TABLE I. Relative abundances of germanium isotopes from measurements on $GeF_3$ and GeI.

| Mass | 70 | 72 | 73 | 74 | 76 |
|---|---|---|---|---|---|
| $GeF_3^+$ | 20.60 | 27.38 | 7.83 | 36.40 | 7.78 |
| Mean Dev. | ±0.06 | ±0.08 | ±0.06 | ±0.10 | ±0.05 |
| $GeI^+$ | 20.65 | 27.43 | 7.86 | 36.34 | 7.72 |
| Mean Dev. | ±0.04 | ±0.02 | ±0.04 | ±0.05 | ±0.01 |
| Aston[a] | 21.2 | 27.3 | 7.9 | 37.1 | 6.5 |

[a] See reference 1.

ment through a capillary leak. Ionization in the Nier-type mass spectrometer was accomplished by slow electron bombardment of the vapors. The ions thus formed were accelerated by a potential of 2000 volts and separated with the conventional sector magnet. The resulting ion currents were measured with an FP54 electrometer tube, balancing the potential developed across the grid resistor with a potential from a precision potentiometer. The ion currents were also recorded by use of a Victoreen VX-41 electrometer tube, driving a standard Brown Electronik Recorder. The vacuum was approximately $5\times10^{-7}$ mm of Hg, and the resolution in all cases was perfect.

In the case of the tetrafluoride, the measurements were made upon the $GeF_3^+$ ion, and the $GeI^+$ ion was used for measurements when the tetraiodide was distilled into the instrument. The results obtained with these compounds are shown in Table I.

The abundances, using $GeF_4$, are the result of three samples in two instruments and the $GeI_4$ measurements of two samples in one instrument. It was found that variations between instruments and between ion sources in the same instrument were larger than variations between samples. Investigation of the mass region, adjacent to the measured ions, revealed it exceptionally free from interfering ions of other materials.

* This document is based on work performed for the AEC by Carbide and Carbon Chemicals Corporation, at Oak Ridge, Tennessee.

[1] Aston, *Mass Spectra and Isotopes* (Edward Arnold and Company, London, 1944), p. 154.

## Erratum: Significance of the Radioactivity of Potassium in Geophysics. II

[Phys. Rev. 74, 831 (1948)]

T. GRÅF
*L. Meitner Laboratory of Nuclear Physics, IVA:s Försöksstation, Stockholm, Sweden*

December 15, 1948

THE sentence at the end of the second paragraph on page 832 should read:

"In addition, to account for the large amounts of atmospheric argon, it should then be assumed that all argon produced in a 40-kilometer deep crustal layer, since the origin of the earth, has escaped into the atmosphere."

*Not* ". . . that all argon was produced in a 40-kilometer deep crustal layer, since the origin of the earth has escaped into the atmosphere."

## Erratum: Line Shapes in Nuclear Paramagnetism

[Phys. Rev. 74, 1184 (1948)]

G. E. PAKE AND E. M. PURCELL
*Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts*

A FACTOR $\frac{1}{2}$ should be supplied to the right member of each of Eqs. (8), Eq. (10), Eq. (11), and the first of Eqs. (1) to make these formulae accurate for frequency $\nu$ in the vicinity of the resonant frequency $\nu_0$. The graphs in Figs. 2 and 3 will then read properly if ordinates are measured in units of $\frac{1}{2}\chi_0\omega_0T_2$.

It should be emphasized that the approximations $\nu/(\nu'+\nu)\cong\frac{1}{2}$ and $\nu'/(\nu'+\nu)\cong\frac{1}{2}$, which hold near resonance and lead to the special forms (3), (4), and (5) of the Kronig-Kramers relations, are evidently not valid at $\nu=0$. Thus the forms of $\chi'(\nu)$ derived from these approximations do not reduce to $\chi_0$ at $\nu=0$ as stated in the paper, but should reduce to $\frac{1}{2}\chi_0$ instead. Supplying the above-mentioned factors $\frac{1}{2}$ takes care of this point, which the authors regrettably overlooked.

## On the Numerical Calculation of the Internal Conversion in the *K*-Shell; the Electric Dipole Case

B. A. GRIFFITH AND J. P. STANLEY
*University of Toronto, Toronto, Canada*

October 25, 1948

THE accurate calculation of internal conversion coefficients is a problem in which recent experimental advances have stimulated considerable interest. Some months ago at the instigation of members of the staff of the National Research Council of Canada at Chalk River, the writers undertook an extensive tabulation of such coefficients in connection with a computational program at the University of Toronto. Recently, it has been learned that similar calculations are being done with the aid of the Harvard Mk II computer. Accordingly, further work here has been suspended pending publication of the Harvard calculations. However, the results which the writers have

TABLE I. Values of the coefficient $2I_k$ for the two electrons of the *K*-shell.

| $\theta$ \ $Z$ | 89 | 84 | 79 | 74 | 69 |
|---|---|---|---|---|---|
| 0.05 | 0.00007429 | 0.00006297 | 0.00005390 | | |
| 0.10 | 0.0002008 | 0.0001667 | 0.0001393 | | |
| 0.15 | 0.0003772 | 0.0003056 | 0.0002530 | | |
| 0.20 | 0.0005938 | 0.0004769 | 0.0003941 | | |
| 0.25 | 0.0008497 | 0.0006773 | 0.0005611 | | |
| 0.50 | 0.002706 | 0.002228 | 0.001825 | 0.001485 | 0.001198 |
| 0.70 | 0.004922 | 0.004112 | 0.003405 | 0.002797 | 0.002280 |
| 1.00 | 0.009736 | 0.008304 | 0.006956 | 0.005810 | 0.004805 |
| 1.50 | 0.02248 | 0.01956 | 0.01687 | 0.01440 | 0.01214 |
| 1.70 | 0.02949 | 0.02593 | 0.02253 | 0.01936 | 0.01642 |

*Note.* In the above Table, $2I_k$ denotes the fraction of photons with energy $h\nu$, emitted by the nucleus, which are absorbed by either of the two electrons in the *K*-shell with ejection of the absorbing electron from the atom. No allowance for screening effects has been made.

The parameters $Z$ and $\theta$ denote, respectively, the atomic number and the ratio $mc^2/h\nu$, where $mc^2$ is the smallest relative energy of a free electron. The value 1/137 has been taken for the fine structure constant.

already obtained for the conversion coefficient ($2I_k$) corresponding to the $K$-shell in the electric dipole case may be of some interest and are tabulated below (Table I).

These values for $I_k$ were calculated with the aid of a Fridén Calculating Machine (Model ST-10) and five significant digits were carried throughout the calculations. Results, however, are quoted to four significant digits only. The formula on which these calculations are based is essentially that given by Hulme.[1] Actually it has been found that Hulme's formula for $I_k$ may be written in the following simplified form:

$$I_k=\frac{\gamma^{3+2\beta}\{[1+\beta+(1/\theta)]^2-1\}^{(\frac{1}{2}+\beta)}e^{b\pi}|\Gamma(1+\beta-ib)|^2}{24(137)(2+\beta)\Gamma(3+2\beta)|(-z)-(1+\beta-ib)|^2}\times[2|P|^2+|Q|^2],$$

where

$$|Q|^2=8(2+\beta)(1-\theta)^2|(1-z^{-1})^{-(1+\beta+ib)}|^2,$$

$$P=P_1+\frac{\Gamma(s+2+\beta)\Gamma(-\beta-1+ib)\Gamma(s+2-ib)}{\Gamma(s+1-\beta)\Gamma(s+1+ib)\Gamma(1+\beta-ib)}\times(-z)^{-(1+\beta-ib)}P_2,$$

and

$$P_1=vF(s+1+ib,\ ib-s-1,\ ib-\beta,\ z^{-1})+wz^{-1}\left[\frac{1-s+i(b+c)}{1-s-i(b+c)}\right]\left[\frac{s+1+ib}{-\beta+ib}\right]\times F(s+2+\beta,\ \beta-s,\ 2+\beta-ib,\ z^{-1}),$$

$$P_2=vF(s+2+ib,\ ib-s,\ ib-\beta+1,\ z^{-1})+w\left[\frac{1-s+i(b+c)}{1-s-i(b+c)}\right]\left[\frac{-\beta-1+ib}{s+1-ib}\right]\times F(s+2+\beta,\ \beta-s,\ 1+\beta-ib,\ z^{-1}),$$

$$v=\gamma(\theta(2+\beta)+1)^{\frac{1}{2}}(\theta-1)+i[-3(\theta(2+\beta)+1)^{\frac{1}{2}}+(\theta\beta+1)^{\frac{1}{2}}(1-\beta+\theta\{2+\beta\})],$$

$$w=-\gamma(\theta(2+\beta)+1)^{\frac{1}{2}}(\theta-1)+i[+3(\theta(2+\beta)+1)^{\frac{1}{2}}+(\theta\beta+1)^{\frac{1}{2}}(1-\beta+\theta\{2+\beta\})].$$

The notation is that of Hulme.

With this simplified formula for $I_k$, it was possible for one of the writers to complete the calculations for the forty values tabulated in approximately eight (8) weeks. A more detailed account of this work has been submitted to the National Research Council of Canada for publication at an early date.

[1] H. R. Hulme, "The internal conversion of radium C," Proc. Roy. Soc. **A138**, 642 (1932).

## Relativistic Formulation of the Quantum Theory of Radiation

S. T. MA
*Dublin Institute for Advanced Studies, Dublin, Eire*
December 10, 1948

IN the usual formulation of the quantum theory of radiation it is customary to eliminate from the theory the time and longitudinal components of the electromagnetic potential, and only the transverse components of the field are quantized.[1] It has recently been attempted by several authors to put the time and longitudinal components on the same footing with the transverse components by dealing with four different kinds of photons. The advantage of the new idea is obvious, but it may be worth while to point out a special situation which arises in a generalized theory of this kind.

Consider an arbitrary Fourier component of the electromagnetic field in vacuum. Let $a_0$ and $a_i(i=1, 2, 3)$ be the Fourier transforms of the time, longitudinal and transverse components of the electromagnetic potential multiplied by suitable numerical factors. Let $\bar{a}_0$ and $\bar{a}_i$ be their Hermitian conjugate operators (not to be confused with the adjoint operators in a theory involving indefinite metric). These operators satisfy the commutation rules

$$[a_0, \bar{a}_0]=-1, \quad [a_i, \bar{a}_i]=+1. \tag{1}$$

A straightforward generalization of the customary theory consists in introducing a representation in which the operators $a_0\bar{a}_0$ and $\bar{a}_ia_i$ are diagonal with non-negative integral eigenvalues $n_0$, $n_i$. The physical interpretation is that $n_0$, $n_1$, $n_2$, $n_3$ are, respectively, the numbers of temporal, longitudinal and transverse photons, $a_0$ and $\bar{a}_0$ are the emission and absorption operators for the temporal photons, $\bar{a}_i$ and $a_i$ are the corresponding operators for the longitudinal and transverse photons. We denote by $\psi(n_0, n_1, n_2, n_3)$ the normalized eigenvectors.

A Hilbert vector $\Psi$ representing a state of the electromagnetic field is subject to the supplementary conditions

$$(a_0-a_1)\Psi=0, \quad (\bar{a}_0-\bar{a}_1)\Psi=0. \tag{2}$$

In accordance with the superposition principle we express $\Psi$ in the form

$$\Psi=\sum_{n_0,n_1,n_2,n_3}^{\infty}c(n_0, n_1, n_2, n_3)\psi(n_0, n_1, n_2, n_3). \tag{3}$$

From (1), (2), (3) it follows that

$$c(n_0, n_1, n_2, n_3)=0, \quad (n_0\neq n_1), \tag{4}$$

$$|c(0, 0, n_2, n_3)|=|c(1, 1, n_2, n_3)|=|c(2, 2, n_2, n_3)|=\cdots. \tag{5}$$

Equations (4) and (5) can also be derived from Dirac's theory of expansors. From these results we can draw the following conclusions: For any given number of transverse photons, $\Psi$ does not represent a pure state but the superposition of an infinite number of states with equal numbers of temporal and longitudinal photons. These two kinds of photons have energies of opposite signs, so they do not contribute to the total energy of the electromagnetic field. They are present even when the total energy vanishes. They have, therefore, quite different properties from those of the transverse photons. A further new feature is that the vector $\Psi$ has an infinite length unless all the $c$'s are zero, and therefore cannot be normalized.

The situation pointed out here should be cleared up in connection with the recent discussions of Schwinger's formulation of quantum electrodynamics.[2]

[1] P. A. M. Dirac, *Principles of Quantum Mechanics* (Oxford University Press, London, 1947), Chap. XII; W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, London, 1944), Chap. II. A relativistic formulation of the elimination has recently been developed by S. Hayakawa, Y. Miyamoto, and S. Tomonaga, by J. Schwinger, and by S. Ashauer.

[2] G. Wentzel, Phys. Rev. 74, 1070 (1948); F. J. Belinfante, Bull. Am. Phys. Soc. 23, No. 7, 17 (1948).