect, are given below:

g	$\sigma(D_2)/\sigma(D)$
0.0	$5.76-1.92\delta+0.0035/g+0.035\delta/g$
0.2	$5.65-1.67\delta$
0.4	$5.23-1.58\delta$
0.6	$4.58-1.36\delta$
0.8	$3.85-0.98\delta$
1.0	$3.13-0.55\delta$
1.1	$2.81-0.36\delta$

In this table g is the relative velocity measured in units of $5\hbar/(2Mr)$ where M is the proton mass and r is the interatomic distance in the deuterium molecule, 0.747×10^{-8} cm. δ is the following function of a_2 and a_4:

$$\delta=(a_4-a_2)^2/(2a_4^2+a_2^2).$$

The cross section becomes infinite at zero velocity. However, the coefficient of the infinite terms is very small.

The Doppler correction was calculated using the formula

$$\sigma_{\text{eff}}=\frac{2\exp(-q^2/q_0^2)}{(\pi)^{\frac{1}{2}}q_0q^2}\int_0^\infty g^2\sigma(g)dg \times\sinh(2qg/q_0^2)\exp(-g^2/q_0^2),$$

where q is the velocity of the neutrons in units $5\hbar/(2Mr)$ and q_0 is the characteristic velocity of the Maxwell distribution written in the form $\exp(-q^2/q_0^2)$. For deuterium at 77.5°K, q_0 is 0.269. The integral is obtained by a numerical integration.

The following values were found for the ratio $\sigma(D_2)/\sigma(D)$ corrected for the Doppler effect:

q	$\sigma(D)_2/\sigma(D)$
0.3	$6.851-2.022\delta$
0.368	$6.047-1.768\delta$
0.4	$5.782-1.680\delta$

The effective wave-length of the neutrons used in the determination of $\sigma(D_2)$ was 5.43A, for which q is 0.346. By interpolation one finds, for $q=0.346$,

$$\sigma(D_2)/\sigma(D)=6.271-1.865\delta.$$

The measured value of this ratio is 21.3/3.44 or 6.19. Consequently, δ is very small.

$$\delta=0.04\pm0.10.$$

Since δ is essentially positive, this result enables one to assign to δ only an upper limit, namely, 0.14. If δ were zero, a_4 would be equal to a_2. To each value of δ there correspond two values of the ratio a_4/a_2, one greater and one less than one. For the upper limit, $\delta=0.14$, the two corresponding values of a_4/a_2 are 0.53 and 2.25. Since δ is between zero and 0.14, the ratio a_4/a_2 must lie between 0.5 and 2.3.

Therefore, it can be concluded that a_4 and a_2 have certainly the same sign. Their magnitudes, however, may differ by somewhat more than a factor 2. The sensitivity of this method is low when the value of a_4 is rather close to that of a_2. Consequently, only the above wide limits can be given for their ratio.

It is notable that deuterium does not share the peculiarities of hydrogen for which the two scattering lengths are of opposite sign and of such magnitudes that the neutron interference effects almost cancel as a result of the almost complete cancelation of the contributions of the two spin orientations.

[3] J. Schwinger and E. Teller, Phys. Rev. **52**, 286 (1937).