The functions $N_n(\rho)$ then depend only on the properties of the material traversed and not on the properties of the particle traversing the material. The functions $A_0(t)$, $A_1(t)$, $A_2(t)$, introduced by Eyges are related to our M_0, M_1, M_2 by

$$A_0(t)=2M_0(t),$$
$$A_1(t)=2[tM_0(t)-M_1(t)],$$
$$A_2(t)=2[t^2M_0(t)-2tM_1(t)+M_2(t)].$$

The mean-square projected scattering angle from R_0 to R is given by

$$\langle\vartheta^2(R_0\rightarrow R)\rangle_{Av}=\tfrac{1}{2}A_0(t)=(M/m)[N_0(\rho)-N_0(\rho_0)],$$

and the mean-square transverse displacement from R_0 to R is given by

$$\langle y^2(R_0\rightarrow R)\rangle_{Av}=\tfrac{1}{2}A_2(t)=t^2M_0(t)-2tM_1(t)+M_2(t)$$
$$=(m/Z^4M)[\rho^2\{N_0(\rho)-N_0(\rho_0)\}$$
$$-2p\{N_1(\rho)-N_1(\rho_0)\}+\{N_2(\rho)-N_2(\rho_0)\}].$$

Consequently,

$$(m/M)\langle\vartheta^2(R_0\rightarrow R)\rangle_{Av} \quad \text{and} \quad (Z^4M/m)\langle y^2(R_0\rightarrow R)\rangle_{Av}$$

are functions only of $\rho_0=(Z^2M/m)R_0$ and $\rho=(Z^2M/m)R$ for a given material within the limits of the approximation made above.

In many cases $G(\rho)$ can be represented fairly well for a given material by an equation of the form $G(\rho)=A\rho^n$ where A and n are constants of the materials. Then N_0, N_1, and N_2 are given, respectively, by $A\rho^{n+1}/n+1$, $A\rho^{n+2}/n+2$, $A\rho^{n+3}/n+3$. Thus, in the case of Ilford C_2 emulsions, $G(\rho)$ is represented within a few percent by the above formula with $A=2.93\times10^{-2}n=-1\cdot128$, with

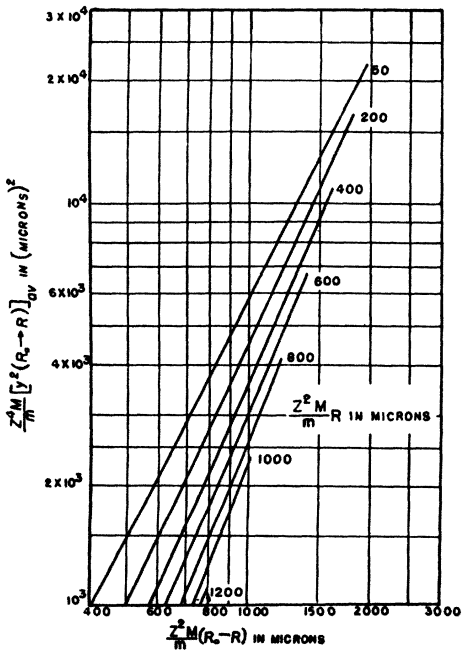

FIG. 2. Mean-square projected transverse displacement, $\langle y^2(R_0\rightarrow R)\rangle_{Av}$, for a particle of charge + or −Ze and mass m in traveling from a residual range R_0 to a residual range R in Ilford C_2 emulsion. (M = proton mass).

M = proton mass and length measured in microns.[3] In Figs. 1 and 2 are plotted $(m/M)\langle\vartheta^2(R_0\rightarrow R)\rangle_{Av}$ and $(Z^4M/m)\langle y^2(R_0\rightarrow R)\rangle_{Av}$ as functions of $\rho_0-\rho=(Z^2M/m)\times(R_0-R)$ for various values of ρ_0 and ρ. These curves should be useful in determining the mass of a particle from the curvature of tracks in an Ilford C_2 emulsion.

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* On leave from Case Institute of Technology during the summer of 1948.

[1] S. Lattimore, Nature **161**, 518 (1948); V. Goldschmidt-Clermond, D. T. King, H. Muirhead, and D. M. Ritson, Proc. Phys. Soc. (London) **61**, 183 (1948).

[2] L. Eyges, Phys. Rev. **74**, 1534 (1948).

[3] These values were obtained by using the range-energy relation for Ilford C_2 emulsions and the formula

$$g(E)=(a/E^2)\ln(bEM)^{\frac{1}{2}}$$

with $a=2.48\times10^{-4}$ (Mev)²/micron, $\ln b=16.58$ with b in (proton mass ×Mev)$^{-1}$, given by S. A. Gousmit and W. T. Scott (Phys. Rev. **74**, 1537 (1948)). The formula is applicable over the range from 200 to 2000 microns for protons.

On the Definition of the "Effective Range" of Nuclear Forces

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December 6, 1948

IN his discussion of the neutron-proton force, Blatt[1] uses the effective range r_e defined by the equation

$$k\cot\delta=-a^{-1}+\tfrac{1}{2}r_ek^2 \tag{1}$$

where k is the wave vector, δ the phase for s-scattering, and a the "scattering radius." Terms proportional to k^4 are neglected.

It is the purpose of this note to point out that the effective range can be expressed by elementary means in terms of the wave function for zero energy.

We start from the well-known relation:[2]

$$\frac{d}{dk^2}\left(\frac{u'}{u_R}\right)=-\frac{1}{u(R)^2}\int_0^R u^2dr \tag{2}$$

where u and u' are the wave function and its derivative, and R is any radius. For sufficiently large distance

$$u=A\sin(kr+\delta) \tag{3}$$

and (2) becomes

$$\cot(kR+\delta)-k\csc^2(kR+\delta)[R+(d\delta/dk)]$$
$$=-2kA^{-2}\csc^2(kR+\delta)\int_0^R u^2dr. \tag{4}$$

Choosing a value of R such that

$$kR+\delta=(n+\tfrac{1}{2})\pi \tag{5}$$

with integer n,

$$\frac{d\delta}{dk}=2A^{-2}\int_0^R u^2dr-R. \tag{6}$$

Using the identity

$$\int_0^R\sin^2(kr+\delta)dr=\tfrac{1}{2}R+(1/4k)\sin2\delta \tag{7}$$

which holds by virtue of (5), we have

$$\frac{d\delta}{dk}=2\int_0^R [A^{-2}u^2-\sin^2(kr+\delta)]dr+(1/2k)\sin 2\delta. \quad (8)$$

By making R tend to infinity, and some simple substitutions

$$2\frac{d}{dk^2}(k\cot\delta)=-2\csc^2\delta\int_0^\infty [A^{-2}u^2-\sin^2(kr+\delta)]dr. \quad (9)$$

According to (1) r_e is equal to (9) taken for $k=0$. For small k

$$\sin(kr+\delta)\sim\sin\delta(1+kr\cot\delta)=\sin\delta[1-(r/a)].$$

Choosing

$$A=\csc\delta, \quad (10)$$

we have finally

$$r_e=-2\int_0^\infty \{u^2(r)-[1-(r/a)]^2\}dr. \quad (11)$$

Here $u(r)$ is the zero-energy wave function, normalized according to (3) and (10) in such a way that the integrand of (11) vanishes at large distances.

[1] J. M. Blatt, Phys. Rev. **74**, 92 (1948), Eq. (1).
[2] H. A. Bethe and R. Peierls, Proc. Roy. Soc. **A149**, 176 (1935), Eq. (7).

Mass Spectrographic Assignment of Rubidium Isotopes*

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WE have used a mass spectrograph to investigate rubidium isotopes produced by bombardment of bromine (ammonium bromide) with helium ions in the Berkeley 60-inch and 184-inch cyclotrons. The 60° deflection spectrograph is similar to that of Lewis and Hayden[1] but with larger dimensions and all-metal construction.[2]

In each bombardment with 20- to 100-Mev helium ions there was a good yield of rubidium activity with half-life about 6 hours. The rubidium activities were separated from the target material using 20–30 micrograms of inactive rubidium carrier and divided into two portions. One part was further purified and used for decay and absorption measurements. The other major portion was placed on the tungsten filament of the mass spectrograph as the nitrate or chloride. The Rb^+ ions produced by heating this filament were analyzed by the instrument and caught on a photographic plate. The mass scale was fixed by the lines of natural Rb^{85} and Rb^{87}. Lines at masses 81 and 82 were shown to be radioactive both by the photographic transfer technique (Fig. 1), and by counting with a Geiger counter provided with a narrow slit. The radioactivity of natural Rb^{87} is far too weak to interfere with these experiments. With 80-Mev helium ions, 5.0-hour Rb^{81} predominated in the mixture, and with 20-Mev helium ions almost pure 6.3-hour Rb^{82} was obtained. Otherwise, the similar half-lives would have made characterization of the radiations, which are listed in Table I, very difficult. The signs of the particles were determined with a crude 180° deflection beta-spectrograph. The energies listed in Table I were obtained with this instrument or from absorption measurements with aluminum, beryllium, or lead, as indicated.

TABLE I. Radiations from rubidium isotopes.

Isotope	Half-life	Radiations	Produced by
Rb^{81}	5.0 hours	β^+ 0.9 Mev (abs. Al) ϵ^- 0.2 Mev (spect.) γ 0.8 Mev (abs. Pb) K x-rays (abs. Al, Be)	Br-α-$2n$ Br-α-$4n$
Rb^{82}	6.3 hours	β^+ 0.9 Mev (abs. Al) γ 1.0 Mev (abs. Pb) K x-rays (abs. Al, Be)	Br-α-n Br-α-$3n$

There are approximately equal numbers of positrons and conversion electrons from Rb^{81}. There are several x-rays and gamma-rays per positron, so that the decay is 60 to 80 percent by electron capture. The possibility of a short-lived krypton daughter complicates the interpretation of these radiations. For Rb^{82} the positron to conversion electron ratio is probably greater than five. Again, there are more x-rays and gamma-rays than positrons, corresponding to 80 to 90 percent electron capture.

The previously reported[3] 6.5-hour rubidium activity assigned to Rb^{84} was presumably Rb^{82}, or a mixture of Rb^{82} and Rb^{81}. No description of the radiations was reported.

Attempts to observe a krypton daughter of Rb^{81} have shown no positive results. This fact is consistent with the recent assignment of the 34-hour krypton (Kr^{79} or Kr^{81})[4] activity to Kr^{79} by Woodward, McCown, and Pool.[5] Our experiments were not very sensitive for radiations as weak as those reported for the 13-second and 55-second krypton activities,[4] and we can make no statement concerning them as daughters of Rb^{81}.

Experiments are under way to characterize some longer-lived activity due to Rb^{83} and Rb^{84} produced in these same bombardments. Barber[6] has reported a 40-day positron emitter which he attributed to Rb^{84}.

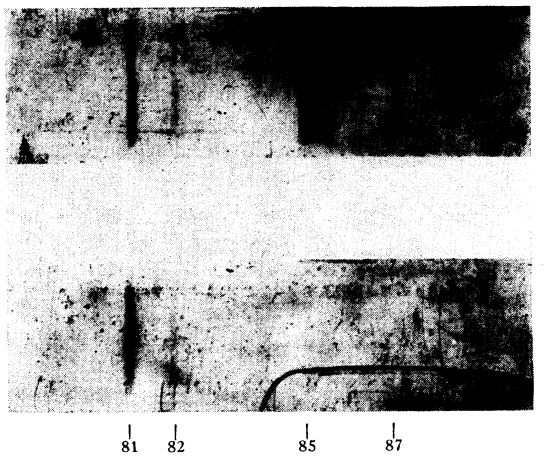

FIG. 1. The original plate shows natural Rb^{85} and Rb^{87} and radioactive Rb^{81} and Rb^{82}. The "transfer" plate is placed emulsion-to-emulsion with the original for several hours before either is developed, to locate radioactive material.