88-Mev Gamma-Ray Cross Sections

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Total cross sections for 88-Mev gamma-rays have been measured in 6 elements by absorption methods to a statistical accuracy of about 1.5 percent. The results expressed in cm^2/gram are 0.0107, 0.0252, 0.0471, 0.0665, 0.0909, and 0.0973 for the elements Be, Al, Cu, Sn, Pb, and U, respectively. Values appear generally lower than predictions of existing theory by a factor proportional to Z^2.

Ratios of pair formation cross sections in different elements have been obtained in a few cases; these data can be used together with the total cross section to obtain non-pair cross sections (presumably Compton scattering). The cross section so obtained at 88 Mev agrees within the experimental accuracy of about 15 percent with the Klein-Nishina prediction for Compton scattering.

INTRODUCTION

EXPERIMENTS have been performed with the gamma-radiation of the 100-Mev betatron in an effort to determine the cross sections for gamma-ray absorption and scattering and to determine the energy spectrum of the radiation. In this paper, the results of some experiments undertaken to determine accurately the cross sections at approximately 90 Mev will be presented; detailed surveys of spectra will be left to future reports.

When this work was started over a year ago it was clear that at least in some cases the theoretical cross section for gamma-rays was in rough agreement with experiment. Semiquantitative information existed from cosmic-ray shower experiments, but perhaps the best evidence came from cloud-chamber experiments by Delsasso, Fowler, and Lauritsen[1] who not only obtained rough information on the total cross section for absorption of 17-Mev radiation but on the individual pair and Compton scattering cross sections. An attempt was therefore made in this work to obtain good statistical accuracy within reasonable limits of effort and to extend the observations to the highest practicable gamma-ray energy currently available.

The total cross section for gamma-rays is probably best determined by measuring the transmission and hence absorption coefficient in particular materials. The measurement of this absorption coefficient at high energies is not as easy as might first be suspected. If one considers the Bremsstrahlung radiation produced by a high energy electron accelerator; e.g., the 100-Mev betatron, one must devise a detector which is sensitive only to the portion of the radiation having the required energy. The pair-detecting spectrum analyzer mentioned below is such a device; the total number of pairs found in a given (small) energy interval is a linear measure of the total number of gamma-quanta in that interval. However, the reduction in intensity near the middle of the Bremsstrahlung energy spectrum caused by intervening absorbing materials is *not* a measure of the true absorption coefficient for that energy. It is a measure of the combined effects of true absorption and scattering (total cross section) and the *increase* of intensity in that interval caused by the scattering of higher energy quanta. This latter effect of the degrading of the spectrum is difficult to evaluate; it depends upon many factors including the particular geometry chosen, etc. However, if one measures the reduction in intensity sufficiently near the upper end point of the Bremsstrahlung spectrum, the degradation of the spectrum can be neglected. This is equivalent to saying that any process for scattering or absorbing the radiation appreciably will reduce the energy enough to fall outside the selected energy region. One needs to know how much of the upper end of the spectrum can be safely used; a large amount is desirable for statistical and stability reasons, but a small amount is desirable to keep degradation correction low. An upper

[1] L. Delsasso, W. Fowler, and C. C. Lauritsen, Phys. Rev. 51, 391 (1937).

TABLE I. Fractional reconverted energy E_c for 100-Mev Bremsstrahlung.

E_c	Compton effect	Pair-Bremsstralung
95 Mev	0.004	0.001
90	0.012	
85	0.019	0.001
80	0.027	0.004
70		0.015

limit to the correction due to degradation can be estimated by making certain assumptions. We shall assume the Bremsstrahlung spectrum given by Heitler[2] with an upper energy limit of approximately 100 Mev. Two processes of absorption; e.g., Compton effect and pair production, are assumed to be appreciable near the upper end point. It will be assumed that the detector responds to radiation above a critical cut-off energy, E_c. We calculate specifically the fraction of initial Bremsstrahlung intensity lying above E_c that is reconverted to radiation of energy greater than E_c for the two absorption processes under consideration. This is a direct indication of the error to be expected in using intensity observations for a calculation of the absorption coefficient. In the case of Compton scattering, one merely calculates the differential cross section for scattering into various energy intervals and compares this with the appropriate total cross section. For the pair process, however, one first must consider the spectrum of electrons and positrons produced from the Bremsstrahlung spectrum. Then one calculates the radiated intensity from these charged particles.

The result of these calculations, shown in Table I, gives specifically the fraction of incident radiation above E_c taking place in the process which is reconverted to energy also above E_c. The table has been constructed for an initial Bremsstrahlung spectrum having an upper limit of 100 Mev.

The permissible cut-off energy for the detector depends upon the final accuracy desired, the ratio of Compton to pair cross section and, of course, on the validity of the above calculations. It is believed that the above figures represent larger errors than are actually found because they neglect the effect of ionization loss (which tends to reduce the energy of the charged particles and hence eliminate them from the energy region under consideration) and they neglect the very important influence of geometrical factors. For example, in the experiments to be described, an angular deflection for a scattered photon of greater than 0.01 radian results in its loss by "geometrical capture;" this effect turns out to greatly reduce the errors shown in Table I; e.g., for a cut-off of 80 Mev the fractional reconverted energy for the Compton effect is reduced to 0.0023 or less than one-tenth the value shown.

These considerations indicate that if one chooses the detector cut-off energy at 80 Mev (for 100-Mev radiation) one can use the results to compute the absorption coefficient with a systematic error of less than $\frac{1}{2}$ percent. This is substantially what has been done in the following experiments; the actual effective cut-off has been placed at about 82 Mev, and it is believed that the systematic error from spectral degradation is less than $\frac{1}{2}$ percent.

In addition to the measurement of total cross section by this absorption method the determination of the individual contributions due to pair formation and Compton scattering has been attempted. This was done very nicely in a cloud chamber by Delsasso, Fowler, and Lauritsen at lower energies[1] by measuring the ratio of electrons to positrons ejected from a radiator illuminated by a monochromatic gamma-ray. This technique cannot be applied directly in this work because the efficiency of collection for a Compton electron is not only different from that of an electron member of a pair but extremely hard to determine. Consequently, the approach has been to measure the ratios of pair cross sections for different materials; this is done by determining the relative numbers of pairs ejected from weighed targets of different materials with known relative gamma-ray dosages. This information allows one to approximately separate the pair formation and Compton scattering; this can be easily seen by writing the ratio of total cross sections in two materials,

$$\frac{\sigma^I}{\sigma^{II}}=\frac{\sigma_p{}^I}{\sigma_p{}^{II}}\frac{(1+\alpha^I)}{(1+\alpha^{II})}. \tag{1}$$

In this equation the superscripts refer to the

[2] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1944), p. 170.

particular material chosen, σ is the total cross section and σ_p the pair cross section. The symbol, α, denotes the ratio of non-pair to pair cross section (theoretically the ratio of Compton scattering to pair formation). Of course this equation is energy dependent, but at a given energy the individual total cross sections can be measured by absorption and the ratio of pair cross sections by the ratio of pairs ejected from pair-forming targets. Furthermore, if one makes material *II* of high atomic number, Z, and chooses a high energy, e.g., 90 Mev the quantity α^{II} can be almost neglected. For example, for Pb at 90 Mev α^{Pb} is theoretically about 0.02; this figure can be assumed without much discomfort even though the theory is in question. Therefore, one can determine essentially α^I by Eq. (1), and if material I is of low Z, this quantity can be determined with fair accuracy. This clearly allows, with the measured value of σ^I, a check on the value of $\sigma_p{}^I$ and, of course, the non-pair part of the cross section. It is obvious that even a rough indication of Compton scattering at an energy of 90 Mev is a sensitive check on the Klein-Nishina formula.

EXPERIMENTAL TECHNIQUE

The radiation used in these experiments is that given by the 100-Mev betatron operated at full energy; this is presumably the Bremsstrahlung radiation whose spectrum is shown in Heitler[2] and with an upper energy limit of 100 Mev. The radiation is emitted in short bursts lasting a few microseconds which are repeated at a uniform rate of 60 per second. The fact that the peak gamma-ray intensity is so large compared with the average rate imposes severe limitations on the detecting equipment; these limitations fortunately can be overcome by careful design and (generally) complicated and extensive circuitry.

It has been shown that the total cross section for gamma-ray absorption can be obtained by measuring the attenuation of the 100-Mev Bremsstrahlung radiation in various materials; the important consideration is to detect only that part of the radiation lying above perhaps 80 Mev in energy. At these energies it is currently believed that the incident photons are absorbed either by (Compton) scattering by electrons or by forming electron-positron pairs in the fields of the nucleus and electrons.[3] The Compton effect could be used as an energy-selective means of detection but, unfortunately, the energy of the ejected electron is not a unique indication of the gamma-ray energy.[4] On the other hand, the total energy of a pair is substantially the energy of the gamma-ray responsible for its production; this feature makes the detection of pairs quite suitable for energy selection.[5]

The easiest system for the detection of high energy pair particles appears to be a set of counters suitably arranged in or around an energy-separating magnetic field. With only a pair of counters, one generally runs into confusion of multiple pairs unless one operates the system at a distressingly low recording rate. An array of counters allows an enormously increased recording rate for two reasons. First, a much greater fraction of all pairs formed are recorded and, second, if the efficiencies are high enough, multiple pairs formed in the target can be detected (by the simultaneous discharge of more than two electron or positron counters) and therefore eliminated in the final recording. A qualitative calculation shows that the use of a multiple counter array in the experiments to be described has resulted in a time saving for data collection of several years. Furthermore, it has resulted in a much clearer understanding of the various "dirt" effects in the apparatus.

The experimental procedure which has been used is to measure the relative intensity of high energy radiation by means of the high energy pairs formed in a thin gold radiator (usually 0.002-inch thick). The intensity attenuation obtained by interposed blocks of various materials is then used to compute the cross sections. The average effective energy of the radiation is

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[3] The photoelectric effect is negligible in all elements at these energies.

[4] The angle of ejection could be used also, but the angles involved are so small that they appear difficult to evaluate.

[5] The pairs formed in the field of the electrons may actually emerge as *triples*, i.e., the electron causing pair formation may itself be ejected with considerable momentum. However, the usual case is that the energy of this third particle will lie below mc^2, and can be screened out by appropriate stopping materials. Furthermore, in the experiments using pair-forming materials, high atomic number Z is chosen so that nuclear pair formation greatly predominates.

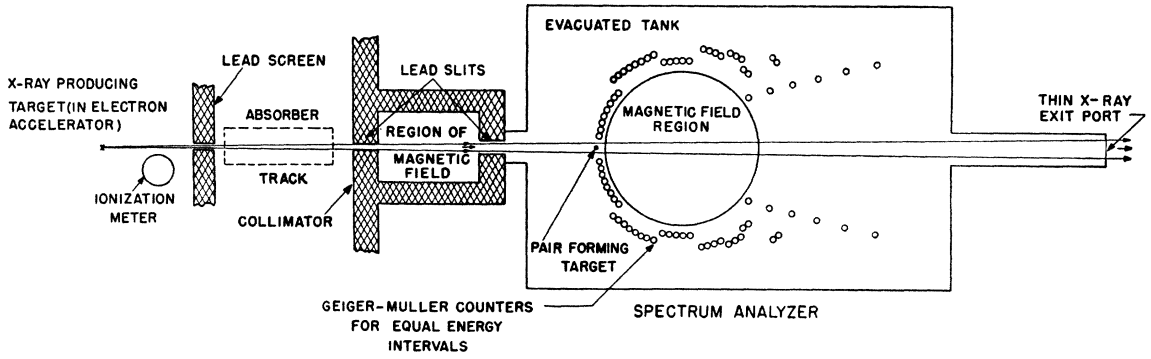

FIG. 1. Schematic diagram of equipment.

computed from the theoretical shape of the Bremsstrahlung spectrum and the expected energy resolution of the pair-forming spectrometer. The assumption of Bremsstrahlung shape can be checked by the intensity ratios found experimentally in adjacent energy channels; the agreement is good enough to assure proper average energy determination to about one percent. This average energy in the experiments to be reported is 88 ± 1 Mev. This inaccuracy in energy is unimportant as cross sections probably do not vary rapidly in this range.

The radiation from the betatron is subject to intensity fluctuations and also requires collimation; the complete experimental equipment is shown schematically in Fig. 1. The beam passes in order through an ionization radiation meter, absorber track, collimator, and pair-detecting spectrum analyzer. The radiation meter in front of the absorber rack is designed to measure the total incident dose during a typical run. It will be noticed that the meter is situated just outside the main beam and also that between the meter and absorbing rack there is placed a thick lead absorber. The purpose of this arrangement is to make the radiation meter insensitive to radiation scattered back from the absorbing blocks; this requirement is obvious if one wishes to have a reading proportional to the incident dose in which the proportionality constant is independent of the presence of absorbing blocks. For the particular geometry chosen the influence of absorber scattering on the meter reading was tested by using a given dense absorber in the two extreme positions on the rack. The ratio of pairs (see discussion below) in the spectrum analyzer to the radiation meter reading was essentially independent of absorber position; this measurement was sufficient to insure a maximum meter correction of less than 0.5 percent for any absorber in the track.

The collimator furnishes a clean gamma-ray beam which does not appear to produce appreciable secondary effects from the various parts of the spectrum analyzer.

The pair-forming spectrum analyzer consists of a thin pair-forming target and a curious array of Geiger-Müller counters about a (circular) magnetic field region. There are many possible field and counter arrangements; the one which has been chosen was designed for pair spectrum measurements up to energies of 200 Mev and is so arranged that each electron (or positron) group of counters represents 15-Mev energy intervals. It is important to note that the region of magnetic field must be large enough that the magnetic deflection between adjacent intervals exceeds comfortably the angular electron or positron scattering in the pair-forming target. This condition requires a magnet structure whose size is not greatly influenced by the highest energy under surveillance; an adequate magnet weighs 5 or 10 tons.[6]

One of the troublesome features of the entire experiment was the nature of the betatron radiation itself. Considerable time was spent in getting a clean x-ray beam; in the early stages of operation the radiation would arise from two

[6] For lower energies where semicircular focusing is feasible, wide-angle particle scattering is permissible; this desirable feature, however, cannot be extended into the several hundred-Mev range without monstrous magnet structures.

or three well separated spots in the betatron. It is clear that the excellence of collimation depends upon a small initial source of radiation. Fortunately, this difficulty finally disappeared when the proper target was installed in the betatron. Secondly, the intensity of the beam would fluctuate enormously from pulse to pulse. There was no obvious cure for this defect, but careful dosage monitors were used for calibrations.[7] Thirdly, the position of the collimated x-ray beam would slowly change due to betatron temperature. These changes, while ordinarily considered negligible, materially altered the ratio of pairs produced from a given target to the monitor dose reading. Therefore, it was necessary to make interpolated short runs (usually 10 min.) for most measurements. Finally, in the measurement of pairs near the upper limit of the spectrum, it is obvious that small changes in betatron energy would cause serious fluctuations in number; actually a one percent change in energy would generally give about a six percent change in pair counting rate. The betatron energy fluctuates with the power supply line voltage and, because of the high Q circuit, also with the line frequency. It was fortunately possible to control the energy to about 0.1 percent by continuously watching the mega-voltmeter and providing appropriate manual corrections.

In the actual use of the spectrum analyzer, one is always confronted with the background correction. The determination of background rates is a very important consideration. At very low recording rates, the background of chance "pairs" from coincident random single events may be negligible, but the statistical accuracy available from these low pair-recording rates is correspondingly poor. At reasonable rates, the background corrections are quite important, and, in the experiments reported here, have been determined usually to within 10 percent. The various background effects have been separated in two or three ways. First, the measured quantities during a normal run are not only the spectrum of pairs, but the spectrum of single events and the number of multiple events. A "single" is defined as an event in which only one counter of given sign (positron or electron) is discharged; a "multiple" is an event where at least two counters of the same sign are discharged. From this information, it is possible to calculate the background correction for pairs. This can be seen as follows.

Let us assume that counters are discharged by one of two mechanisms—either by true pair particles from the pair-forming target or by random single events, whether or not these singles be caused by room background of scattered radiation, cosmic rays, or by fragments of true pairs (the other member of the pair does not, for some reason, produce a corresponding discharge in a counter—it may be lost by geometry, absorption, or scattering). Denote for these two mechanisms probabilities (per betatron pulse) p_l and s_j^- and s_k^+, where p_l is the probability for a recorded pair in the lth energy interval, s_j^- is the probability of recording an electron single in the jth energy channel and s_k^+ is the corresponding probability for the kth positron channel. The actual recorded rates for "zero" events (no discharges), Z, electron singles, S_j^-, positron singles, S_k^+, and pairs, P_l, will then evidently be

$$Z=e^{-\phi} \tag{2}$$

where ϕ denotes the sum of all the p_l, s_j^- and s_k^+.

$$S_j^-=Z(e^{s_j^-}-1) \tag{3}$$

$$S_k^+=Z(e^{s_k^+}-1) \tag{4}$$

$$P_l=Z[p_l+\delta_l+\sum(e^{s_j^-}-1)(e^{s_k^+}-1)] \tag{5}$$

where δ_l involves higher terms in p_l. Equations (3) and (4) contain the curious exponential factor; this is necessary to take into account the events in which two or more random events occur in the same counter channel actually registering as a single. The summation term in Eq. (5) is the "background" term for pairs; the summation must be made over all energy combinations j and k adding up to the energy interval l. From these equations, it is clear that the probabilities p, s^-, and s^+ can be easily calculated from the observed rates Z, S^-, S^+ and P.

This mathematical formulation can be trusted only as far as the original assumptions can be justified, i.e., that only the two stated processes

[7] The fluctuations are still troublesome because the calibrated monitor reads only the average intensity. For many of the measurements, one needs to know the mean intensity, mean square intensity, mean cube intensity, etc.

contribute towards counter discharges. This has actually been tested by a sample experiment without the pair-forming target and determining a calculated chance pair spectrum (the summation term of Eq. (5)). This has been compared with the measured "pair" spectrum and the agreement has been excellent for all high energy pair channels. Tests have also been made of the way in which the net p_l varies with target thickness, and, although this is complicated by particle scattering in the target, it is found that p_l is proportional to target thickness. Nevertheless, in the cross-section experiments to be reported, the background correction has been held to less than 10 percent of the true pairs; this is done simply by limiting the intensity of x-ray beam.

DESCRIPTION OF EQUIPMENT

A detailed description of the equipment in this paper is not necessary; however, a few points of interest can be mentioned. The dose integrating meter at the front of the absorber rack consists of an argon-filled ion chamber connected to a vacuum condenser having a capacitance of about 10^{-9} farad. The storage time constant of this condenser is noteworthy; even with a one-mg radium source one foot away the leakage time constant was measured to be greater than 50 years. Connections to the storage condenser are made in the vacuum at the beginning and end of the run. For reading the condenser potential a Dersham electrometer is used as a null device; the condenser voltage is balanced by an external accurately measured potential. In this way an accuracy of 5 mv is obtained even when reading potentials up to 10 volts. The ion chamber is specially made to have constant sensitivity even when the collector changes potential.

The collimator consists essentially of two thick (about 10 inches) lead slits with an intervening region of magnetic field. The first slit defines the beam aperture and the magnetic field sweeps all charged particles aside. The second lead slit does not quite touch the beam; it is designed to remove only the (wide) secondary radiation produced at the first slit. The entire collimator is, of course, evacuated.

The spectrum analyzer tank is shown schematically in Fig. 1; the actual box is shown in Fig. 2. It is essentially a large evacuated box containing the Geiger-Müller tubes and pair-forming target. While the Geiger-Müller tubes can be operated very easily in an induction of 5000 gauss[8] the tubes associated with the electronic amplifier must be located well out of the magnetic field. The leads in the evacuated tank to the individual Geiger-Müller channels must be mutually shielded; low capacity coaxial tubes are satisfactory. The target consists of a thin 8-mm ribbon of the selected material; actually a target rack containing several elements is used. Selection of the desired element is made by an external control.

An important part of the spectrum analyzer is the magnetic field. In this model a moderate flux density (4000 gauss) was used with a large (2-foot diameter) pole face. The field was controlled automatically by a device using the proton resonance principle described by Purcell *et al.*[9] In this control apparatus the frequency of the radio signal instead of the value of magnetic field was modulated over the resonance; this necessitated the construction of a balanced bridge nearly insensitive to frequency and microphonics. Such a bridge was actually made and is in use but the author cannot recommend this procedure for general use. Nevertheless the control apparatus was finally made reliable and could easily control the field to 0.01 percent over long periods of time. The chief trouble encountered in its actual use with the betatron was the microphonics from the high noise level (about 105 db) in the betatron room during x-ray generation.

The Geiger-Müller counter information from preamplifiers located at the spectrum analyzer tank is fed to an analyzing circuit by means of several coaxial cables. The analyzing circuit only gives an output coincident with the betatron x-ray pulse. The circuit puts directly on registers

[8] The operation of the Geiger-Müller tubes in a magnetic field was investigated by shining light through an end seal liberating photoelectrons from the cathode. The counting rate and pulse shape from this source was found independent of magnetic induction at least up to 5000 gauss. The photoelectrons are ideal for this test. They are liberated in numbers independent of magnetic flux and are at the same time of very low energy and far from the anode, so that they are most easily influenced by magnetic fields.

[9] E. M. Purcell, H. C. Torrey, and R. V. Pound, Phys. Rev. 69, 37 (1946).

the spectrum of single events of both signs, spectrum of pairs, and number of multiple events. This operation unfortunately requires about 200 vacuum tubes and associated parts.

In spite of the large number of tubes required (about 500) for proper operation of the entire equipment the overall electronic reliability has been very gratifying. Electronic failures have prevented operation for less than one percent of scheduled time; the chief reasons for this record are conservative ratings on components and built-in test-equipment.

EXPERIMENTAL RESULTS

Total Cross Section

The total cross section for 88-Mev gamma-rays has been obtained by absorption methods in 6 elements—Be, Al, Cu, Sn, Pb, and U. The preparation of the absorbing samples was found quite difficult because of the purity requirement (varies with the element but ought to be at least 99.5 percent) and the degree of homogeneity required. For example, in one of the early aluminum castings which had been rolled to decrease porosity, the measured sample density was about 0.5 percent too low; this led to the discovery of an internal longitudinal cavity which would have completely invalidated an absorption measurement. The final samples have, however, been carefully checked for density, uniformity and purity; in all cases except that for Be the purity has been higher than 99.85 percent. In the case of Be, the purity is not known precisely; the chief contaminant is oxygen and is believed to contribute about 0.5 percent by weight. All other contaminants total less than 0.5 percent, including the effective porosity factor. The exact oxygen content is now being determined.

The absorbing samples were designed to provide two transmission factors of about 0.3 and 0.1. The smaller figure gives more accurate statistical results, but the larger figure was used in a few instances to provide a check. In no case was there observed a discrepancy in measured absorption coefficient with the two absorber thicknesses.

The determination of an absorption coefficient at one energy for a given material to an accuracy of about two percent requires an operating time of about one day. During this run, the intensity is usually adjusted in the betatron so that at least one counter in the array is discharged on about one-half of the betatron bursts; of these events about one-third are true pairs and the rest are background single events. The background singles are largely caused by scattered low energy radiation (perhaps one to 10 Mev) in the betatron room. It is perhaps surprising that the amount of this softer radiation in the room is so high; however, it probably is caused chiefly by the betatron electron beam after striking the target which causes the main x-ray beam. In the figures which have already been quoted, the spectrum analyzer counter array is already shielded from the room by about one ton of lead arranged to give the lowest effective singles background per recorded pair.

The results of several measurements made on various absorbing samples is shown in Table II, and a weighted average of the individual measurements on each material is given. The cross section is quoted for convenience both in cm^2 per gram and in cm^2 per atom. The errors which are quoted are in all cases the probable statistical error and do not include possible systematic errors. It is believed, however, that for all cases, except that for Be, the systematic error is so small that it will not affect the result. In the case for Be, a correction of -0.7 ± 0.3 percent is shown because of contamination; it is clear that a small contamination of a heavy element in a light element is much more serious than a small contamination of a light element in a heavy

FIG. 2. Spectrum analyzer.

TABLE II. Experimental values of total cross section at 88 Mev.

Sample	Approx. transmission	cm²/g	cm²/atom	Probable statistical error	Date
Be	0.28	0.0107_6	0.161×10^{-24}	1.2%	7/15/48
(corrected for contaminant)		*0.0106_9*	*0.160×10^{-24}*	*1.5%*	
Al	0.4	0.0247	1.10×10^{-24}	13.5%	11/24/47
	0.4	0.0271	1.21×10^{-24}	11.0%	11/25/47
	0.18	0.0256	1.15×10^{-24}	3.5%	12/15/47
	0.18	0.0249	1.11×10^{-24}	4.7%	5/11/47
	0.075	0.0251	1.12×10^{-24}	1.8%	5/13/47
weighted average		*0.0252*	*1.128×10^{-24}*	*1.5%*	
Cu	0.12	*0.0471*	*4.971×10^{-24}*	*1.5%*	5/21/48
Sn	0.08	0.0662	13.1×10^{-24}	2.2%	5/13/48
	0.08	0.0695	13.7×10^{-24}	2.4%	5/21/48
	0.08	0.0664	13.1×10^{-24}	1.7%	5/27/48
	0.08	0.0654	12.9×10^{-24}	1.6%	5/28/48
weighted average		*0.0665*	*13.11×10^{-24}*	*0.95%*	
Pb	0.4	0.0868	29.8×10^{-24}	13.0%	11/24/47
	0.1	0.1010	34.7×10^{-24}	13.0%	11/24/47
	0.4	0.0720	24.8×10^{-24}	17.0%	11/25/47
	0.1	0.0910	31.3×10^{-24}	11.0%	11/25/47
	0.14	0.0919	31.6×10^{-24}	3.7%	12/10/47
	0.14	0.0900	30.9×10^{-24}	3.3%	12/12/47
	0.14	0.0910	31.3×10^{-24}	2.2%	5/11/48
weighted average		*0.0909*	*31.27×10^{-24}*	*1.6%*	
U	0.1	*0.0973*	*38.46×10^{-24}*	1.1%	7/16/48

element. It can be seen that the results are internally consistent even though they were obtained with a variety of absorbers. Some of the early results were obtained in the latter part of 1947 with a different counter geometry in the spectrum analyzer. The change in geometry was not required for absorption measurements but was necessary in order to improve the low energy interval efficiency for spectrum measurements.

It does not appear profitable to attempt a much more accurate statistical result except perhaps for a single test case. In order that the systematic errors be kept smaller than the statistical error one must operate at a correspondingly low intensity; the result appears to be that in order to halve the listed errors an increase in operating time of about tenfold is required.

Ratios of Pair Cross Sections

The ratios of pair cross sections at 88 Mev in different elements has been measured by counting directly the ratios of 88 Mev pairs ejected from weighed targets of the test elements with equal x-ray bombardment. The accuracy with which a ratio is determined is not as high as for the total cross section chiefly because the statistics are not as favorable, energy fluctuations in the betatron are more serious, and scattering corrections in the spectrum analyzer are more important. The statistics can be improved slightly by considering larger energy intervals; this procedure is probably quite safe since the ratio of pair cross sections in two elements is not expected to be violently dependent on energy. The chief expected variation with energy comes from the different screening corrections applied principally at the high energies. For this reason the experimental results will be quoted in two ways; no subscript will signify the value obtained directly at 88 Mev, while the subscript, w, will signify the result at 88 Mev of the best theoretical curve through the experimental points above 50 Mev. This theoretical curve is not a constant only because of screening; however, in practice the screening corrections are very small.

In this experiment one must be quite sure that the measured number of pairs is proportional to the pairs actually produced. For targets of different thicknesses, the angular distribution of pair fragments varies because of multiple scattering in the target itself; unless the efficiency of the spectrum analyzer is independent of this scattering one can easily obtain incorrect pair cross-section ratios. The procedure used in the following experiments has been to first choose targets sufficiently thin that these scattering corrections are small; this is tested by observing directly the correspondence between observed pairs and target thickness. Second, the test targets were made thick enough to give about the same theoretical multiple scattering; this thickness incidentally gives nearly the same yield of pairs. This was not quite possible in the case of a heavy element for mechanical reasons, but was sufficiently well approximated that corrections are negligible.

It was hoped that information could be obtained on the same elements for which total cross sections were obtained, but it was clear from Eq. (1) that the principal information was to be obtained on light elements in comparison with a

TABLE III. Ratio of pairs ejected from equal target weights.

Combination	Date	R	error	R_w	error
Be/Au	8/ 5/48	0.089	3%	0.093	2.0%
Al/Au	8/18/48	0.2665	2.6%	0.274	1.0%
Cu/Au	8/12/48	0.482	3%	0.505	1.5%
Be/Cu	8/ 6/48	0.1875	3%	0.192	1.2%

TABLE IV. Experimental values of α at 88 Mev.

Element	Combination	α	error	α_w	error
Be	Be/Au	0.38	0.05	0.32	0.04
Al	Al/Au	0.082	0.05	0.052	0.04
Cu	Cu/Au	0.116	0.06	0.07	0.04
Be	Be/Cu	0.29	0.05	0.26	0.04

single heavy element. An attempt was made to construct a thin radiator of lead for the comparison heavy element, but for the required thickness of about 10^{-3} cm the mechanical properties of such a foil were inadequate. Consequently a foil was made of pure gold (99.97 percent) of about the proper thickness. This foil is strong and fairly uniform. Pure foils were also made of Be, Al, Cu, and Sn for these experiments. The chief problem in the construction of a suitable foil appears to be the determination of thickness in a specified region. This was done by cutting a large sheet of the material into uniform pieces around the region under consideration and plotting the measured density contours. In this way the effective thickness of the target was determined to about one percent even though some of the foils were thin as 0.001 cm.

The results of several measurements are shown in Table III which lists the ratios, R, of pairs ejected per unit weight of target. As already explained, the symbol R_w represents the 88-Mev value obtained by the best fit for data above 50 Mev.

This information can be used in Eq. (1) to determine the experimental ratio of non-pair to pair cross sections at 88 Mev if one first finds the total cross section in Au and then assumes a value for α for the heavier element. The total cross section in Au can be inferred from the value found in Pb; the only assumption is that the cross section in cm^2 per atom is a smooth function of atomic number, Z. This interpolation procedure yields an "experimental" total cross section in Au of 0.0896 cm^2 per gram ± 1.6 percent. For the two heavier elements, Au and Cu, appearing in Table III, the theoretical values of α at 88 Mev have been assumed (see discussion below); these are 0.024 for Au and 0.06 for Cu. The result is shown in Table IV, where for convenience the combination is shown from which the determination was made.

COMPARISON OF EXPERIMENTAL RESULTS WITH THEORY

The theory of gamma-ray absorption is not yet complete so that precise comparison with experiment is impossible. However, the theory of Compton scattering is complete and yields the Klein-Nishina formula[10] easily evaluated for any energy and material. The theory of pair formation in the field of the nucleus exists,[11] but the numerical values available are limited by the accuracy of reading some curves not designed for high precision.[12] Furthermore, this theory has led to curves of cross-section values as a function of Z;[13] these curves are, however, calculated under the assumptions of the Fermi-Thomas atomic model and the Born approximation. It is unfortunate that in the case of heavy elements one distrusts the Born approximation and in the case of light elements one distrusts the Fermi-Thomas atom. In fact, Wheeler and Lamb have shown[14] that in the elements hydrogen and nitrogen the results using the Fermi-Thomas model can differ appreciably from those obtained using the exact wave functions. Consequently, there does not appear to be any theoretical numerical values for pair formation which can be compared precisely to the experimental results. Therefore, for definiteness in the following discussion the theoretical pair cross section will be defined as that obtained by the Fermi-Thomas atom and Born approximation; it is hoped that in the future some pertinent numerical theoretical values for exact wave functions become available.

In addition to pair formation in the field of

[10] See, for example, reference 2, p. 157.
[11] H. Bethe and W. Heitler, Proc. Roy. Soc. **146**, 83 (1934).
[12] See, for example, reference 11, p. 93.
[13] See, for example, reference 2, p. 201. These cross-section curves are apparently not very accurate; they have recently been recalculated and plotted by P. V. C. Hough who has furnished the author with the new curves.
[14] J. A. Wheeler and W. E. Lamb, Phys. Rev. **55**, 858 (1939).

TABLE V. Theoretical cross sections at 88 Mev.*

	Be		Al		Cm	
	cm²/g	cm²/atom	cm²/g	cm²/atom	cm²/g	cm²/atom
Pairs (nucleus)	0.005928	0.0888×10^{-24}	0.02035	0.9113×10^{-24}	0.04186	4.418×10^{-24}
Pairs (electron)	0.001565	0.0234×10^{-24}	0.00167	0.0748×10^{-24}	0.00153	0.161×10^{-24}
Compton scatt.	0.00240	0.0359×10^{-24}	0.00261	0.1169×10^{-24}	0.00247	0.261×10^{-24}
Total	0.00989	0.1482×10^{-24}	0.02463	1.103×10^{-24}	0.04586	4.840×10^{-24}
$\alpha_{theor.}$	0.321		0.118		0.057	

	Sn		Pb		U	
	cm²/g	cm²/atom	cm²/g	cm²/atom	cm²/g	cm²/atom
Pairs (nucleus)	0.06511	12.83×10^{-24}	0.09803	33.72×10^{-24}	0.1068	42.21×10^{-24}
Pairs (electron)	0.00138	0.27×10^{-24}	0.00127	0.44×10^{-24}	0.0012	0.49×10^{-24}
Compton scatt.	0.00228	0.45×10^{-24}	0.00214	0.74×10^{-24}	0.0021	0.83×10^{-24}
Total	0.06877	13.55×10^{-24}	0.1014	34.90×10^{-24}	0.1101	43.53×10^{-24}
$\alpha_{theor.}$	0.0343		0.0237		0.0193	

* There is also shown in this table, labeled $\alpha_{theor.}$, the ratio of cross sections for Compton scattering and pair production.

the nucleus there is, especially in light elements, pair formation in the field of the electrons. Wheeler and Lamb[14] have calculated the cross section for this process, with the usual assumption of Fermi-Thomas atom and Born approximation (which should be very good for electrons), and in addition neglecting the effect of high-momentum transfers to the electron. They have also furnished exact results for hydrogen which deviate appreciably from the Fermi-Thomas hydrogen atom. There appear in the literature other papers dealing with pairs formed in the field of the nucleus[15] but for definiteness the Wheeler and Lamb treatment will be considered here. In addition to the Compton effect and pairs formed in the fields of the nucleus and electrons, there are conceivably other processes which might contribute to the total cross section. Hamilton and Peng[16] have found that the scattering of a light quantum by the nuclear meson charge cloud can give a substantial cross section up to 1000 Mev; however, effects of this type of scattering at 90 Mev are not calculated[17] and in the following discussion will be assumed negligible. The possible contribution of gamma-nucleon cross section is not known, but information exists on some absolute cross sections which indicates that at energies near the resonance the cross section can be as large as the Compton effect.[18] However, it appears likely that in most elements the contribution above 50 Mev is small.

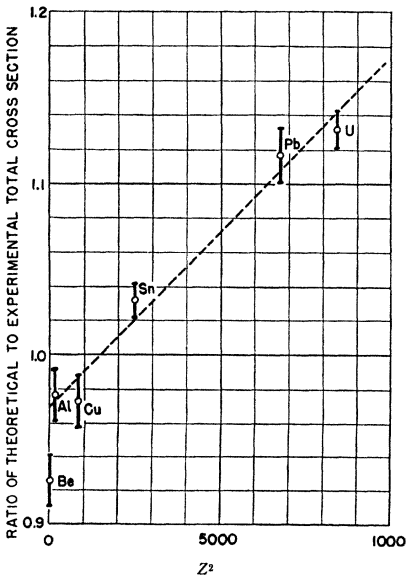

FIG. 3. Ratio of theoretical to experimental cross section *vs.* the square of atomic number.

We have therefore three processes which theoretically contribute appreciably to the total cross section. The cross sections for these at 88

[15] First mention was by F. Perrin, Comptes Rendus **197**, 1100 (1933); K. M. Watson, Phys. Rev. **72**, 1060 (1947) (results do not agree with Wheeler and Lamb); P. Nemirovsky, J. Phys. U.S.S.R. **11**, 94 (1947) (considers only energies <3 Mev); A. Borsellino, Helv. Phys. Acta **20**, 136 (1947); V. Vortruba, Phys. Rev. **73**, 1468 (1948).

[16] Hamilton and Peng, Proc. Roy. Ir. Acad. **49A**, 197 (1944).

[17] Such calculations would probably not be very significant anyway because of present uncertainties in meson theories.

[18] J. L. Lawson and M. Perlman, Phys. Rev. **74**, 1190 (1948).

Mev are shown in Table V; for convenience the cross sections are listed both in cm^2 per gram and cm^2 per atom.[19]

We may now compare the experimental and theoretical total cross sections from Tables II and V. Discrepancies clearly exist especially for heavy elements; this fact leads one to suspect the failure of the Born approximation in giving the proper theoretical result. One would expect the failure of the Born approximation to be proportional to $(Z/137)^2$; consequently a plot of the ratio of theory to experiment *vs.* Z^2 should be linear.[20] Such a plot is shown in Fig. 3 where the ordinate is expanded for easy visibility. The errors shown on the points are the probable statistical errors of the experimental results only; the error associated with the theoretical values is probably caused mainly by the theoretical approximations used. It is clear that the curve in Fig. 3 is mainly linear in Z^2 indicating that the present theory gives a larger cross section than is actually measured by about $Z^2/500$ percent. It is surprising that at lower energies (<3 Mev) the result of exact calculation[21] gives a correction of opposite sign. It is not clear whether or not at high energies the size and sign of the correction can be inferred from the low energy results. It is hoped that when exact calculations of pair production in heavy elements are made available this discrepancy can be understood.

The intercept of the curve in Fig. 3 for small Z^2 is approximately unity indicating that the theoretical predictions for pair production and Compton scattering are approximately correct. The experimental results are actually somewhat larger than those from theory possibly because of slight theoretical failure, experimental inaccuracy, or perhaps because of some other contributing process. The experimental point for Be is unfortunately distinctly off the linear plot. It was suspected that this might be in part caused by the Fermi-Thomas model. Exact calculations have now been made by R. Ehrlich and H. Hurwitz,[22] using the known wave functions for Be, to check this possibility. The exact results increase the theoretical cross section to 0.00996 cm^2 gram which accounts for only about 0.2 of the discrepancy.

With respect to the ratio of non-pair to pair cross sections, the values of α obtained by experiment in Table IV can be compared to those obtained theoretically in Table V. In no case does a large discrepancy occur; indeed it is highly reassuring that even for the most sensitive case of beryllium agreement within the experimental error exists. This is an indication that the Klein-Nishina formula for Compton scattering is correct within perhaps 15 percent at energies of 88 Mev.

CONCLUSIONS

It has been shown that at an energy of 88 Mev the experimental cross sections for gamma-ray absorption are nearly those predicted by theory. Discrepancies occur of magnitude proportional to Z^2 and, in addition, some discrepancy exists in the measured case of beryllium. The Compton scattering cross section at 88 Mev has been approximately verified by the combined use of total cross sections and ratios of pair cross sections. It is encouraging that observed discrepancies have been small; in fact they are probably caused mainly by known approximations in present theory.

It is a pleasure to acknowledge the cooperation of the betatron group, and especially the services of the operators, McNamara and Martin. The sample of uranium was kindly fabricated and furnished by the Metallurgical Section of the Knolls Atomic Power Laboratory. It is a privilege to acknowledge the help of R. Ehrlich and H. Hurwitz, who not only have shown continued interest and activity in the theoretical treatment of radiation problems, but have materially

[19] The author is indebted to Miss E. Coe who has performed the appropriate numerical integrations of the screening functions given in Bethe and Heitler and in Wheeler and Lamb. The numbers shown in this table are accurate integrations but probably cannot be trusted to better than one percent because of possible inaccuracies in the screening functions. They differ slightly from the values calculated independently by P. V. C. Hough, e.g. for lead a difference of nearly 2 percent exists.

[20] The plot should strictly not be quite linear because the failure of the Born approximation applies only to the part of the cross section caused by pairs in the nuclear field. However, it happens that where Z^2 is appreciable the contribution to cross section by other processes is very small (see Table V).

[21] Hulme and Jaeger, Proc. Roy. Soc. **153**, 443 (1936).

[22] R. Ehrlich and H. Hurwitz, private communication.

assisted in computations pertinent to the spectrum analyzer equipment itself. Finally, it must be obvious that the success of experiments involving extensive equipment depends upon painstaking tests and circuit development; without the able assistance of C. M. Bishop, the equipment used in these experiments would not have been built and operated satisfactorily.

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The Angular and Lateral Spread of Cosmic-Ray Showers*

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The mean square angular and lateral spreads in showers have been evaluated for electrons as well as photons as functions of their energy. Analytical expressions have been obtained for high and medium energies down to the critical shower energy. The calculations have been extended by numerical methods for air down to low energies (~4 Mev). Radiative effects and ionization losses have been taken into account simultaneously for all energies, and we believe that no factors of physical significance have been omitted.

1. INTRODUCTION

THE calculation of the sidewise spread of a shower constitutes a problem of importance comparable to the evaluation of its development in depth. Particularly, the discussion of the nature of large air showers cannot be carried out without knowledge of their lateral evolution.

The first treatment of this problem has been given by Euler and Wergeland.[1] These authors discussed the mechanism of the spreading and its general features. Their numerical results, as pointed out by Bethe[2] and by the present authors[3] were, however, quite unsatisfactory and gave an extension of showers far too small. L. Landau[4] set up diffusion equations for the sidewise development in extension of the well-known Landau-Rumer[5] treatment of shower theory. However, his results are invalidated by numerical errors. The most extensive investigation was made by G. Molière.[6] Unfortunately, only an abbreviated version of his work is available. Molière uses an extension of Landau's method and carries it through to an actual evaluation of the radial density distribution in a shower. Because of the complications of the process, he is forced to neglect ionization losses for energies higher than the ionization limit and he takes low energy electrons into account according to a rather inadequate method, as he points out himself. His function will thus be subject to later revision. An evaluation of the mean square angular spread of electrons as function of energy has been given by S. Z. Belenky.[7] He starts with the Landau diffusion equations and obtains from them a set of integro-differential equations for various moments of the distribution. They are evaluated with the help of the method of Tamm

* This paper is based on the Ph.D. Thesis by Jane Roberg, Duke University, 1942. Its publication has been delayed because of the war. The results have been announced previously at several meetings of the Am. Phys. Soc. and have been communicated privately on request.

** Now at Yale University.

[1] H. Euler and H. Wergeland, Astrophys. Nor. 3, 165 (1940); Naturwiss. 28, 41 (1940).

[2] H. A. Bethe, Phys. Rev. 59, 684 (A) (1941).

[3] L. W. Nordheim, Phys. Rev. 59, 929 (A) (1941); Jane Roberg, Phys. Rev. 62, 304 (A) (1942).

[4] L. Landau, J. Phys. U.S.S.R. 2, 234 (1940).

[5] L. Landau and G. Rumer, Proc. Roy. Soc. A166, 213 (1938).

[6] G. Molière, Naturwiss. 30, 87 (1942); more fully reported in W. Heisenberg, *Cosmic Radiation* (Dover Publications, New York, 1946).

[7] S. Z. Belenky, J. Phys. U.S.S.R. 8, 9 (1944).

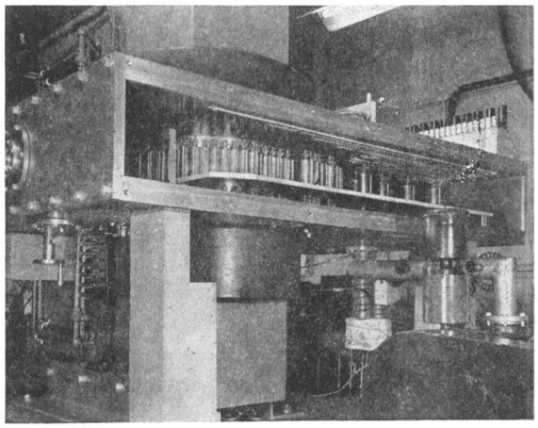

FIG. 2. Spectrum analyzer.