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Magnetic Refraction of Neutrons at Domain Boundaries

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In the course of experiments on the production of polarized neutrons, small-angle scattering (of the order of one minute) of neutrons in unmagnetized iron was observed. Further detailed experiments on the angular distribution, and variation with iron thickness, of the scattering have been performed. The results are compared with theory and are shown to be consistent with the hypothesis that the scattering is caused by magnetic refraction of the neutrons at domain boundaries.

INTRODUCTION

IN connection with a series of experiments on the polarization of neutrons,[1] a new effect was found which complicated the polarization results for some time. When the single transmission effect (E, the percentage increase in transmission of an iron block upon magnetization) was measured under conditions of good geometry, it was observed to increase appreciably. When the geometry was improved as much as possible with the equipment then in use (Fig. 1 of reference 1), the measured value of E for a block 1 cm thick became 13 percent instead of the 3 percent to be expected from neutron polarization alone. By using iron blocks of smaller thickness, d, it was found that the excess single transmission effect became relatively even more predominant, being about 1 percent for a 0.1-cm block for which the real E (being proportional to d^2) would be expected to be only 0.03 percent. The spurious single transmission effect could also be obtained for small magnetizing fields H, unlike the true effect which requires extremely high magnetization.

These early results made it quite clear that the new effect differed from the increased transmission caused by neutron polarization, and that it should be eliminated in order to study the latter. Measurements with different geometries showed that if the geometry were made bad enough so that all neutrons scattered by as much as 1° were included in the transmitted beam, then only the real transmission effect remained. It was thus shown that the new effect was a small-angle (less than 1°) scattering which took place in unmagnetized, but not in magnetized, iron. As the main interest at the time was in neutron polarization, the small-angle scattering was not investigated in any more detail than was necessary to eliminate its influence on the polarization effects. After the polarization work was finished, however, the scattering was investigated in more detail in order to determine its cause.

THEORETICAL CONSIDERATIONS

The fact that the small-angle scattering disappears with moderate magnetizing fields must mean that it is associated with the lining up of domains along the crystal axes (which takes place at low fields) and not with the rotation of the magnetic vector toward the applied field and away from the crystal axes (which takes place at high fields and which is necessary for neutron polarization). The cause of the scattering then must lie in the domain structure of unmagnetized iron.

As neutrons pass through polycrystalline iron they are scattered when they encounter a crystal grain at a Bragg angle. For thermal neutrons, however, the Bragg scattering occurs at angles of the order of 30° and greater, and hence will have nothing to do with the small-angle scattering. In addition, the neutron wave will be refracted at crystal boundaries and at domain boundaries. Of course the crystal boundaries do not change with magnetization and hence refraction at them cannot contribute to the effect. Refraction at domain boundaries, however, could cause the scattering and deserves more detailed investigation.

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[1] D. J. Hughes, J. R. Wallace, and R. H. Holtzman, Phys. Rev. **73**, 1277 (1948).

The index of refraction of a crystal for neutron waves (for absorption small compared to scattering) is given[2,3] by

$$n^2-1=(\lambda^2Na/\pi), \tag{1}$$

where λ is the neutron wave-length, N the number of nuclei per cm^3, and a the amplitude of coherent scattering. In the case of iron a is partly nuclear and partly magnetic in origin, being given by

$$a=a_n\pm a_m, \tag{2}$$

where a_n is the nuclear scattering amplitude (not spin-dependent) and a_m is the magnetic scattering amplitude which adds to, or subtracts from, the nuclear amplitude, depending on the orientation of the neutron spin with respect to the domain magnetization. In the general case the evaluation of a_m is quite complicated[2] because it involves the evaluation of the atomic form factor for the various scattering angles. However, if the neutron wave-length is so long that all the scattering is forward, or if only the forward direction is under consideration, as in the present case, the index is given simply by

$$n^2-1=(\lambda^2Na_n/\pi)\pm(\mu B/E), \tag{3}$$

where μ is the neutron moment, B the magnetic induction, and E the neutron energy. Equation (3) gives the indices for the case of magnetization along the direction of incidence, for which case the difference of the indices is a maximum.

Thus there will be two indices of refraction in the iron, and the deviation experienced by a particular neutron at a domain boundary will depend on its orientation with respect to the magnetizations of the adjacent domains and on the orientation of the boundary. Upon magnetization, the boundaries, and hence the deviations, will disappear. The effect of the successive deviations will be to spread an initial collimated beam into a Gaussian whose width, σ, will increase with the square root of the thickness of iron traversed,

$$\sigma=\sigma_0(d/\delta)^{\frac{1}{2}}, \tag{4}$$

where σ_0 is the average (R.M.S.) deviation per domain boundary and δ the domain size. The average deviation depends on the distribution of shape, orientation, and magnetization direction of the domains, and it would be exceedingly difficult to compute accurately. The maximum deviation is, of course, given by twice the critical glancing angle for total reflection (θ_c) at a domain boundary where a_m changes sign. It follows from (3) that

$$\sin\theta_c=\theta_c=(2\mu B/E)^{\frac{1}{2}}, \tag{5}$$

a result which is independent of a_n and which agrees with the simple classical picture of a particle of energy E reflected at a boundary where its potential energy changes by $2\mu B$. Insertion of numerical values in Eq. (5) shows that the maximum deviation will be $2\theta_c=21.2'$ for thermal neutrons. The average deviation, σ_0, will depend on domain shape and orientation but will certainly be only a small fraction of $\theta_{\max}$. The average deviation might be expected to be exceedingly small if the domains were oriented in some regular manner, say with boundaries nearly normal to the neutron motion. In general, the multiply refracted neutrons will be unpolarized because of the random nature of the deviations. Only in the special case of single scattering at the critical angle would the neutron spin states be separated.[4]

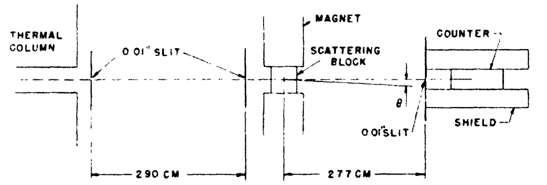

FIG. 1. Apparatus for measurement of magnetic small-angle scattering using slit geometry.

MEASUREMENTS

The first detailed investigations of the small-angle scatterings were made with apparatus very similar to that used for the polarization experiments, that is, with neutron beams of cylindrical cross section. It was found that if the direct beam were blocked by a cadmium disk at the neutron counter, so that only scattered neutrons were measured, then a negative single transmission

[2] O. Halpern, M. Hamermesh, and M. Johnson, Phys. Rev. **59**, 981 (1941).

[3] M. Goldberger and F. Seitz, Phys. Rev. **71**, 294 (1947).

[4] Scattering at the critical angle as a means of production of polarized neutrons has been considered by A. Achieser and J. Pomeranchuk, J. Exper. and Theoretical Physics, U.S.S.R. **18**, 475 (1948).

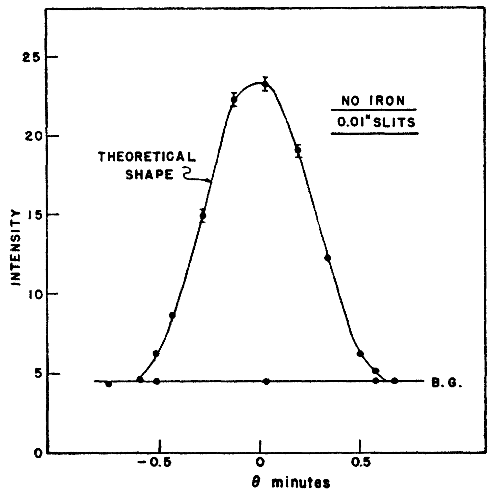

FIG. 2. Angular distribution of neutrons in the beam of Fig. 1 as detected with a 0.01-in. counter slit with no iron block in the beam.

resulted. However, attempts to measure the actual distribution of neutrons scattered outside the cadmium disk, using annular cadmium rings as diaphragms, showed only that the angles were much smaller than 1°. Further refinements decreased the upper limit until it was necessary to change to a slit geometry to obtain sufficient intensity for narrow angles.

The apparatus used to study the distribution of the scattered neutrons with the slit geometry is diagrammed in Fig. 1. A beam of neutrons from the thermal column of the Argonne heavy water pile is formed by two cadmium slits, 0.01″ wide, 1″ high, and 290 cm apart. The horizontal angular distribution in this beam is measured with a proportional counter 284 cm past the second slit. The counter and a third 0.01″-slit are moved in a horizontal direction by a micrometer screw to trace out the neutron distribution. The 0.01″-slits were the smallest that could be used without getting into undue background trouble.

The neutron distribution was first checked with no iron in the beam to see how it compared with that expected from the geometry of the slit system, the background being determined by placing cadmium over the central slit. Figure 2 shows the experimental points compared with the expected "theoretical" shape calculated from geometry. The agreement is excellent, and it is clear that any scattering present with the slits alone, such as air or room scattering, is negligible. The width of the direct beam pattern (half-width about $\frac{1}{4}'$) will complicate the determination of the true scattering distribution if the latter is of the order of 1′ or less. Contrary to early expectations, the scattering proved to be small enough so that the finite width of the direct beam complicated the analysis.

The distribution of the small-angle magnetic scattering was measured by placing a block of cold-rolled steel at the second slit between the poles of an electromagnet, as shown in Fig. 1, and measuring the neutron intensity with the iron magnetized and unmagnetized. The results, with background subtracted, are given in Fig. 3 for a 0.57-cm block. The points for "*H* on" fit the smooth curve which is the same curve as that of Fig. 2 calculated for the shape of the direct beam. In other words, there is no change in the shape of a beam caused by passage through a magnetized iron block, any small-angle scattering being negligible. About half the neutrons are scattered in the block, of course, but practically all go into Debye-Scherrer rings at large angles and do not change the observed beam shape. In addition, there is a slight amount of incoherent, isotropic scattering, but the fraction contained in the small angles studied here is negligible.

The results for the unmagnetized iron ("*H* off" in Fig. 3), however, show a very definite spreading of the beam extending out to several minutes. It is quite clear that the small-angle scattering is not much larger in magnitude than the geo-

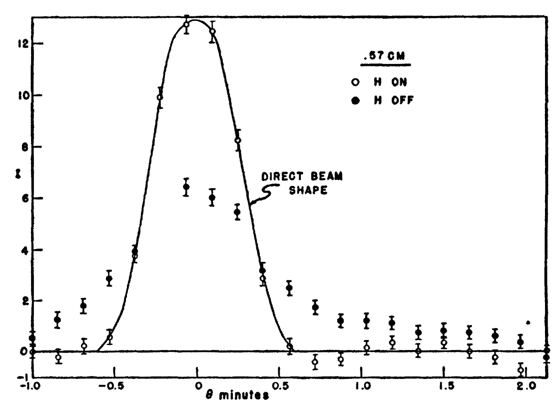

FIG. 3. Neutron distribution after passage through a magnetized ("*H* on") and an unmagnetized ("*H* off") iron block. The smooth curve is the same curve as shown in Fig. 2, adjusted by an intensity scale factor.

metrical spread of the direct beam, hence the shape of the curve of Fig. 3 will depend both on the distribution of the scattered neutrons and on the shape of the direct beam. It is noteworthy that intensity at the center of the pattern is much greater for H on than for H off. The ratio of these two intensities would be interpreted as the single transmission effect, related to neutron polarization, if the effect of small-angle scattering were unknown. It was just this increased single transmission effect with good geometry that led to the discovery of the small-angle scattering. For the 0.57-cm block, for instance, for which the true single transmission effect is about 1 percent, the good geometry of Fig. 3 would have led to an apparent single transmission effect of 100 percent. The errors shown on the points of Fig. 3 are based only on the statistics of the number of counts observed. It is likely that there is some additional error caused by irregular changes in background, for a uniform background was subtracted in plotting the points of Fig. 3.

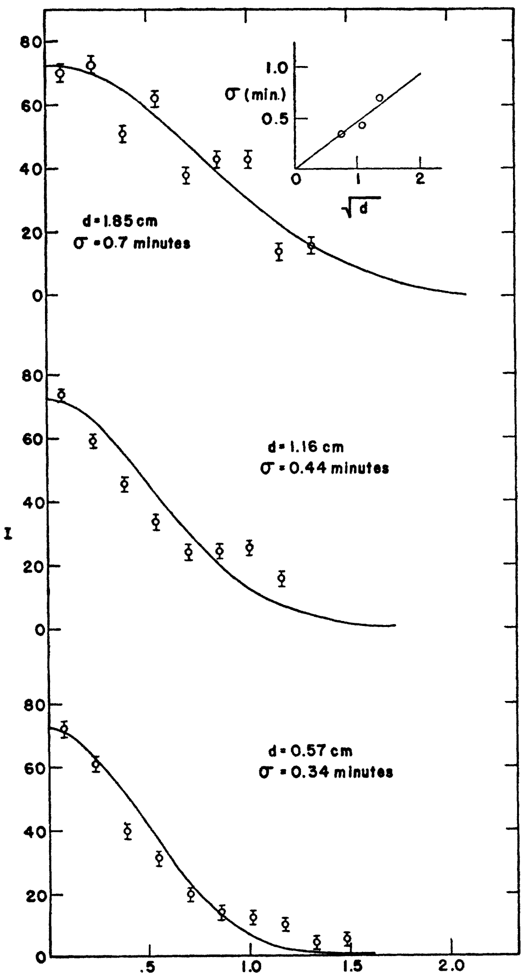

FIG. 4. Experimental neutron distributions for different block thicknesses, d, compared to curves calculated on the assumption that each part of the direct beam is spread into a Gaussian of width σ. The variation of the σ used for each block with d is also shown.

Even though it was difficult to obtain intensity above background, and sufficiently well separated from the direct beam at the same time, measurements were made as a function of block thickness d. In addition to the 0.57-cm block, thicknesses of 1.1 and 1.8 cm were used. Curves similar to Fig. 3 were obtained for both the other blocks, and it was qualitatively clear that the spread of the scattered neutrons increased with d. A more quantitative analysis of the different distributions will be given in the next section. Rough measurements made as a function of magnetizing field showed that the scattering disappeared with magnetizing fields of only a few hundred oersteds, thus verifying the earlier finding that the scattering disappeared as the domain walls disappeared.

DISCUSSION OF RESULTS

According to the theory that the scattering is caused by multiple refraction, it should be possible to fit the results by a Gaussian distribution whose width increases with the square root of d. The comparison with the experimental data was made by assuming that each part of the direct beam was split into a gauss and that the sum of these Gaussians would represent the scattered beam. In this way a series of curves for the scattered distribution was calculated, each corresponding to a particular width, σ, for the Gaussian. It was found that the scattered distributions for the various thicknesses were consistent with the shapes of the calculated curves and that σ increased with d. The experimental points for each block are shown in Fig. 4 with the calculated curves which best fit the results. Although the data are quite rough, the calculated shapes adequately account for the scattered distributions. The Gaussian width assumed for each block thickness is plotted as a function of $d^{\frac{1}{2}}$ in Fig. 4 also. As the errors involved in fixing σ for each curve are quite large, it can only be said that the variation of σ with d is consistent with a $d^{\frac{1}{2}}$ relation and hence with the multiple refraction theory.

The most significant test of the refraction hypothesis is probably that of the numerical value of the scattering. From the curve of Fig. 4 it is seen that a σ of 0.46 corresponds to 1 cm of iron. For the particular iron used the domain size is known to be 3.4×10^{-3} cm, so that $(d/\delta)^{\frac{1}{2}}$ will be 17. From Eq. 4, σ_0 then turns out to be 0.027′. The question then is whether this value of σ_0, which is inferred from the experimental distributions, is consistent with the magnetic refraction hypothesis. Although the details of domain structure are unknown, it is possible to make an approximate calculation of the expected σ_0 without undue difficulty.

The maximum angle of deviation (21.2′) at a single scattering is, of course, much larger than the σ for a d of 1 cm (0.46′), hence the possibility must be considered in the calculation that the observed distribution is caused by single scattering rather than a superposition of small deviations. However, single deviations larger than 1′ occur only for glancing angles less than 0.3°. Because the probability of a domain boundary being oriented at such angles is very small, and because the area presented to the beam in such a case is small, the contribution of large single deviations is negligible. As the glancing angle increases, the deviation rapidly decreases (deviation $=E/B\cot\theta$ for glancing angle θ over 10′), but the probability of occurrence of the angle increases (as $\sin\theta\cos\theta$). The R.M.S. deviation per domain, σ_0, can be calculated from Eq. (3) if it is assumed that the domain boundaries are oriented at random and that the directions of magnetization on opposite sides of the boundary are at random. The resulting value, for one component of the deviation to correspond to the slit geometry, is

$$\sigma_0=0.029',$$

in satisfactory agreement with the observed value of 0.027′, considering the approximate nature of the calculation.*

Although the exceedingly low intensities caused by the necessity for fine beam collimation has made it difficult to get accurate data, it seems as if the hypothesis that the scattering is caused by multiple refraction at the domain boundaries adequately accounts for the results. The work thus demonstrates the presence of two indices of refraction for neutrons in iron, the difference in the indices being caused by magnetic scattering. It is possible that the small-angle scattering will be useful in investigating domain structure because of its relation to domain sizes, shape, and orientation.

We wish to express our thanks to O. Halpern, M. Hamermesh, W. Selove, and H. Snyder with whom we have had interesting discussions concerning the magnetic refraction of neutrons.

* *Note added in proof:* Since this article was written, it has been pointed out by O. Halpern (at the Physical Society Meeting in Chicago December 1948) that the refraction calculations are a special case of the more complete theory (to be treated in a forthcoming publication of Professor Halpern) of small angle effects including diffraction. The refraction treatment gives correct results when the angle of deviation is larger than the angle of diffraction; in the present experiments the angle of refraction is slightly larger than the angle of diffraction.