Multiple Compton Scattering

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An experimental and theoretical investigation has been made of the γ-radiation in water produced by a uniform distribution of radioactive cobalt. The intensity and spectral distribution of the radiations are determined essentially by the multiple Compton scattering that takes place in the liquid. An approximate quantitative treatment is presented in which the multiple scattering is considered as a succession of steps which are the same for every emitted γ-ray. Although the method is quite crude, the numerical results both for the absolute counting rate of a submerged counter and for the effect of shielding around the counter are in close agreement with experiment.

I. INTRODUCTION

WHILE much experimental work[1] has been done on the scattering of γ-radiation, the study of multiple scattering has been neglected. In the arrangements usually considered, a plane parallel beam falls upon a target. The geometrical factors thereby introduced make an analysis of the results difficult.[2]

A substantial simplification is possible by use of a γ-ray source which is homogeneously distributed throughout an infinite medium. The effect of multiple scattering then manifests itself in the intensity and spectral distribution of the radiation which is the same at every point in the medium. In the next section of this paper, we will show that these quantities can be related, in a simple way, to the strength of the source and the density of electrons in the medium. In Section III an equally simple experimental arrangement is described which effectively provides a γ-ray source distributed throughout an infinite medium. In Section IV the measurements of intensity and spectral distribution are compared with the theoretical values.

II. THEORY

Suppose that there are η-γ-ray sources per cubic cm distributed throughout a uniform medium and that each source emits λ-quanta of energy E_0 per second. Each quantum is scattered many times by the electrons of the medium,[3] losing energy in each scattering process according to the familiar Compton formula, until it is finally absorbed photoelectrically. The quanta existing in the medium can then be divided into groups which are determined by the number of scatterings experienced, i.e., quanta in the k'th group have been scattered k times. The quanta in each group will be distributed in energy about some mean energy, E_k.

Every quantum will travel a mean distance, l, between scattering processes. This distance depends upon the energy and can be computed from the well-known Klein-Nishina formula.[4] It is plotted as a function of the energy for the electron density of water (3.37×10^{23} electrons/cm^3) in Fig. 1. It is inversely proportional to the electron density.

The Klein-Nishina cross section, σ, can be decomposed into the sum $\sigma_a+\sigma_s$ where σ_s/σ is the fraction of incident energy given to the scattered quantum while σ_a/σ is the fraction of incident energy given to the recoil electron. The ratio σ_s/σ can be obtained by integrating the scattered intensity as given by the Klein-Nishina formula over all angles and dividing the result by the product of the total cross section and the incident energy. A graph of this ratio against energy is shown in Fig. 2.

We will now assume that each quantum passes through a succession of energy values given by the mean energies of the various groups. Thus the actual problem is replaced by one in

[1] A. H. Compton and S. K. Allison, *X-Rays in Theory and Experiment* (D. Van Nostrand Company, Inc., New York, 1930), Ch. III.
[2] Hirshfelder, Magee, and Hull, Phys. Rev. **73**, 8 (1948).
[3] It is assumed that the energy E_0 is sufficiently low that pair production is negligible.
[4] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1944), Ch. III.

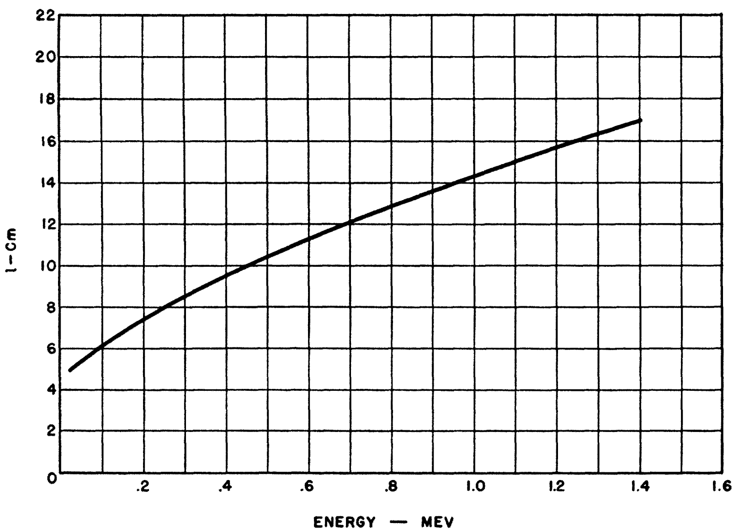

FIG. 1. Mean distance, l, between scatterings.

which all quanta of the kth group have the mean energy E_k. The accuracy of this procedure will be discussed in Section IV. With this assumption the mean energy of the $k+1$st group is determined from the energy of the kth group by

$$E_{k+1} = (\sigma_s/\sigma)E_k, \tag{1}$$

where σ_s/σ is evaluated for the energy E_k. The succession of energy values determined by Eq. (1) for an initial energy $E_0 = 1.2$ Mev is given in Table I.

A detector immersed in the medium will, according to an obvious definition of efficiency, ϵ, register quanta of the kth group at a rate R_k given by

$$R_k = \int_{\Delta E_k} \epsilon(E)dE \int \bar{F}_k \cdot d\bar{\sigma}, \tag{2}$$

where $\bar{F}_k$ is the incident flux of quanta of the k'th group, and ΔE_k is an energy interval that

TABLE I. Scattered energies.

Energy Mev	σ_s/σ	ϵl	$\epsilon l t_{\text{Al}}$	$\epsilon l t_{\text{Pb1}}$	$\epsilon l t_{\text{Pb2}}$
1.2	0.545	106	105	100	95
0.654	0.63	37	36.2	34.6	32.1
0.412	0.64	17	16.5	12.4	9.3
0.284	0.73	11	10.5	6.0	3.42
0.207	0.79	9.5	9.0	3.24	1.17
0.164	0.825	10.5	9.9	2.38	0.57
0.135	0.845	11	10.3	1.55	0.24
0.113	0.86	12	11.2	1.01	0.09
0.098	0.875	12	11.1	0.22	
0.086	0.89	11	10.1		
		237	229	161.4	141.1

contains all quanta of the k'th group. The surface integral is extended over the active surface of the detector. It may be readily evaluated for a detector of axial symmetry whose area projected perpendicularly to the axis of symmetry is A. Let $N_k = \int n_k dE$ where n_k is the number of quanta of the kth group per unit energy interval generated in each cubic cm per second. Then it is readily shown that

$$\int \bar{F}_k \cdot d\bar{\sigma} = \frac{\pi}{4} n_k l A. \tag{3}$$

Substituting Eq. (3) in (2) and summing over k, we find

$$R = \sum_0^\infty R_k = \frac{\pi}{4} A \sum_0^\infty \int_{\Delta E_k} \epsilon n_k l dE$$

$$= \frac{\pi A}{4} \sum_0^\infty N_k l_k \epsilon_k. \tag{4}$$

In the last step the assumption that all quanta of the kth group have the same energy has been used, and the subscripts on l and ϵ mean that values of these quantities at the energy E_k are to be used.

Usually the photoelectric absorption of the detector walls causes the response of the detector to vanish at an energy sufficiently high that the photoelectric absorption of the medium is negligible. Then, in the region of interest, N_k is constant and must be equal to $\eta\lambda$, the number of quanta emitted by the γ-ray source per cm^3

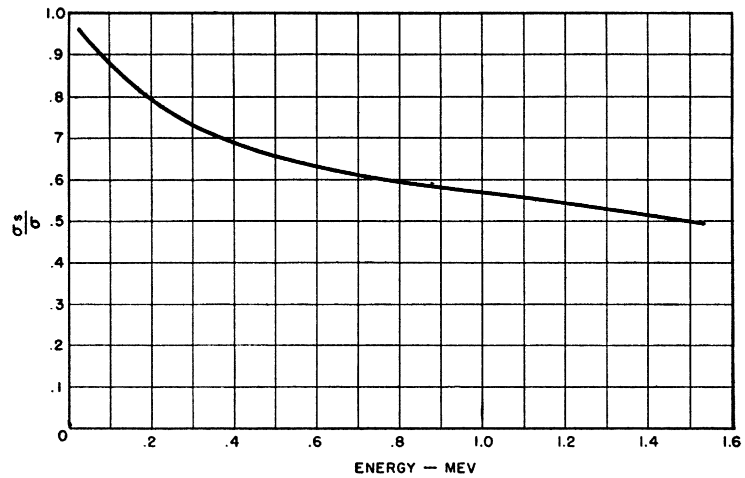

FIG. 2. Fraction of incident energy, σ_s/σ, transmitted to the scattered quantum.

per sec. Hence, Eq. (4) may be written as

$$R=(\pi A/4)\eta\lambda \sum_0^m l_k\epsilon_k. \quad (5)$$

The upper limit of the sum is determined by setting E_m equal to the energy at which the detector ceases to respond.

Equation (5) is the desired relation between the intensity of the radiation, the source strength, and the properties of the medium. Of course it also involves the properties of the detector through the quantity ϵ. If we were only interested in the flux of γ-radiation through a cylinder, ϵ_k should be set equal to unity. The various terms in the sum of Eq. (5) correspond to the mean energies of the successive groups. Hence, the part of the response that arises from quanta in a given energy interval is comprised by the terms in this sum which lie within this energy interval. Hence, Eq. (5) also contains the spectral distribution of the radiation.

III. EXPERIMENT[5]

A homogeneously distributed source of γ-radiation may be realized by dissolving a salt of radio-isotope in water. We have used Co^{60}, which emits two γ-rays per disintegration of 1.2-Mev mean energy. Thus the number of γ-rays emitted per cm^3 per second is $2\eta\lambda$, which should replace $\eta\lambda$ in our previous equations.

A Cu Geiger-Müller counter of $\frac{1}{32}$-in. wall thickness, 1-in. diameter and 8.83-in. effective length was used for a detector. The anode was a 3-mil tungsten wire, and the tube was filled with a five to one ratio of argon to ether to a pressure of 30 cm Hg. The efficiency was measured with several standard sources with the results shown in Table II. The last column of the table gives the results of Bradt *et al.*[6] for a Cu counter, and our values differ by less than the experimental error. Bradt's results for a Cu tube are shown in Fig. 3.

Measurements were first made of the background counting rate at different positions in a

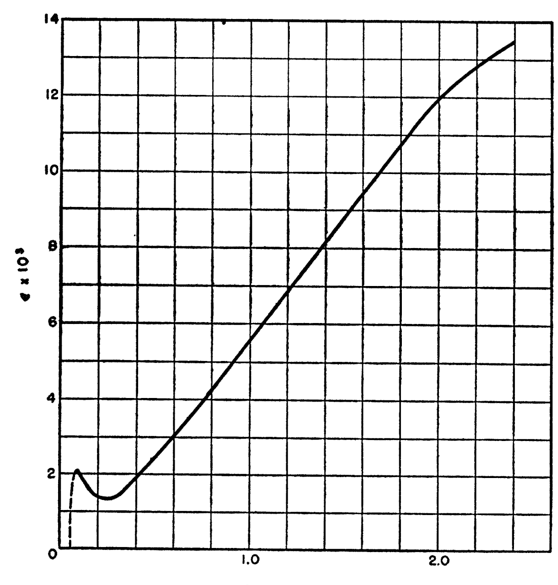

FIG. 3. Efficiency of a Cu counter.

[5] We are indebted to C. Fiddler for assistance in the experimental work.

[6] H. Bradt, *et al.* Helv. Phys. Acta **19**, 77 (1946).

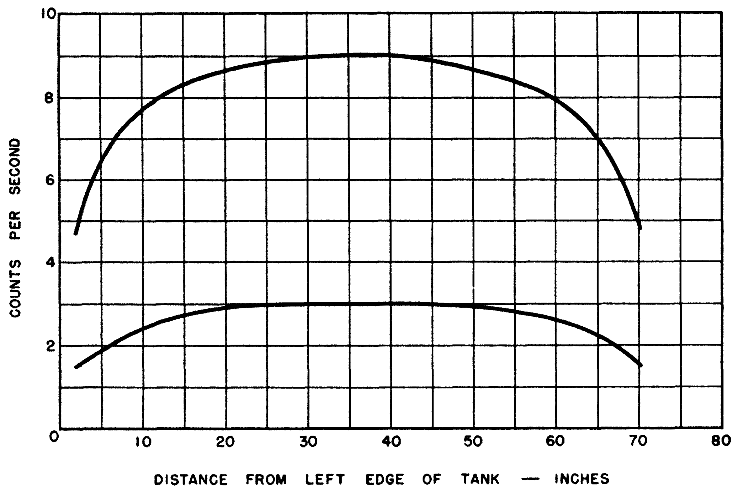

FIG. 4. Counting rate at points within the tank.

tank 6′ in diameter and 6′ high filled with water. These measurements were repeated with two concentrations of Co^{60}, whose activity was measured before being put in solution. One concentration had a specific activity of 4.0 $\mu\mu C$ (10^{-12} curies) per cm^3, the other 11.9 $\mu\mu C$ per cm^3.

For the larger Co^{60} concentration $2\eta\lambda$ is $2\times 11.9\times 10^{-12}\times 3.7\times 10^{10}=0.88$ γ-ray per second per cm^3. The results with background subtracted are shown in Fig. 4. The ordinates are precisely in the ratio of the specific activities. The flat portion of the curves at the center of the tank indicates that the effect at the walls does not extend inward more than 60 cm. Thus measurements made at the center are representative of an infinitely extended medium.

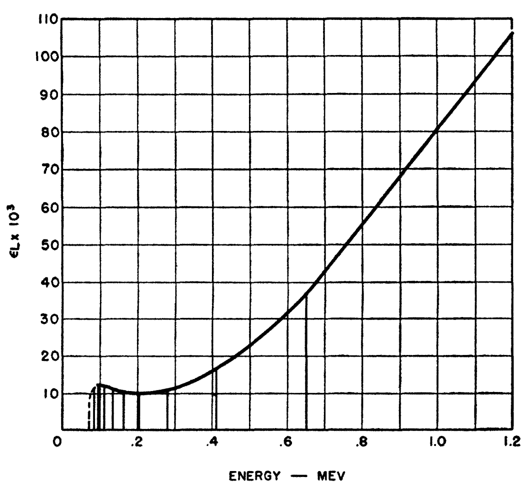

FIG. 5. Product of efficiency and path length.

The counter was always enclosed by an aluminum shield (0.4 g/cm²) to protect it from the water. The effect of introducing additional shields around the counter are shown in Table III. These measurements were made to obtain some information about the spectral distribution of the radiation.

IV. DISCUSSION

The counting rate can readily be determined from Eq. (5). The numerical values for the factors before the summation have been given in the previous section. The summation may be effected graphically by plotting ϵl against energy, as shown in Fig. 5. The values of $\epsilon_k l_k$ for energies of the scattered quanta given in Table I are represented by the ordinates of the figure and

TABLE II. Efficiency of copper counter.

Source	Mean energy of γ-rays-Mev	Measured efficiency	Bradt, *et al.*
Co^{60}	1.2	0.70%	0.68%
RaC*	0.78	0.39%	0.40%
X-rays	0.12	0.14%	0.14%

* The mean energy and γ-ray spectrum is given by C. D. Ellis and G. H. Aston in Proc. Roy. Soc. **A129**, 180 (1930).

TABLE III. Effect of shielding on submerged counter.

Shield	Counts/sec.	Experimental reduction in counting rate	Calculated reduction in counting rate
Lead—0.8 g/cm²	6.52 ± 0.10	(27 ± 3)%	29.5%
Lead—1.6 g/cm²	6.17 ± 0.15	(31 ± 5)%	38%
Al—0.8 g/cm²	8.52 ± 0.14	4.5 ± 3%	7%
No shield	8.92 ± 0.14	0	0

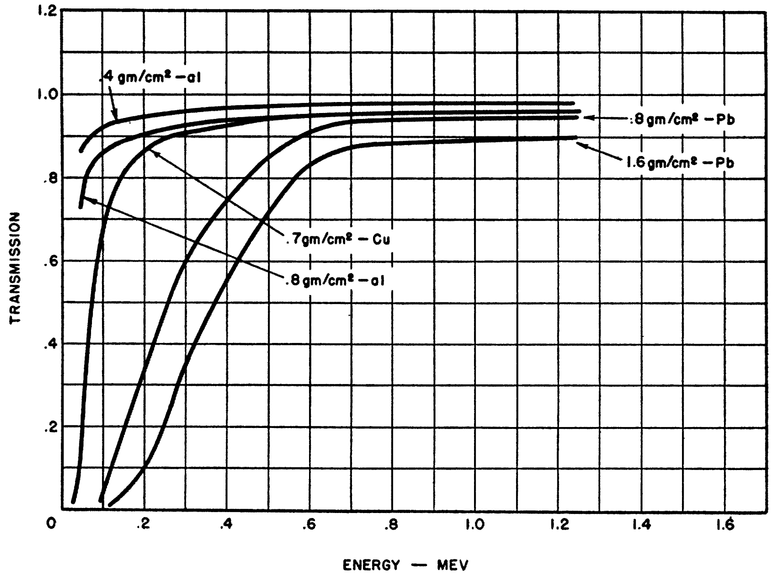

FIG 6. Transmission of shields.

their sum, including the attenuation introduced by the 0.4-g/cm aluminum shield, is 0.229 cm. From Eq. (5) we find for the 11.9-$\mu\mu C$/cm^3 concentration of Cl^{60} that the counting rate should be 9.04/sec., which is 1.3 percent higher than the observed value in Table II.

The effect of introducing shields around the counter may be understood from the transmission curves shown in Fig. 6. They are computed from the known mass absorption coefficients[7] of the various metals used. The efficiency of a counter with a shield around it is now reduced at each energy by the transmission coefficient for this energy. It will be noted that the mass absorption coefficient includes scattering as well as photoelectric absorption. Our use of this coefficient should overestimate the effect of shielding because not all of the quanta scattered in the shield are deflected through a large enough angle to miss the counter. The error is small for Pb shields where photoelectric absorption is predominant. If the indicated calculation is made for 0.80-g/cm^2 and 1.60-g/cm^2 lead shields, $\epsilon_k l_k$ is found to be 0.161 cm and 0.141 cm, giving 29.5 percent and 38 percent reduction in the counting rate. Similarly, a 0.80-g/cm^2 Al shield gives a 7 percent reduction. Comparison with experiment is shown in Table III.

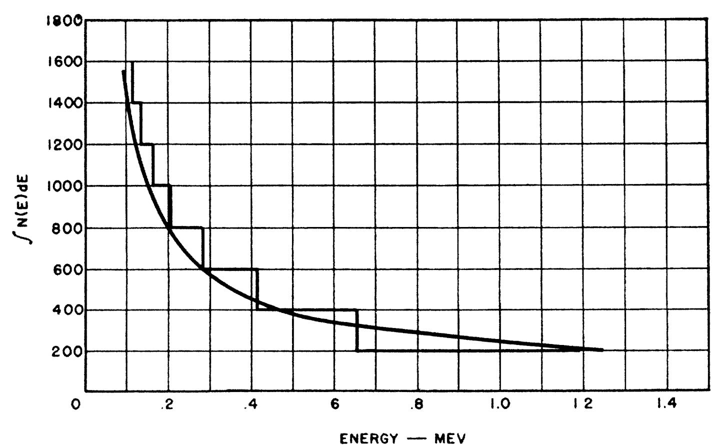

FIG. 7. Energy distribution of quanta.

[7] A. H. Compton and S. K. Allison, *X-Rays in Theory and Experiments* (D. Van Nostrand Company, Inc., New York, 1936), Appendix IX.

It is surprising that the assumption that all scattered quanta of one group have the same energy yields results so closely in agreement with experiments on both intensity and spectral distribution of the radiation. The meaning of this assumption in terms of the energy distribution is illustrated in Fig. 7, in which the number of quanta with energy greater than E has been plotted. This number increases by N whenever E passes through one of the energies E_k. The same number has been computed from a statistical study of 200 calculated γ-ray tracks by Dr. Fano and Mr. Karr[8] at the National Bureau of Standards and is also shown in Fig. 7. The similarity of the two curves accounts for the close agreement between our experimental and calculated results.

[8] We are indebted to Dr. Fano and Mr. Karr for permission to quote the results of their work on an Office of Naval Research contract.

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Conductivity Induced by Electron Bombardment in Thin Insulating Films

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When thin films of insulating material are bombarded by high velocity electron beams that can penetrate through the thickness of the film, steady currents can be obtained that are as much as 100 times that in the bombarding beam. These currents vary with the gradient across the film and are proportional to the fraction of the beam energy absorbed in the film. Typical data obtained with amorphous silica are presented, together with a description of the experimental procedures that were used.

INTRODUCTION

ALTHOUGH considerable work has been done with the bombardment of insulators by electron beams, most of it has been related to emission of secondary electrons. Relatively few papers have dealt with the problem of the currents induced by the bombarding beam, even though this has been known for some time,[1] and most of these dealt with semiconducting and photo-conductive materials.[2–4] More recently some work has been done with crystals of silver chloride and diamond, as reported by Van Heerden.[5] In these cases pulses of current were observed to flow through the crystal under bombardment by pulses of ionizing particles like alpha-particles[6] or pulsed electron beams.[7] The magnitudes of these currents are very sensitive to the structure of the crystal.

An effect has been found, however, which is not dependent on the crystalline structure and has been found in all of the several insulating films tried to date. Among these are silica, magnesium fluoride, electrolytic aluminum oxide, and mica. It was found that electrons can flow continuously through thin films of the insulator when bombarded by an electron beam of sufficient velocity to penetrate through the thickness of the film. These currents have been observed to be as large as 100 times that of the bombarding beam and are proportional to the energy absorbed from the bombarding beam.

EXPERIMENTAL PROCEDURE

The tests described below refer to thin films of amorphous silica on Nichrome base metal plates two inches square. The films were in the range of thickness from 2500A to 15,000A and were prepared by heating the plates in an atmosphere of ethyl silicate vapor, which decomposes

[1] A. Becker, Ann. d. Physik **13**, 394 (1904).
[2] R. Kronig, Phys. Rev. **24**, 377 (1924).
[3] R. Frerichs, Phys. Rev. **72**, 594 (1947).
[4] E. S. Rittner, Phys. Rev. **73**, 1212 (1948).
[5] P. J. Van Heerden, *The Crystal Counter* (N. V. Noord-Hollandsche Uitgevers Maatschappij, Amsterdam, 1945).
[6] A. J. Hearn, Phys. Rev. **23**, 524 (1948).
[7] K. G. McKay, Phys. Rev. **74**, 1606 (1948).