already obtained for the conversion coefficient ($2I_k$) corresponding to the K-shell in the electric dipole case may be of some interest and are tabulated below (Table I).

These values for I_k were calculated with the aid of a Fridén Calculating Machine (Model ST-10) and five significant digits were carried throughout the calculations. Results, however, are quoted to four significant digits only. The formula on which these calculations are based is essentially that given by Hulme.[1] Actually it has been found that Hulme's formula for I_k may be written in the following simplified form:

$$I_k=\frac{\gamma^{3+2\beta}\{[1+\beta+(1/\theta)]^2-1\}^{(\frac{1}{2}+\beta)}e^{b\pi}|\Gamma(1+\beta-ib)|^2}{24(137)(2+\beta)\Gamma(3+2\beta)|(-z)-(1+\beta-ib)|^2}\times[2|P|^2+|Q|^2],$$

where

$$|Q|^2=8(2+\beta)(1-\theta)^2|(1-z^{-1})^{-(1+\beta+ib)}|^2,$$

$$P=P_1+\frac{\Gamma(s+2+\beta)\Gamma(-\beta-1+ib)\Gamma(s+2-ib)}{\Gamma(s+1-\beta)\Gamma(s+1+ib)\Gamma(1+\beta-ib)}\times(-z)^{-(1+\beta-ib)}P_2,$$

and

$$P_1=vF(s+1+ib,\ ib-s-1,\ ib-\beta,\ z^{-1})+wz^{-1}\left[\frac{1-s+i(b+c)}{1-s-i(b+c)}\right]\left[\frac{s+1+ib}{-\beta+ib}\right]\times F(s+2+\beta,\ \beta-s,\ 2+\beta-ib,\ z^{-1}),$$

$$P_2=vF(s+2+ib,\ ib-s,\ ib-\beta+1,\ z^{-1})+w\left[\frac{1-s+i(b+c)}{1-s-i(b+c)}\right]\left[\frac{-\beta-1+ib}{s+1-ib}\right]\times F(s+2+\beta,\ \beta-s,\ 1+\beta-ib,\ z^{-1}),$$

$$v=\gamma(\theta(2+\beta)+1)^{\frac{1}{2}}(\theta-1)+i[-3(\theta(2+\beta)+1)^{\frac{1}{2}}+(\theta\beta+1)^{\frac{1}{2}}(1-\beta+\theta\{2+\beta\})],$$

$$w=-\gamma(\theta(2+\beta)+1)^{\frac{1}{2}}(\theta-1)+i[+3(\theta(2+\beta)+1)^{\frac{1}{2}}+(\theta\beta+1)^{\frac{1}{2}}(1-\beta+\theta\{2+\beta\})].$$

The notation is that of Hulme.

With this simplified formula for I_k, it was possible for one of the writers to complete the calculations for the forty values tabulated in approximately eight (8) weeks. A more detailed account of this work has been submitted to the National Research Council of Canada for publication at an early date.

[1] H. R. Hulme, "The internal conversion of radium C," Proc. Roy. Soc. **A138**, 642 (1932).

Relativistic Formulation of the Quantum Theory of Radiation

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In the usual formulation of the quantum theory of radiation it is customary to eliminate from the theory the time and longitudinal components of the electromagnetic potential, and only the transverse components of the field are quantized.[1] It has recently been attempted by several authors to put the time and longitudinal components on the same footing with the transverse components by dealing with four different kinds of photons. The advantage of the new idea is obvious, but it may be worth while to point out a special situation which arises in a generalized theory of this kind.

Consider an arbitrary Fourier component of the electromagnetic field in vacuum. Let a_0 and $a_i (i=1, 2, 3)$ be the Fourier transforms of the time, longitudinal and transverse components of the electromagnetic potential multiplied by suitable numerical factors. Let $\bar{a}_0$ and $\bar{a}_i$ be their Hermitian conjugate operators (not to be confused with the adjoint operators in a theory involving indefinite metric). These operators satisfy the commutation rules

$$[a_0, \bar{a}_0]=-1, \quad [a_i, \bar{a}_i]=+1. \tag{1}$$

A straightforward generalization of the customary theory consists in introducing a representation in which the operators $a_0\bar{a}_0$ and $\bar{a}_i a_i$ are diagonal with non-negative integral eigenvalues n_0, n_i. The physical interpretation is that n_0, n_1, n_2, n_3 are, respectively, the numbers of temporal, longitudinal and transverse photons, a_0 and $\bar{a}_0$ are the emission and absorption operators for the temporal photons, $\bar{a}_i$ and a_i are the corresponding operators for the longitudinal and transverse photons. We denote by $\psi(n_0, n_1, n_2, n_3)$ the normalized eigenvectors.

A Hilbert vector Ψ representing a state of the electromagnetic field is subject to the supplementary conditions

$$(a_0-a_1)\Psi=0, \quad (\bar{a}_0-\bar{a}_1)\Psi=0. \tag{2}$$

In accordance with the superposition principle we express Ψ in the form

$$\Psi=\sum_{n_0, n_1, n_2, n_3}^{\infty} c(n_0, n_1, n_2, n_3)\psi(n_0, n_1, n_2, n_3). \tag{3}$$

From (1), (2), (3) it follows that

$$c(n_0, n_1, n_2, n_3)=0, \quad (n_0\neq n_1), \tag{4}$$

$$|c(0, 0, n_2, n_3)|=|c(1, 1, n_2, n_3)|=|c(2, 2, n_2, n_3)|=\cdots. \tag{5}$$

Equations (4) and (5) can also be derived from Dirac's theory of expansors. From these results we can draw the following conclusions: For any given number of transverse photons, Ψ does not represent a pure state but the superposition of an infinite number of states with equal numbers of temporal and longitudinal photons. These two kinds of photons have energies of opposite signs, so they do not contribute to the total energy of the electromagnetic field. They are present even when the total energy vanishes. They have, therefore, quite different properties from those of the transverse photons. A further new feature is that the vector Ψ has an infinite length unless all the c's are zero, and therefore cannot be normalized.

The situation pointed out here should be cleared up in connection with the recent discussions of Schwinger's formulation of quantum electrodynamics.[2]

[1] P. A. M. Dirac, *Principles of Quantum Mechanics* (Oxford University Press, London, 1947), Chap. XII; W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, London, 1944), Chap. II. A relativistic formulation of the elimination has recently been developed by S. Hayakawa, Y. Miyamoto, and S. Tomonaga, by J. Schwinger, and by S. Ashauer.

[2] G. Wentzel, Phys. Rev. **74**, 1070 (1948); F. J. Belinfante, Bull. Am. Phys. Soc. **23**, No. 7, 17 (1948).