Beta-Decay of Be^{10} and Theory of Beta-Decay

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If a spin change of three units is involved in the $Be^{10}\rightarrow B^{10}$ transition—as seems to be indicated at present—theoretical predictions of the minimum half-life lead to an essentially unique beta-ray spectrum very different from the allowed shape. Measurement of this spectrum will provide a crucial test of existing theories of beta-decay.

THE nucleus Be^{10} decays to B^{10} with the emission of electrons having a maximum energy of 570±10 kev and with a half-life of $2.7\pm0.4\cdot10^6$ years; no γ-ray is observed.[1] On the other hand, C^{10} decays to B^{10} with the emission of positrons having a maximum energy of 2.2 Mev and with a half-life of 19.1 sec.; in this case a γ-ray is observed.[2] Recently, the spin of B^{10} was measured[3] and found to be 3. Finally, it is almost certain[4] that both Be^{10} and C^{10} possess spin 0. It would thus appear that the great difference in half-life between Be^{10} and C^{10} can now be understood: the long half-life of Be^{10} is due to a spin change of three units associated with a transition to the ground state[5] of B^{10}, whereas the short half-life of C^{10} is made possible by the transition of C^{10} to an excited level of B^{10} (which may be assigned spin 0 or 1, depending on whether the Fermi or Gamow-Teller selection rules are presumed to hold). However, a correct theory of beta-decay must explain not only the half-life but also the energy spectrum of the beta-rays. A closer examination of the Be^{10} situation reveals that a reasonably accurate measurement of the beta-ray spectrum from Be^{10} will provide a crucial test of all existing theories of beta-decay—provided that the spin of Be^{10} is really 0.

We have calculated the energy spectra corresponding to a spin change of 3 for the $Be^{10}\rightarrow B^{10}$ transition on the basis of the five possible interactions. Since current nuclear theory does not account for the measured spin of B^{10}, we shall also disregard its parity predictions. Calculations have therefore been performed on both suppositions, assuming that the parities of the ground states of Be^{10} and B^{10} are the same and that they are different. A beta-transition with a spin change of 3 and no parity change requires at least a second forbidden tensor or axial vector transition or a third forbidden pseudoscalar transition. Parity change requires at least a third forbidden scalar, axial vector, tensor, or vector transition. Using a notation similar to that of a previous paper,[6] we write:
no parity change,

$$C_{2T}=D_2|Q_3(\beta\sigma,r)|^2;\quad C_{2A}=D_2|Q_3(\sigma,r)|^2;\quad C_{3P}=D_1'|Q_3(\beta\gamma_5 r,r)|^2.$$

Parity change,

$$C_{3S}=D_1'|Q_3(\beta r,r)|^2;\quad C_{3A}=D_4|Q_3(\sigma\times r,r)|^2;$$
$$C_{3T}=D_1|Q_3(\beta\sigma\times r,r)|^2+D_2|Q_3(\beta\alpha,r)|^2-D_3[Q_3(\beta\sigma\times r,r)Q_3^*(\beta\alpha,r)+\mathrm{cc}];$$
$$C_{3V}=D_1''|Q_3(r,r)|^2+D_2|Q_3(\alpha,r)|^2+iD_3[Q_3(\alpha,r)Q_3^*(r,r)-\mathrm{cc}].$$

Figure 1 gives the results, taking $W_0=2.118$ (in units of mc^2) as the upper limit of the spectrum. We have not plotted D_1' and D_1'' since they do not differ from D_4 and D_1, respectively, by more than 17 percent over the entire range. The first five transitions yield unique energy spectra, whereas the last two yield arbitrary linear combinations of three different types of spectra, since the values of the matrix elements are unknown.

[1] D. J. Hughes and C. Eggler, Phys. Rev. **74**, 1239 (1948); also private communication from Dr. Hughes.
[2] R. Sherr, H. R. Muether, and M. G. White, Phys. Rev. **74**, 1239 (1948); also private communication from Dr. White.
[3] W. Gordy, H. Ring, and A. B. Burg, Phys. Rev. **74**, 1191 (1948).
[4] All measured spins of even-even nuclei have been found to be 0, including the homologous radioactive nucleus C^{14}; we assume spin 0 for Be^{10} and C^{10} for the purposes of our discussion.
[5] M. Goldhaber, Phys. Rev. **74**, 1194 (1948).
[6] R. E. Marshak, Phys. Rev. **70**, 980 (1946); see also E. Greuling, Phys. Rev. **61**, 568 (1942).

TABLE I. Values of L_i.

L_1	L_1'	L_1''	L_2	L_3	L_4
0.070	0.046	0.074	0.052	0.058	0.042

TABLE II. Maximum values of the distinct $|Q_3|^2$.

$\lvert Q_3(\sigma, r)\rvert^2$	$\lvert Q_3(\beta\gamma_5 r, r)\rvert^2$	$\lvert Q_3(r, r)\rvert^2$	$\lvert Q_3(\sigma\times r, r)\rvert^2$	$\lvert Q_3(\alpha, r)\rvert^2$
$\frac{28}{15}\rho^4$	$\frac{4}{5}\alpha^2\rho^6$	$\frac{4}{5}\rho^6$	$\frac{16}{15}\rho^6$	$\frac{28}{15}\alpha^2\rho^4$

Thus far, we have not eliminated any of the interactions by comparing the minimum theoretical half-life with the observed half-life. If we write $\tau_i = \tau_0/|Q_3|^2 L_i$, where

$$L_i = \int_1^{W_0} D_i W(W^2-1)^{\frac{1}{2}}(W_0-W)^2 dW,$$

we shall obtain the minimum value of τ_i by choosing the maximum value of $|Q_3|^2$. The values of the six L_i's are listed in Table I. It is seen that the variation in L is less than a factor 2, so that the differences in half-lives will arise chiefly from the variation of $|Q_3|^2$ (see Table II below). The maximum value of $|Q_3|^2$ is arrived at by setting the square of the matrix element equal to the isotopic number of Be^{10} (i.e., 2) and by inserting the proper statistical weight of the ground state of B^{10} (i.e., 7). Table II lists the maximum value[7] of the distinct $|Q_3|^2$. In Table II, ρ is the radius of Be^{10} (in units of $\hbar/mc$), namely, $8.6\cdot 10^{-3}$, while α^2 is the square of the matrix element of the Dirac α-operator taken between the ground states of Be^{10} and B^{10}. We have not distinguished between matrix elements involving the Dirac β-operator since we may set $\beta=1$ in the non-relativistic approximation for the nucleons. Moreover, we have not included values for the cross-terms in C_{3T} and $C_{\delta A}$ since we may use the geometric means of the other two terms in each interaction.

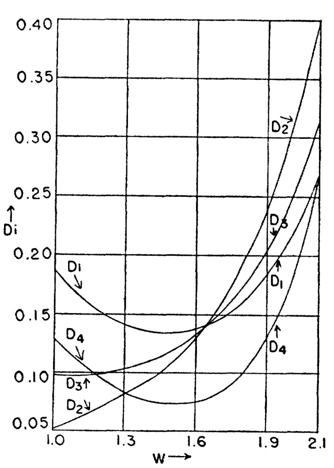

FIG. 1. Values of D_i as a function of energy. The upper limit of the spectrum W_0 is taken as 2.118 in units of mc^2.

The value of τ_0 depends on the radioactive nucleus chosen as the norm. In principle, τ_0 should be taken from the decay of the neutron. However, since the measured half-life of the neutron[8] is quite unreliable at the present time, the comparison nucleus most suitable for our purposes is He^6. The maximum energy of the electron associated with the $He^6 \rightarrow Li^6$ transition is 3.7 ± 0.2 Mev, and the half-life is 0.82 ± 0.06 sec.; there is no evidence for a γ-ray.[9] If the decay of He^6 is to the ground state of Li^6, the Gamow-Teller selection rules apply, and τ_0 becomes $5.8\cdot 10^3$ sec.[10] If the decay of He^6 is to a very low excited level (less than 100 kev[9]) of Li^6 possessing spin 0, so that the Fermi selection rules apply, τ_0 is three times smaller, namely, $1.9\cdot 10^3$ sec., for the scalar and vector interactions, and $1.9\cdot 10^4$ for the pseudoscalar interaction (setting $\alpha^2 = 1/10$). Table III lists the minimum half-lives for the five transitions which

[7] E. Greuling, see reference 6; see also R. E. Marshak, Phys. Rev. 61, 431 (1942).

[8] A very preliminary value for the half-life of the neutron is about 30 minutes (A. H. Snell and L. C. Miller, Phys. Rev. 74, 1217 (1948)). If this value is adopted, τ_0 becomes $2.4\cdot 10^3$ sec. for the scalar, axial vector, tensor, and vector interactions, and $1.5\cdot 10^4$ times smaller for the pseudoscalar interaction (see J. Tiomno and J. A. Wheeler, forthcoming publication). The value of τ_0 obtained for the S, A, T, and V interactions is essentially identical with that obtained from the He^6 decay. The value obtained for the P interaction is very different and is a consequence of the extremely small (and known) value of the Dirac α-operator found in the neutron decay as contrasted with the large value (for lack of specific information) used for the He^6 decay (see footnote 11 below).

[9] W. J. Knox, Phys. Rev. 74, 1192 (1948).

[10] This value is obtained by choosing the maximum value for the square of the matrix element, i.e., 2. It is conceivable that the ground state wave functions of He^6 and Li^6 are much more orthogonal to each other than those for Be^{10} and B^{10}, making τ_0 several orders of magnitude smaller, but very unlikely. Other beta-ray evidence, especially the mirror nuclei, leads to approximately the same value for τ_0. Of course, an accurate determination of the half-life of the neutron will fix τ_0.

TABLE III. Minimum half-lives for the five transitions associated with the single matrix elements.

2T	2A	3A	3P	3S
$3.5\cdot10^5$ yr.	$3.5\cdot10^5$ yr.	$1.0\cdot10^{10}$ yr.	$4.1\cdot10^9$ yr.	$4.1\cdot10^9$ yr.

are associated with single matrix elements, the appropriate τ_0 being used in each case. It is seen from Table III that the minimum half-lives for $3A$, $3P$, and $3S$ are much too long to be reconciled with the observed half-life; these three transitions must therefore be discarded.[11] The minimum half-lives associated with $2T$ and $2A$ *are* consistent with the observed half-life. If we now examine the two remaining transitions, $3T$ and $3V$, which involve two distinct matrix elements apiece, we find that the first matrix element of $3T$ yields a minimum half-life identical with that listed for $3A$, whereas the first matrix element of $3V$ yields a minimum half-life identical with that listed for $3S$. The second matrix element of $3T$ yields a minimum half-life of $3.5\cdot10^6$ yr. (choosing the rather large value of 1/10 for α^2). The second matrix element of $3V$ leads to a minimum half-life three times shorter. Both of these latter half-lives are consistent with the observed half-life if we are willing to stretch the numbers somewhat (for W_0, ρ, etc.). It is clear that the cross-terms in both $3T$ and $3V$ yield minimum half-lives which are much too long.

We may therefore conclude that the observed half-life of Be^{10} requires the rejection of *all* matrix elements[11] except the four ($Q_3(\beta\sigma, r)$, $Q_3(\sigma, r)$, $Q_3(\beta\alpha, r)$, $Q_3(\alpha, r)$) which are associated with the energy spectrum given by D_2. Of these four, three ($2T$, $2A$, $3T$) are Gamow-Teller-type interactions, and one ($3V$) is a Fermi-type interaction. The D_2 spectrum is so different from the allowed spectrum that a careful measurement of the beta-spectrum from Be^{10} should be decisive for present-day theories of beta-decay. If the spectrum turns out to be allowed, as seems to be indicated by a preliminary measurement,[1] it will follow that present theories of beta-decay will have to undergo serious modification.

The author is indebted to Messrs. Albert Simon and Clayton Black for assistance with the numerical calculations.

[11] There is one loophole in this argument: *if* the half-life of the neutron is 30 minutes and *if* α^2 for the $Be^{10}\rightarrow B^{10}$ transition is as much as 1/10, the minimum half-life for the $3P$ transition would turn out to be $2.8\cdot10^6$ yr., consistent with the observed half-life. Other beta-ray evidence does not lend support to the pseudoscalar interaction (E. P. Wigner, private communication). However, even accepting this possibility, the predicted spectrum would be very different from the allowed spectrum (see curve D_4 in Fig. 1), and the measurement of the beta-spectrum from Be^{10} would still provide a crucial test of beta-theory. It is interesting to note that above $W=1.6$, D_4 and D_2 have roughly the same shape.

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The Beta-Spectrum of Be^{10}

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RECENT investigations have thrown much light on the disintegration of Be^{10}, long an outstanding problem of nuclear structure and beta-theory. The accompanying paper of Marshak reviews the recent evidence and points out that, while the large spin change involved in the disintegration explains the long half-life, it also predicts a surprisingly unique spectrum shape. Because of the resulting importance of the experimental Be^{10} spectrum to beta-theory, it is desirable to report some results already obtained on the spectrum using an absorption method.

Measurements[1] made since the activity was first isolated by McMillan and Ruben[2] in 1940

[1] J. Levinger and E. Meiners, Phys. Rev. **71**, 586 (1947); D. J. Hughes, C. Eggler, and C. M. Huddleston, Phys. Rev. **71**, 269 (1947); A. K. Pierce and F. W. Brown, III, Phys. Rev. **70**, 779 (1946); E. M. McMillan, Phys. Rev. **72**, 591 (1947).

[2] E. M. McMillan and S. Ruben, Phys. Rev. **70**, 123 (1946).