Note on the Bremsstrahlung Produced by Protons

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The bremsstrahlung produced by protons is discussed on the assumption that the anomalous magnetic moment can be described in terms of a Pauli type interaction with the electromagnetic field. The result of a perturbation theoretical calculation of the cross section is given, and a comparison is made to the approximate method of Weizsäcker and Williams. Reasons are given for the failure of the Weizsäcker method in special cases.

IN calculations concerning the radiative energy loss of mesons of spin $\frac{1}{2}$[1] it has been shown that the bremsstrahlung cross section increases rapidly with increasing energy if the particle has an anomalous magnetic moment of the Pauli type.[2] The assumption that the *proton* is correctly described by Pauli's equation is admittedly questionable. However, it is of interest to apply the calculations to the proton, inasmuch as the bremsstrahlung produced by primary cosmic particles may be of importance in accounting for the production of showers.

The aforementioned calculations were based upon the approximate method of Weizsäcker and Williams,[3] although it has been found that this approximation can lead to error in cases for which small impact parameters are important.[4] On the other hand, the method is known to give a correct result for the Dirac particle with no anomalous moment. The question therefore arises as to whether the method is applicable in the case of the Dirac particle (proton?) with arbitrary moment. It is the purpose of this note to compare the Weizsäcker-Williams result to a straightforward calculation by the method of perturbation theory and to point out that the approximate procedure can give misleading results in this problem, in particular for the case of the pure Coulomb field.

We assume that the only interaction between proton and nucleus arises from the electrostatic field of the nuclear charge, and that it is of the form

$$H' = e\Phi - i\lambda\frac{e\hbar}{2Mc}\beta\boldsymbol{\alpha}\cdot\nabla\Phi.$$

(Φ = spherically symmetric potential of nuclear electrostatic field, $\boldsymbol{\alpha}$, β = Dirac matrices, $\lambda(e\hbar/2Mc) = 1.79$ nuclear magnetons.) The time dependent perturbation theory may be applied, in the manner described by Heitler,[5] to the calculation of the cross section for bremsstrahlung. For convenience, we restrict the discussion to proton energies which are large compared to Mc^2. For the pure Coulomb field, ($\Phi = Ze/r$) one obtains the result

$$d\sigma = dk\lambda^4 Z^2\alpha(e^2/Mc^2)\{E_0/(Mc^2)^2\}F(k/E_0), \quad (1)$$

where $d\sigma$ = total cross section for emission of a quantum in the energy range $(k, k+dk)$, α = fine structure constant, and E_0 = initial energy of proton. The function F, defined by

$$F(x) = -\tfrac{1}{3}\{(4-x)(1-x)^2\ln(1-x) + x[6(1-x)+x^2]\ln x + x(1-x)\}$$

is zero for $x=0$ and $x=1$ and has the maximum value 0.67 for $x\approx 0.26$. Integrating, one obtains for the energy loss cross section

$$\Sigma = \int_0^{E_0} kd\sigma = \frac{5}{36}\lambda^4 Z^2\alpha(e^2/Mc^2)^2\{E_0^3/(Mc^2)^2\}$$

This result is remarkable for its very strong dependence upon E_0, as compared to the case of the electron ($\lambda = 0$), for which[6]

$$\Sigma_{(\lambda=0)} = Z^2\alpha(e^2/Mc^2)^2 4E_0[\ln(2E_0/Mc^2) - \tfrac{1}{3}].$$

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[2] W. Pauli in Handbuch d. Physik (Julius Springer, Berlin, 1933), Vol. XXIV/1, p. 233.
[3] C. F. v. Weizsäcker, Zeits. f. Physik. 88, 612 (1934); E. J. Williams, Phys. Rev. 45, 729 (1934).
[4] R. F. Christy and S. Kusaka, Phys. Rev. 59, 405 (1941).
[5] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, New York, 1944), p. 161.
[6] See reference 5, p. 172.

However, this behavior is characteristic of the Coulomb field and is essentially a result of the divergence of Φ for $r \rightarrow 0$. When this divergence is removed by the introduction of a nuclear radius, Σ is found to depend less strongly upon the energy. Thus, if one carries through the perturbation calculation for a potential Φ which is Ze/r for $r>a$ but approaches a value $\approx Ze/a$ for $r=0$, one obtains

$$d\sigma = dk \cdot \lambda^4 I Z^2 (e^2/Mc^2)^2 (e^2/aMc^2) \times \{(1-k/E_0)/Mc^2\}, \quad (2)$$

where I is a number of order unity which depends upon the detailed shape of the function assumed for Φ. For the energy loss cross section, this gives

$$\Sigma = \tfrac{1}{6}\lambda^4 I Z^2 (e^2/Mc^2)^2 (e^2/aMc^2)(E_0^2/Mc^2).$$

The introduction of the nuclear radius therefore reduces the energy dependence of the cross section by a factor $\sim 1/E_0$.

The formula (1) is essentially that derived by Batdorf and Thomas[1] by the Weizsäcker-Williams method, in which the problem is treated in the coordinate system in which the proton is at rest, and the scattering of the virtual quanta in the contracted Coulomb field is calculated from the Compton scattering cross section. The total cross section is expressed as an integral over the impact parameters r, extending from $r_{\min}$ to $r_{\max}$. The lower limit for the integration, which enters on account of the semiclassical nature of the method, is an essential feature and must be at least as large as the Compton wave-length of the proton. Equation (2) is obtained if one takes $r_{\min}=a$, which amounts to the neglect of impacts within the nuclear radius. The above calculation shows, however, that it is precisely for these small radii that one obtains significant contributions to the cross section for the pure Coulomb field.

It must be concluded, therefore, that the formula (1) represents the actual situation only to the extent that the nuclear field can be considered to be produced by a static distribution of charge. In a realistic treatment of the problem, fluctuations of nuclear charge density would have to be considered, and the problem of individual proton-proton collisions within the nucleus would therefore present itself. Also, in close collisions with nucleons, the specifically nuclear forces would come into play and would influence the production of radiation. Finally, the recoil of the nucleons, which has been neglected in the above, would have the effect of reducing the cross section.

It should be noted that the Weizsäcker-Williams approximation gives a complete answer in the case of a particle of normal moment (e.g., the electron). The reason for this difference in the two problems is easily understood. The neglect of small impact parameters implies the neglect of Compton scattering of the high energy virtual quanta characteristic of this region. For the electron this is justified by the small Compton cross section for quanta of energy greater than Mc^2. For the Pauli particle, however, the Compton cross section increases linearly with the energy in the extreme relativistic limit (see Appendix), so that the contributions from virtual quanta of high energy are of decisive importance. Since the Weizsäcker-Williams method in its customary form does not apply to the region in the immediate neighborhood of the nucleus, it does not take proper account of these high energy quanta, and cannot be used to obtain formula (1). The approximate result is in agreement with the formula (2) for the "cut-off" Coulomb field, since for this field the high energy quanta are present with negligible intensity.

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APPENDIX, COMPTON SCATTERING FORMULAE

The differential cross section for Compton scattering by a Pauli particle of magnetic moment $(1+\lambda)e\hbar/2Mc$ is[7]

$$\begin{aligned} d\sigma = d\Omega\tfrac{1}{2}(e^2/Mc^2)^2(k^2/k_0^2)\{&[k_0/k+k/k_0-\sin^2\theta] \\ &+\lambda[(2k_0k/M^2c^4)(1-\cos\theta)^2] \\ &+\lambda^2[(k_0k/M^2c^4)\{4(1-\cos\theta)+\tfrac{1}{2}(1-\cos\theta)^2\}] \\ &+\lambda^3[(k_0k/M^2c^4)\{2(1-\cos\theta)+\sin^2\theta\}] \\ &+\lambda^4[(k_0k/2M^2c^4)(1+\tfrac{1}{2}\sin^2\theta)]\}, \end{aligned}$$

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[7] Compare W. Pauli, Rev. Mod. Phys. **13**, 203 (1941), and reference (1), where this result is derived for the special case $\lambda=-1$. Note also that the expression given by Pauli in Table III, formula III, should be corrected to read

$$d\Omega(\gamma-1)^4(e^2/Mc^2)^2\frac{1}{4}\frac{k}{k_0}\left(\frac{k}{Mc^2}\right)^2(1+\tfrac{1}{2}\sin^2\theta)+\cdots.$$

where e and M are the charge and mass of the scatterer, k_0 and k are the initial and final quantum energies, and θ is the scattering angle. The first term is the Klein-Nishina formula, and the remaining terms arise from the "Pauli part" of the magnetic moment. The total cross section is

$$\sigma=\pi(e^2/Mc^2)^2\Bigg[\left\{-\frac{(2+2\gamma-\gamma^2)}{\gamma^3}\ln(1+2\gamma)+\frac{4}{\gamma^2}+\frac{2(1+\gamma)}{(1+2\gamma)^2}\right\}+\lambda\left\{\frac{2}{\gamma}\ln(1+2\gamma)-\frac{4(1+3\gamma)}{(1+2\gamma)^2}\right\}$$
$$+\lambda^2\left\{\frac{1}{2\gamma}\ln(1+2\gamma)-\frac{(1+3\gamma-8\gamma^2)}{(1+2\gamma)^2}\right\}+\lambda^3\left\{-\frac{1}{\gamma}\ln(1+2\gamma)+\frac{2(1+3\gamma+4\gamma^2)}{(1+2\gamma)^2}\right\}$$
$$+\lambda^4\left\{-\frac{1}{4\gamma}\ln(1+2\gamma)+\frac{(1+3\gamma+4\gamma^2+2\gamma^3)}{2(1+2\gamma)^2}\right\}\Bigg],\quad (\gamma=k_0/Mc^2).$$

In the extreme relativistic limit, the term in λ^4 is dominant:

$$\sigma\approx\pi(e^2/Mc^2)^2(\gamma/4).$$

It is this term which leads, by the method of Weizsäcker and Williams, to formula (2) above.

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Radiations of Uranium Y*

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The radioactivity of UY (Th_{231}) has been re-investigated. Samples were prepared by growth from U_{235} largely freed of U_{238}, and absorption, decay, and coincidence experiments were run. The presence of the 210-kev beta-ray was re-confirmed, and a 35-kev gamma-ray 82 percent converted in the L shell was discovered. The half-life of UY was determined as 25.5 hours.

INTRODUCTION

URANIUM Y is the second member of the naturally occurring actinum series of radioactive isotopes. It rapidly approaches equilibrium with the parent U_{235}, having a half-life of about one day, and is isotopic with thorium, ionium, and UX_1. Since UX_1 is formed at a considerably greater rate from the U_{238} in natural uranium, it is not possible to prepare UY in radioactively pure form from this source. Thus, despite its discovery by Antonoff[1] in 1911, relatively little work has been reported on its radioactive properties.

The International Radium Standards Commission[2] reported in 1931 a half-life of 24.6 hours and a beta-ray absorption coefficient of about 300 cm^{-1} in aluminum. This absorption coefficient, reported by Kirsch,[3] corresponds to an energy of 200 kev on the basis of recent range-energy-absorption curves of Libby.[4] Gratias and Collie[5] redetermined the half-life in 1932, arriving at a value of 24.0 hours after discounting a 25.4-hour determination obtained by an electroscope method. These authors give references to several earlier investigations of the half-life. In 1937 Erchova[6] reported extensive absorption measurements on UY. She found two components of the radiation with absorption coefficients of 19.6 and 216 cm^2 per gm. These were both considered as beta-radiation and correspond[4] to beta-ray energies of 160 kev and 0.6 Mev. (The more penetrating component, if considered as electromagnetic radiation, would have an energy of about 11 kev.)

In 1945 at the Metallurgical Laboratories in

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