# A Precision Study of the Tungsten $K$ Spectrum Using the 2-Meter Focusing Curved Crystal Spectrometer* 

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#### Abstract

Using the 2 -meter focusing curved crystal gamma-ray spectrometer, careful measurements were made of the wave-lengths of the $K$ series lines and $K$ absorption edge of tungsten for the dual purpose of establishing a precision linkage between wave-length measurements in the gamma- and x-ray regions and of improving our knowledge of the tungsten wave-lengths. The high precision, resolution, and luminosity of the instrument made. possible the complete resolution of the $\beta$-doublet, the partial resolution of the $\gamma$-doublet, and the detection and measurement of a $\delta$-line close to the absorption edge. Absolute determinations of the Bragg angles for Mo $K \alpha_{1}$ and W $K \alpha_{1}$ reflected from the (310) planes of quartz by W. J. West in this laboratory, using a precision 2-crystal spectrometer, are described. These were undertaken (a) to standardize the tungsten $K$ spectrum with higher accuracy in terms of the Siegbahn scale of x-ray wave-lengths and (b) to yield on this scale a precision determination of the grating constant of the quartz (310) planes. Adopting W. J. West's value of $208.575 \pm 0.008$ x.u. (Siegbahn scale) for the wave-length of the $\alpha_{1}$ line, the wave-lengths of the $K$ series lines and $K$ edge were found to be:


| Siegbahn notation | $\alpha_{2}$ | $\alpha_{1}$ | $\beta_{3}$ | $\beta_{1}$ | $\beta_{2}{ }^{\text {II }}$ | $\beta_{2}{ }^{I}$ |  | Abs. edge. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sommerfeld notation | $\alpha_{2}$ | $\alpha_{1}$ | $\beta_{2}$ | $\beta_{1}$ | $\gamma^{2}$ | $\gamma_{1}$ | $\delta$ | Abs, edge. |
| Transition | $K-L_{\text {II }}$ | $K-L_{\text {III }}$ | $K-M_{1 I}$ | $K-M_{I I I}$ | $K-N_{I I}$ | K-NIII | K-OII, III | $K-\infty$ |
| Wave-length x.u. (Siegbahn scale) | $\begin{array}{r} 213.387 \\ \pm 0.010 \end{array}$ | $\begin{array}{r} 208.575 \\ \pm 0.008 \end{array}$ | $\begin{array}{r} 184.772 \\ \pm 0.020 \end{array}$ | $\begin{array}{r} 183.967 \\ \pm 0.020 \end{array}$ | $\begin{aligned} & 179.212 \\ & \pm 0.020 \end{aligned}$ | $\begin{aligned} & 179.038 \\ & \pm 0.020 \end{aligned}$ | $\begin{array}{r} 178.052 \\ \pm 0.020 \end{array}$ | $\begin{array}{r} 177.947 \\ \pm 0.020 \end{array}$ |
| Wave-length A | $\begin{array}{r} 0.213818 \\ \pm 0.000010 \end{array}$ | $\begin{array}{r} 0.208996 \\ \pm 0.000008 \end{array}$ | $\begin{array}{r} 0.185145 \\ \pm 0.000020 \end{array}$ | $\begin{array}{r} 0.184339 \\ \pm 0.000020 \end{array}$ | $\begin{array}{r} 0.179574 \\ \pm 0.000020 \end{array}$ | $\begin{array}{r} 0.179400 \\ \pm 0.000020 \end{array}$ | $\begin{array}{r} 0.178412 \\ \pm 0.000020 \end{array}$ | $\begin{array}{r} 0.178306 \\ \pm 0.000020 \end{array}$ |

West's 2 -crystal spectrometer study gives for the (310) planes of quartz at $18^{\circ} \mathrm{C}$ the grating constant

$$
d=1177.637 \pm 0.020 \text { x.u. (Siegbahn scale). }
$$

## INTRODUCTION

THE 2-meter curved crystal focusing spectrometer ${ }^{1}$ used in the precision determina$\operatorname{tion}^{2}$ of the wave-length of the $0.41-\mathrm{Mev}$ line of $\mathrm{Au}^{198}$ was designed so that it could be used for measurements in the x-ray as well as in the gamma-ray regions of the spectrum in order to make possible the direct comparison of gammaray wave-lengths with the better known x-ray wave-lengths. The calibration of the instrument has now been carried out using the $K$ series lines of tungsten reflected from the (310) planes of the curved quartz lanina.

The tungsten lines were chosen for this calibration work, in spite of the fact that the 2 -meter

[^0]curved crystal instrument admits of studying wave-lengths as long as the silver $K$ spectrum, (a) because it was deemed wise to utilize portions of the precision wave-length screw not too far removed from the $\gamma$-ray region for whose study the instrument was primarily designed, and (b) because of the ready availability of a tungsten target tube suitable for this purpose.
Since the $K$ series lines of tungsten have themselves not been studied as carefully or as extensively as some of the softer x-ray lines, a separate study with the 2-crystal spectrometer by W. J. West was also simultaneously undertaken in this laboratory to relate the $K \alpha_{1}$ line of tungsten to the $K \alpha_{1}$ line of molybdenum with all possible precision. Since the (310) planes of quartz were also used in this 2-crystal spectrometer study, a precision value of the grating constant, $d$, for (310) planes of quartz was thus also obtained.

The high resolving power of the 2 -meter


Fig. 1. Schematic illustration of the geometry used in two-crystal spectrometer precision Bragg angle determinations of $W K \alpha_{1}$ and Mo $K \alpha_{1}$ reflected from the (310) planes of quartz by transmission of the x-ray beam through 1-mm thick quartz plates. $X$ represents the focal spot of the x-ray tube, $S$ the first slit or stop. The full lines show the general position for the parallel rocking curve. The dotted lines show the position for the antiparallel rocking curve. The xenon-filled counter is at $C$ and is supported on an arm pivoted about the axis of crystal 2 . The instrument $I$ can as a whole be rotated on a pivot coincident with the pivot of crystal 1.
curved crystal spectrometer has made possible the complete resolution of the $K \beta$-doublet (Siegbahn notation: $K \beta_{1}, K \beta_{3}$ ) and the partial resolution-reported previously by Hudson and Vogt and by E. Ingelstam ${ }^{3}$ of the $K \gamma$-doublet (Siegbahn notation: $K \beta_{2}$ ). The high precision afforded by the instrument has made possible a more precise determination than has heretofore
been reported of the wave-lengths of the $K$ series lines relative to that of the $\alpha_{1}$ line, and of the various doublet separations. The high luminosity of the instrument has endowed the $K \delta$-line, resulting from a $K-\mathrm{O}_{I I} \mathrm{O}_{I I I}$ transition, with sufficient intensity to make possible an accurate measurement of its wave-length. In addition, a precise measurement of the position of the tungsten $K$ edge was made with a result that is appreciably different from that of previous measurements by Mack and Cork and by I. Manescu. ${ }^{4}$

## APPARATUS

The spectrometer and the multicellular counter used for the detection of the diffracted beam have been described by DuMond. ${ }^{1}$ In the present case the gamma-ray source holder which occupies a position on the focal circle was replaced by a narrow slit behind which, supported on the upper beam of the spectrometer, was situated a tungsten "shock proof" type x-ray tube operated at 140 kv and 5.0 ma . The measurements on the lines were made with 0.016 inch of lead in the beam in order to keep the peak counting rates from being excessively high. ${ }^{5}$ For the deter-

Table I. 2-crystal spectrometer Bragg angle determinations of Mo $K \alpha_{1}$ and $W K \alpha_{1}$ on quartz (310) planes.

| Mo $K \alpha_{1}$ line |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Deviation of beam by crystal No. 1 to | $\begin{aligned} & \text { Run } \\ & \text { No. } \end{aligned}$ | Parallel position reading | Worm wheel correction | Parallel corrected | Antiparallel position reading | Worm wheel correction | Antiparallel position corrected | $2 \theta$ | Bragg angle |
| Left | 1 | $73^{\circ} 13^{\prime} 54^{\prime \prime}$ | +4 $\frac{1}{4}^{\prime \prime}$ | $73^{\circ} 13^{\prime} 58 \frac{1}{4}^{\prime \prime}$ | $108^{\circ} 12^{\prime} 31 \frac{1}{2}^{\prime \prime}$ | +3 ${ }^{\frac{1}{\prime \prime}}$ | $108^{\circ} 12^{\prime} 343^{\prime \prime}{ }^{\prime \prime}$ | $34^{\circ} 58^{\prime} 36 \frac{1}{2}^{\prime \prime}$ | $17^{\circ} 29^{\prime} 18 \frac{1}{\prime \prime}^{\prime \prime}$ |
| Left | 2 | $73^{\circ} 13^{\prime} 55 \frac{1}{2}^{\prime \prime}$ | +4 ${ }^{\frac{1}{4}}{ }^{\prime \prime}$ | $73^{\circ} 13^{\prime} 599^{\frac{3}{4}}{ }^{\prime \prime}$ | $108^{\circ} 12^{\prime} 32 \frac{3}{3}^{\prime \prime}$ | +3 ${ }^{\frac{1}{4} \prime \prime}$ | $108^{\circ} 12^{\prime} 36^{\prime \prime}$ | $34^{\circ} 58^{\prime} 36 \frac{1}{2 \prime \prime}^{\prime \prime}$ | $17^{\circ} 29^{\prime} 188^{\prime \prime}{ }^{\prime \prime}$ |
| Left | 3 | $73^{\circ} 13^{\prime} 55 \frac{1}{2}^{\prime \prime}$ | +411 | $73^{\circ} 13^{\prime} 593^{\prime \prime}{ }^{\prime \prime}$ | $108^{\circ} 12^{\prime} 322^{\prime \prime}{ }^{\prime \prime}$ | +3 ${ }^{\prime \prime}$ | $108^{\circ} 12^{\prime} 35 \frac{1}{\frac{1}{\prime \prime}}$ | $34^{\circ} 58^{\prime} 35{ }^{\frac{3}{\prime \prime}}$ | $17^{\circ} 29^{\prime} 17 \frac{7^{\prime \prime}}{}$ |
| Left | 4 | $893{ }^{\circ} 14^{\prime} 1^{\prime \prime}$ | $-2^{\prime \prime}$ | $893^{\circ} 13^{\prime} 59^{\prime \prime}$ | $928^{\circ} 12^{\prime} 35 \frac{3}{}{ }^{\prime \prime}$ | $+1^{\prime \prime}$ | $928^{\circ} 12^{\prime} 363^{\frac{3}{4 \prime}}$ | $34^{\circ} 58^{\prime} 37 \frac{3}{4}{ }^{\prime \prime}$ | $17^{\circ} 29^{\prime} 18 \frac{7}{\prime \prime}^{\prime \prime}$ |
| Right | 5 | $928^{\circ} 12^{\prime} 39 \frac{3}{4}{ }^{\prime \prime}$ | $+1^{\prime \prime}$ | $928^{\circ} 12^{\prime} 403^{\frac{3}{4}}$ | $893^{\circ} 14^{\prime} 8^{\prime \prime}$ | $-2^{\prime \prime}$ | $893^{\circ} 14^{\prime} 6^{\prime \prime}$ | $34^{\circ} 58^{\prime} 34 \frac{3}{\frac{3}{4}}{ }^{\prime \prime}$ | $17^{\circ} 29^{\prime} 173^{\prime \prime}{ }^{\prime \prime}$ |
| Right | 6 | $927^{\circ} 58^{\prime} 433^{\prime \prime}$ | +1' ${ }^{\prime \prime}$ | $927^{\circ} 58^{\prime} 44 \frac{1}{4}^{\prime \prime}$ | $893{ }^{\circ} 0^{\prime} 8 \frac{1^{\prime \prime}}{}{ }^{\prime \prime}$ | $-2^{\prime \prime}$ | $893{ }^{\circ} 0^{\prime} 6 \frac{1}{2}{ }^{\prime \prime}$ | $34^{\circ} 58^{\prime} 37 \frac{3}{4}{ }^{\prime \prime}$ | $17^{\circ} 29^{\prime} 18 \frac{7^{\prime \prime}}{}$ |


| WK $\alpha_{1}$ line |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Right | 1 | $95^{\circ} 48^{\prime} 5.6^{\prime \prime}$ | $+5^{\prime \prime}$ | $95^{\circ} 48^{\prime} 10.6^{\prime \prime}$ | $85^{\circ} 38^{\prime} 35.3^{\prime \prime}$ | -3.5 ${ }^{\prime \prime}$ | $85^{\circ} 38^{\prime} 31.8^{\prime \prime}$ | $10^{\circ} 9^{\prime} 388^{\prime \prime}$ | $5^{\circ} 4^{\prime} 49.4{ }^{\prime \prime}$ |
| Right | 2 | $95^{\circ} 45^{\prime} 35.0^{\prime \prime}$ | $+5^{\prime \prime}$ | $95^{\circ} 45^{\prime} 40^{\prime \prime}$ | $85^{\circ} 36^{\prime} 5.5^{\prime \prime}$ | $-3.5^{\prime \prime}$ | $85^{\circ} 36^{\prime} 2^{\prime \prime}$ | $10^{\circ} 9^{\prime} 38.0^{\prime \prime}$ | $5^{\circ} 4^{\prime} 49.0^{\prime \prime}$ |
| Right | 3 | $95^{\circ} 25^{\prime} 4.6^{\prime \prime}$ | $+5^{\prime \prime}$ | $95^{\circ} 25^{\prime} 9.6^{\prime \prime}$ | $85^{\circ} 15^{\prime} 36.6^{\prime \prime}$ | -4.1" | $85^{\circ} 15^{\prime} 32.5^{\prime \prime}$ | $10^{\circ} 9^{\prime} 37.1^{\prime \prime}$ | $5^{\circ} 4^{\prime} 48.6^{\prime \prime}$ |
| Right | 4 | $915^{\circ} 13^{\prime} 4.0^{\prime \prime}$ | $+2^{\prime \prime}$ | $915^{\circ} 13^{\prime} 6.0^{\prime \prime}$ | $905^{\circ} 3^{\prime} 37.8^{\prime \prime}$ | -9.4" | 905 ${ }^{\circ} 3^{\prime 28.4}{ }^{\prime \prime}$ | $10^{\circ} 9^{\prime} 37.6^{\prime \prime}$ | $5^{\circ} 4^{\prime} 48.8^{\prime \prime}$ |
| Right | 5 | $915^{\circ} 13^{\prime} 4.8^{\prime \prime}$ | +2' | $915^{\circ} 13^{\prime} 6.8^{\prime \prime}$ | $905^{\circ} 3^{\prime} 38.4 \prime$ | -9.4' | $905^{\circ} 3^{\prime} 29.0^{\prime \prime}$ | $10^{\circ} 9^{\prime} 37.8^{\prime \prime}$ | $5^{\circ} 4^{\prime} 48.9^{\prime \prime}$ |
| Left | 6 | $905^{\circ} 3^{\prime} 36.6^{\prime \prime}$ | -9.4' | $905^{\circ} 3^{\prime} 27.2^{\prime \prime}$ | $915^{\circ} 13^{\prime} 1.3^{\prime \prime}$ | $+2^{\prime \prime}$ | $915^{\circ} 13^{\prime} 3.3^{\prime \prime}$ | $10^{\circ} 9^{\prime} 36.1^{\prime \prime}$ | $5^{\circ} 4^{\prime} 48.1^{\prime \prime}$ |
| Left | 7 | $905^{\circ} 3^{\prime} 34.6^{\prime \prime}$ | -9.4' | $905^{\circ} 3^{\prime} 25.2^{\prime \prime}$ | $915^{\circ} 13^{\prime} 2.6^{\prime \prime}$ | $+2^{\prime \prime}$ | $915^{\circ} 13^{\prime} 4.6^{\prime \prime}$ | $10^{\circ} 9^{\prime} 39.4{ }^{\prime \prime}$ | $5^{\circ} 4^{\prime} 49.7^{\prime \prime}$ |
| Left | 8 | 85 ${ }^{\circ} 3^{\prime} 29.6^{\prime \prime}$ | $-4.6^{\prime \prime}$ | $85^{\circ} 3^{\prime} 25.0^{\prime \prime}$ | $95^{\circ} 12^{\prime} 58.6^{\prime \prime}$ | +5.4" | $95^{\circ} 13^{\prime} 4^{\prime \prime}$ | $10^{\circ} 9^{\prime} 39.0^{\prime \prime}$ | $5^{\circ} 4^{\prime} 49.5^{\prime \prime}$ |
| Left | 9 | $85^{\circ} 3^{\prime} 30.3^{\prime \prime}$ | -4.6" | $85^{\circ} 3^{\prime} 25.7^{\prime \prime}$ | $95^{\circ} 12^{\prime} 59.0^{\prime \prime}$ | +5.4' | $95^{\circ} 13^{\prime} 4.4^{\prime \prime}$ | $10^{\circ} 9^{\prime} 38.7^{\prime \prime}$ | $5^{\circ} 4^{\prime} 49.3^{\prime \prime}$ |

Average value of $\theta$ for $W K \alpha_{1}$ on (310) planes of quartz $5^{\circ} 4^{\prime} 49.0^{\prime \prime} \pm 0.2^{\prime \prime}$.

[^1]

Fig. 2. Typical 2-crystal spectrometer rocking curve of molybdenum $K \alpha_{1}$ line (run No. 1) reflected from (310) planes of quartz in first-order, parallel, and antiparallel positions. The ordinate numbers on the parallel and antiparallel rocking curves give the number of counts in two minutes. The vertical heights of the rectangular dots on the antiparallel curve are indicative of the statistical uncertainty of counting (square root of total counts). On the parallel rocking curve the statistical uncertainty is not thus known.
mination of the $K$ edge the lead absorber was replaced by tungsten foil 0.0034 inch thick. The multicellular counter consisted of four sections with partitions (cathodes) of 100 -mesh monel metal gauze plated with a layer of silver about 0.001 inch thick.

In addition to the 2 -meter curved crystal spectrometer, a precision 2-crystal spectrometer ${ }^{6}$ with specially lapped and calibrated worm wheels was used by W. J. West to determine with high absolute precision the Bragg angles of reflection for both the tungsten and molybdenum $K \alpha_{1}$ lines. Two optically flat $1-\mathrm{mm}$ thick plates of quartz (in all respects identical to, and cut from, the same crystal specimen as the lamina used in the 2 -meter curved crystal spectrometer) were mounted on the 2-crystal spectrometer, one over each pivot, so that the (310) reflecting planes of the quartz (normal to the surfaces of the lamina) were accurately parallel to the axis of each pivot (as shown by photographic tests of reflections from both sides of these planes with films placed some 2 meters distant from the crystals). Great care was taken to mount the quartz plates without subjecting them to mechanical strain

[^2]which might bend them. The reflections were studied with the beam transmitted through the plates. A Geiger-Müller counter filled with 10 cm of xenon and a small amount (about 0.3 cm ) of petroleum ether was used and found very satisfactory for the intensity measurements. Crystal No. 1 was left stationary in the appropriate position to reflect the $K \alpha_{1}, \alpha_{2}$ lines, the geometry of lead stops and $x$-ray tube being such as to select these lines alone. Crystal No. 2 was rotated so that rocking curves both in the $(1,1)$ and $(1,-1)$ or "antiparallel" and "parallel" positions were observed, the antiparallel curves being the profiles of the $K \alpha_{1}$ line, and the angular displacement between the $(1,1)$ and $(1,-1)$ settings then gave the precision data from which the Bragg angle could be computed. The primary purpose of this 2-crystal spectrometer work was to establish with high precision the ratio of the wave-lengths of the tungsten $K \alpha_{1}$ line to that of the Mo $K \alpha_{1}$ line. This last has been determined with such care that it may well be regarded as the standard for the entire Siegbahn scale of wavelengths.

## PRECISION STANDARDIZATION OF TUNGSTEN $K \alpha_{1}$ WITH THE 2-CRYSTAL SPECTROMETER

Six complete and independent sets of measurements of the Bragg angle for Mo $K \alpha_{1}$, and nine


Fig. 3. Typical 2-crystal spectrometer rocking curve of tungsten $K \alpha_{1}$ line (run No. 4) reflected from (310) planes of quartz in first-order, parallel, and antiparallel positions. The ordinate numbers on the parallel rocking curve give the number of counts in 2 minutes, those on the antiparallel curve the number in 5 minutes. The vertical heights of the rectangular dots on the antiparallel curve are indicative of the statistical uncertainty of counting (square root of total counts). On the parallel rocking curve the statistical uncertainty is not thus shown.
similar measurements for $\mathrm{W} K \alpha_{1}$, were made in such a way as to utilize different parts of the calibrated precision worm wheels for the angle measurements. Each Bragg angle measurement consisted in the complete delineation of a parallel and an antiparallel position rocking curve. Figure

1 shows schematically the geometry of the parallel and antiparallel settings. In some of the measurements the beam was deviated to the left by crystal No. 1, and in others to the right. The number and variety of the worm wheel settings was also increased by turning crystal No. 2

Table II. Tungsten $K$ spectrum measured with 2-meter curved crystal spectrometer.

|  | Siegbahn notation | $\alpha_{2}$ | $\alpha_{1}$ | $\beta_{3}$ | $\beta_{1}$ | $\beta_{2}{ }^{I I}$ | $\beta_{2}{ }^{I}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Sommerfeld notation | $\alpha_{2}$ | $\alpha_{1}$ | $\beta_{2}$ | $\beta_{1}$ | $\gamma_{2}$ | $\gamma_{1}$ | $\delta$ |
|  | Transition | $K-L_{1 I}$ | $K-L_{111}$ | $K-M_{11}$ | $K-M_{I I I}$ | $K-N_{I I}$ | $K-N_{1 I I}$ | $K-O_{1 I, I I I}$ |
| 1 | Run 1 | 213.451 | 208.639 | 184.824 | 184.020 |  |  | K Or,ill |
| 2 | Run 2 | 213.425 | 208.611 | 184.811 | 184.002 | 179.242 | 179.068 | 178.081 |
| 3 | Run 3 |  | 208.619 |  | 184.006 |  |  | 178.081 |
| 4 | Average x.u. nominal |  |  |  |  |  |  |  |
| 5 | Average x.u. assuming | 213.387 | 208.575 | 184.772 | 183.967 | 179.242 179.212 | 179.068 179.038 | 178.081 178.052 |
|  | West's value $\alpha_{1}$ | $\pm 0.010$ | $\pm 0.008^{*}$ | $\pm 0.020$ | $\pm 0.020$ | $\pm 0.020$ | $\pm 0.020$ | $\pm 0.020$ |
| Other determinations for comparison: |  |  |  |  |  |  |  |  |
| 6 | E. Ingelstam ${ }^{\text {a }}$ | 213.382 | 208.571 | 184.795 | 183.991 | 179.232 | 179.049 | 178.073 |
| 7 | Hudson and Vogt ${ }^{\text {a }}$ | 213.38 | 208.57 | 184.73 | 183.93 | 179.13 | 178.95 | 177.99 |
| 8 | J. H. Williams ${ }^{\text {b }}$ | 213.37 | 208.56 | 184.75 | 183.97 |  |  |  |
| 9 | Duane and Stenström ${ }^{\text {c }}$ | 213.48 | 208.67 | 184 | 6 |  |  |  |
| 10 | Siegbahn ${ }^{\text {d }}$ | 213.52 | 208.85 |  |  |  |  |  |
| 11 | Stephenson and Cork ${ }^{\text {e }}$ | 213.45 | 208.62 |  |  |  |  |  |
| 12 | Rechou ${ }^{\text {f }}$ | 213.41 | 208.50 | 184 |  |  |  |  |

[^3]${ }_{b}$ S See reference 3 .

- See reference 13.
${ }^{d}$ M. Siegbahn, Physik. Zeits. 20, 533 (1919).
- B. R. Stephenson and J. M. Cork, Phys. Rev. 27, 138 (1926).
${ }^{1}$ G. Rechou, Comptes Rendus 180, 1107 (1925).
Note.-Line 4 in the above table gives wave-lengths in instrumental screw divisions averaged by columns. These values were not used, however
in computing line 5 . The values in line 5 (which we regard as our final best results) were computed in the following way. The wave-length value of $\alpha_{1}$ in line 4 in nominal screw divisions was compared with $W$. J. West's absolute determination of this wave-length so as to yield the instrumenta scale factor, 1.00023 screw divisions per x.u. Wave-length differences for each run between all lines and the WK $\alpha$, line of that run were then determined in nominal scale units. The mean values of these differences for the different runs in which they occurred were then converted to $x$.u. using the above scale factor. These differences were then combined by adding or subtracting them from West's value of $\alpha_{1}$ to give the other values in We
 residual irreproducibility in the readings, exert, by this procedure, a minimum effect on the results.
through $180^{\circ}$ for some of the Bragg angle determinations so that in such cases the entry and exit sides of the quartz plate were interchanged. This 2-crystal instrument ${ }^{6}$ is provided with four independent motions of rotation actuated by worm wheel drives which permit (a and b) precision settings of the 2 -crystal tables to about $\frac{1}{4}$ second of arc, (c) angular setting of the instrument as a whole about an axis coincident with that of crystal No. 1 relative to the primary x-ray beam, and (d) angular setting of the arm supporting the ion chamber or counter about an axis coincident with crystal No. 2. This extreme flexibility greatly facilitates the determinations here described since, for any desired configuration, the correct angular position for all the elements of the instrument can be approximately calculated and set up on the four worm wheels as soon as the settings for one particular configuration have been accurately established.
The very small grating content of the quartz (310) planes (only slightly more than one-third as great as that for the cleavage planes of calcite,
for example) is of considerable value in increasing the precision of the work since the angles measured are for this reason much larger relative to the precision of the worm wheels. Table I lists the results of the separate measurements of the Bragg angles. The corrections made for the residual errors of the worm wheels are shown. The final result of these measurements gives for the wave-length of the tungsten $K \alpha_{1}$ line in x-units (on the Siegbahn scale for which the $K \alpha_{1}$ line of molybdenum has the wave-length value 707.831 x.u.)

$$
\lambda w K \alpha_{1}=208.575 \pm 0.008 \text { x.u. }
$$

This result agrees satisfactorily with a determination by Ingelstam, ${ }^{3}$ who found $\lambda_{W K \alpha_{1}}$ $=208.571$ x.u., and it compares favorably also with measurements by J. H. Williams ${ }^{7}-208.56$ x.u.

## the grating constant for the quartz (310) PLANES

It can be shown that no correction for refractive index is needed for the case of a beam


Fig. 4. Tungsten $K \boldsymbol{\alpha}$ lines obtained by reflection from opposite sides of the (310) planes of quartz with the 2 -meter focusing curved crystal spectrometer. The effective "window curve" of the spectrometer is narrow-of the order of 0.1 x.u.-resulting in line widths only slightly greater than the natural line widths. The vertical heights of the small rectangles are indicative of the statistical uncertainty of counting-square root of the number of counts. The abscissa scale readings are in nominal $x$-units as read on the wave-length drum of the spectrometer. The "zero" of these readings is not exactly centered on the zero wave-length position, and the true wave-lengths are to be obtained by taking half the difference between the right and left-hand positions. A multiplying factor differing slightly from unity must also be applied to these figures to convert nominal drum x.u. to true x.u. The wave-length scale is broken between the $\alpha_{1}$ and $\alpha_{2}$ lines which would otherwise be separated more than three times as far as here shown.
${ }^{7}$ J. H. Williams, Phys. Rev. 40, 791 (1932).


Fig. 5. Tungsten $K \beta_{1}, \gamma$ - and $\delta$-lines obtained by reflection from opposite sides of the (310) planes of quartz with the 2 -meter focusing curved crystal spectrometer. The general remarks in the caption of Fig. 4 apply here also.
of x -rays transmitted through a plate with reflection from internal planes if these latter lie normal to the entry and exit faces. ${ }^{8}$ The observations were made at a temperature of $22^{\circ} \mathrm{C}$. They indicate for this temperature a grating constant for the (310) planes of quartz, $d_{22^{\circ}}=1177.705$ x.u. Reducing this to the usually accepted standard value of $18^{\circ} \mathrm{C}$ by applying the thermal expansion coefficient ${ }^{9}$ of $14.5 \times 10^{-6}$ per degree C for expansion normal to the optic axis, we obtain for the (310) planes

$$
d_{18^{\circ}}=1177.637 \pm 0.02 \text { x.u. (Siegbahn scale). }
$$

This result agrees well with a value of 1177.64 x.u. calculated from measurements made by Bergquist ${ }^{10}$ on the (100) or prism planes of quartz, and it is also in agreement with the value 1177.6 given by E. Ingelstam. ${ }^{11}$

Figures 2 and 3 show two sets of the 2-crystal spectrometer rocking curves in (1,1) and ( $1,-1$ ) positions typical, of those from which the data of Table I were obtained, Fig. 2 being taken from the runs with molybdenum radiation and Fig. 3 from the runs with tungsten radiation. The extreme narrowness of the parallel position rocking curves for these quartz planes (about 2.8 seconds of arc full width at half-maximum

[^4]height in the case of tungsten) is remarkable. ${ }^{12}$ The breadths of the lines in the antiparallel case are so much greater than the parallel position rocking curves that they may be regarded as virtually perfect delinations of the spectral line profiles for $\mathrm{W} K \alpha_{1}$ and Mo $K \alpha_{1}$.

## CALIBRATION OF THE 2-METER CURVED CRYSTAL SPECTROMETER AND RESULTING WAVELENGTH MEASUREMENTS OF THE TUNGSTEN $K$ SPECTRUM

Measurements on the diffracted x-ray beams were made on both sides of the (310) planes of the curved quartz crystal.
The curved crystal gamma-ray spectrometer is so designed ${ }^{1}$ that the wave-length of the diffracted radiation entering the detector may be determined directly from the reading of a drum and vernier attached to a precision screw which drives the source holder along the focal circle. The geometry of the mechanism is such that the displacement of this screw is directly proportional to the sine of the Bragg angle of the diffracted radiation entering the detector and hence to its wave-length. Thus, the difference between the reading on the scale when the spectrometer is set to detect a given wave-length after reflection from one side of the crystal planes and that for reflection from the opposite side of these planes gives twice the wave-length of the radia-

[^5]tion. The dimensions of the spectrometer were so chosen that one turn of the screw is almost exactly equal to one $\mathrm{x} . \mathrm{u}$. By means of the vernier, readings of wave-length to 0.001 x.u. may easily be made. The screw of the spectrometer has been calibrated for deviations from linearity over the entire range from +300 x.u. to -300 x.u. by comparison with a National Bureau of Standards calibrated glass scale using a 100 -power microscope and working to $0.001 \mathrm{x} . \mathrm{u}$. The chief departures from linearity are (a) a quasi-periodic error (with one rotation of the screw as the fundamental period) whose profile changes slowly as one progresses along the screw and (b) an aperiodic error. It may be said, in general, that the deviation from linearity from both these errors nowhere exceeds $0.020 \mathrm{x} . \mathrm{u}$. over the range +300 x.u. to $-300 \mathrm{x} . \mathrm{u}$. By using the calibration curve to make corrections, it is estimated that readings are probably correct to within $0.020 \mathrm{x} . \mathrm{u}$.

The precise calibration of the scale of the instrument was made by observing the displacement of the screw between the two settings for the $\mathrm{WK} \alpha_{1}$ line. An average of three such determinations yielded a wave-length in scale divisions of $208.623 \pm 0.008$. If the wave-length of the $\mathrm{W} K \alpha_{1}$ line is taken as $208.575 \mathrm{x} . \mathrm{u}$. (W. J. West's 2-crystal spectrometer value), the scale factor for the spectrometer becomes 1.00023 divisions per x.u. Using this value, the wave-lengths of the other $K$ series lines and that of the $K$ edge may be determined. The results obtained in this way are shown in Table II. For comparison, the values obtained by other observers are indicated. The measurements of Hudson and Vogt were relative measurements based on the assumption of the $\alpha_{1}$ and $\alpha_{2}$ wave-length values obtained by Duane and Stenström. ${ }^{13}$ For the purpose of this comparison they have been restated, assuming the validity of the results obtained by W. J. West for the $\mathrm{W}-K \alpha_{1}$ wave-length. All values shown in the table are in x.u. (Siegbahn scale) based on the convention that the effective grating space in the first order for calcite at $18^{\circ} \mathrm{C}$ is 3029.04 x.u. Figures 4 and 5 show graphically the results of one complete run over the tungsten $K$ spectrum on the two sides of the atomic reflecting planes. The vertical strokes at each observed point are

[^6]indicative of the statistical uncertainty (the square root of the total number of counts observed). Large gaps are necessarily left in the wave-length scale between lines to permit reproducing the plot in reasonable compass.

## DOUBLET SEPARATIONS

The $4.812 \pm 0.007-\mathrm{x} . \mathrm{u}$. separation between the components of the $\alpha$-doublet is in good agreement with that obtained by Ingelstam by Williams and by Duane and Stenström, and it is to be expected from the energy difference between the $L_{I I}$ and $L_{I I I}$ levels.
The separation of $0.805 \pm 0.001 \mathrm{x} . \mathrm{u}$. between the components of the $\beta$-doublet agrees well with that obtained by Seemann, ${ }^{14}$ and it is also in good agreement with the measurements of Hudson and Vogt, and of Ingelstam. It differs from the 0.78 x.u. separation reported by Williams, whose measurements may be in error because of a failure completely to resolve the doublet. The wave-length separation of this doublet computed from the more accuately


Fig. 6. Separation of $\mathrm{W}-K$ edge and $\mathrm{W}-K \delta$-line, using metallic tungsten foil absorber 0.0034 inch thick. Curve 1 is the spectrum without any external absorber. The effect of the $K$ absorption edge of the tungsten in the target itself can be clearly seen superposed on the $\delta$-line. Curve 2 is the spectrum with the absorbing foil in the beam normalized to coincide with curve 1 on the left-hand or soft side. Curve 3 plots the ratio of 2 to 1 . The wave-length position assigned as the "edge" has been chosen as nearly as possible at the point of inflection of 3 . Curve 4 is corrected to remove target absorption, using curve 3 for this purpose.
${ }^{14}$ H. Seemann, Zeits. f. Physik 73, 87 (1931).

Table III.

| Siegbahn notation | $\alpha_{1}$ | $\alpha_{1}$ | $\boldsymbol{\beta}_{3}$ | $\beta_{1}$ | $\beta_{2}{ }^{\text {II }}$ | $\beta_{2}{ }^{I}$ |  | Abs. edge. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sommerfeld notation | $\alpha_{2}$ | $\alpha_{1}$ | $\beta_{2}$ | $\beta_{1}$ | $\gamma 2$ | $\gamma_{1}$ | $\delta$ | Abs. edge. |
| Transition | $K-L_{1 I}$ | $K-L_{\text {III }}$ | $K-M_{1 I}$ | $K-M_{1 I I}$ | $K-N_{I I}$ | $K-N_{\text {III }}$ | $K-O_{I I, ~}^{1 I I}$ | $K-\infty$ |
| Wave-length x.u. (Siegbahn scale) | $\begin{array}{r} 213.387 \\ \pm 0.010 \end{array}$ | $\begin{array}{r} 208.575 \\ \pm 0.008 \end{array}$ | $\begin{array}{r} 184.772 \\ \pm 0.020 \end{array}$ | $\begin{array}{r} 183.967 \\ \pm 0.020 \end{array}$ | $\begin{array}{r} 179.212 \\ \pm 0.020 \end{array}$ | $\begin{array}{r} 179.038 \\ \pm 0.020 \end{array}$ | $\begin{array}{r} 178.052 \\ \pm 0.020 \end{array}$ | $\begin{array}{r} 177.947 \\ \pm 0.020 \end{array}$ |
| Wave-length A | $\begin{array}{r} 0.213818 \\ \pm 0.000010 \end{array}$ | $\begin{array}{r} 0.208996 \\ \pm 0.000008 \end{array}$ | $\begin{array}{r} 0.185145 \\ \pm 0.000020 \end{array}$ | $\begin{array}{r} 0.184339 \\ \pm 0.000020 \end{array}$ | $\begin{array}{r} 0.179574 \\ \pm 0.000020 \end{array}$ | $\begin{array}{r} 0.179400 \\ \pm 0.000020 \end{array}$ | $\begin{array}{r} 0.178412 \\ \pm 0.000020 \end{array}$ | $\begin{array}{r} 0.178306 \\ \pm 0.000020 \end{array}$ |

measured $L_{I}-M_{I I}$ and $L_{I}-M_{I I I}$ transitions is 0.807 x.u.

The partial resolution of the $K \gamma$-doublet is apparent in Fig. 5. E. Richard Cohen has obtained the two components shown in the figure which give a composite curve closely fitting the experimental curve except for a slight deviation at the wings. The best fit was obtained for components which differ in wave-length by $0.17 \mathrm{x} . \mathrm{u}$. and which have an intensity ratio of 2 to 0.94 . The small departure from the expected 2 to 1 ratio may be due to slightly greater absorption of the longer wave-length component in the target, lead filter, and counter window. Our value for the doublet separation is in good agreement with the value of $0.18 \mathrm{x} . \mathrm{u}$. reported by Hudson and Vogt and 0.183 x.u. reported by Ingelstam, and in excellent agreement with the value of 0.171 x.u. computed from the measured wave-lengths of the $L_{I}-N_{I I}$ and $L_{I}-N_{I I I}$ transitions.

## $K$ EDGE

The effect of passage through the $K$ absorption edge of tungsten is apparent from the distorted profile of the $K \delta$-line and the obviously lower background value on the short wave-length side. Since there was no tungsten in the x-ray beam after its emergence from the tube, the observed effect must be associated with the passage of the beam through a thin layer of the target. In order to correct the line profile for the purpose of determining the wave-length value at its center and to determine the position of the absorption
edge, a 0.0034 -inch thick tungsten foil was placed in the beam, and readings of the intensity were made both with and without this additional tungsten. The ratio of the intensity measured with the tungsten foil in the beam to that without the foil was plotted against wave-length, and the point of steepest slope of the curve was taken as the position of the edge. Figure 6 illustrates how this was done. In this way the edge was found to be 0.105 x.u. from the center of the $\delta$-line, giving the wave-length associated with the tungsten $K$ absorption discontinuity as 177.947 x.u. This value differs by 0.27 x.u. from the value of 178.22 obtained by Mack and Cork, ${ }^{4}$ and by 0.063 x.u. from the value 178.01 of Manescu. ${ }^{4}$

## CONCLUSIONS

The curved crystal spectrometer employed in this work is easily capable of giving wave-length separations to within 0.01 x.u. Since the wavelength of the $\mathrm{WK} \alpha_{1}$ line as measured by W. J. West is of comparable precision, we may tabulate (Table III) values of the tungsten $K$ series lines good to 0.02 x.u. or better. The values are also given in angström units, following the recent recommendation of the X-Ray Analysis Group of the Institute of Physics (England) that $\lambda(\mathrm{A})=1.00202 \lambda(k \mathrm{x} . \mathrm{u}$.$) . This factor differs only$ slightly from that given by DuMond and Cohen ${ }^{15}$ based chiefly on the work of J. A. Bearden for converting wave-lengths in $\mathrm{x} . \mathrm{u}$. to grating wavelengths.

[^7]
[^0]:    * Work supported by contract with the Office of Naval Research.
    ${ }^{* *}$ Now at Temple University, Philadelphia, Pennsylvania.
    ${ }^{1}$ J. W. M. DuMond, Rev. Sci. Inst. 18, 626 (1947).
    ${ }^{2}$ J. W. M. DuMond, D. A. Lind, and B. B. Watson, Phys. Rev. 73, 1392 (1948).

[^1]:    ${ }^{3}$ H. C. Hudson and H. G. Vogt, Proc. Nat. Acad. Sci. 19, 444 (1933); E. Ingelstam, (4) No. 5 Nova Acta Reg. Soc. Sci., Upsal. (1936).
    ${ }^{4}$ J. E. Mack and J. M. Cork, Phys. Rev. 30, 741 (1927); I. Manescu, Comptes Rendus 216, 732 (1943).
    ${ }^{5}$ This lead filter can be shown to produce a change in absorption with wave-length in going across the profile of a line (say 0.2 x.u.) which results in a difference of less than 1 percent in the observed intensity on the two extreme sides, and the shift in line position will hence be negligible from this cause.

[^2]:    ${ }^{6}$ For a complete description of this instrument, together with the methods of correcting the worm wheels and calibrating their residual errors, see Jesse W. M. DuMond, Rev. Sci. Inst. 8, 112 (1937).

[^3]:    * This value with its probable error is from the 2-crystal spectrometer work of W. J. West.

[^4]:    ${ }^{8}$ See, for example, Jesse DuMond, Rev. Sci. Inst. 18, 629 (1947), Fig. 4 and accompanying text.
    ${ }^{9}$ Determined by A. H. Jay, Proc. Roy. Soc. 142, 237 (1933).
    ${ }^{10}$ O. Bergquist, Zeits. f. Physik 66, 496 (1930).
    ${ }^{11}$ E. Ingelstam, Arkiv. f. Mat., Astr. o. Fys. 27B, No. 4 (1939).

[^5]:    12 Work is now in progress to determine reflection coefficients of these 310 planes, using the set-up here described.

[^6]:    ${ }^{13}$ W. Duane and W. Stenström, Proc. Nat. Acad. Sci. 6, 477 (1920).

[^7]:    ${ }^{15}$ J. W. M. DuMond and E. R. Cohen, Rev. Mod. Phys. 20, 82 (1948).

