Note on Dirac's Theory of Magnetic Poles

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DIRAC'S result that $e_0g_0=hc/4\pi$ [1] where e_0 is the elementary electric point charge and g_0 the elementary point magnetic pole may be obtained very simply from classical electrodynamics and quantum mechanics.

The electromagnetic momentum per unit volume in a vacuum due to an electric field F and a magnetic field H is equal to the vector product of F and H divided by $4\pi c$. With $F=e/r^2$ and $H=g/r^2$ this gives for the angular momentum in the field around a charge e and pole g the value eg/c. This result was obtained by J. J. Thomson about the year 1900.[2]

According to quantum-mechanical theory any angular momentum must be an integral or half-odd integral multiple of $h/2\pi$ so that $eg=nhc/2\pi$ or $(n+\frac{1}{2})hc/2\pi$ where $n=0, 1, 2, 3\cdots$.

If we take the half-odd value, then $hc/4\pi$ is the smallest possible value of eg and so $e_0g_0=hc/4\pi$.

[1] P. A. M. Dirac, Phys. Rev. **74**, 817 (1948).
[2] J. J. Thomson, *Elements of the Mathematical Theory of Electricity and Magnetism* (1900).

The Magnetic Moments of the Neutron and Proton

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THE method of calculating the correction to the magnetic moment of the electron given recently by the author[1] has been applied to the calculation of the magnetic moments of the neutron and proton in the pseudoscalar meson theory. The calculations are carried out for the charged and uncharged theories, using the theory of holes to describe the behavior of the heavy particles. A consistantly relativistic treatment has been used throughout, the only approximation being in the use of second-order perturbation theory. Finite results are obtained for all the quantities calculated.

One may write for the interaction energy of the nucleon and meson fields in the presence of a static magnetic field:[2]

Neutral mesons

$$\mathcal{H}_{\text{int}}=(4\pi)^{\frac{1}{2}}f[\psi_P^*(\sigma\cdot\nabla\phi-\gamma_S\pi)\psi_P+\psi_N^*(\sigma\cdot\nabla\phi-\gamma_S\pi)\psi_N]+2\pi f^2[(\psi_P^*\gamma_S\psi_P)(\psi_P^*\gamma_S\psi_P)+(\psi_N^*\gamma_S\psi_N)(\psi_N^*\gamma_S\psi_N)].$$

Charged mesons

$$\mathcal{H}_{\text{int}}=(8\pi)^{\frac{1}{2}}f[\psi_P^*(\sigma\cdot D\phi-\gamma_S\pi^*)\psi_N+\psi_N^*(\sigma\cdot D^*\phi^*-\gamma_S\pi)\psi_P]+4\pi f^2[(\psi_P^*\gamma_S\psi_N)(\psi_N^*\gamma_S\psi_P)+(\psi_N^*\gamma_S\psi_P)(\psi_P^*\gamma_S\psi_N)].$$

Here f is the coupling constant; ψ_P, ψ_N the (quantized) wave functions of the proton and neutron, respectively; ϕ, π the (quantized) meson field and its conjugate momentum; $D\equiv\nabla-ieA_0$, A_0 is the vector potential of the static magnetic field, and finally $\gamma_S=i\alpha_1\alpha_2\alpha_3$. It may be mentioned that the terms quadratic in the coupling constant are not unique;[2] however, both possible versions of the Hamiltonian lead to the same magnetic moments.

Using now the exact solutions of the Dirac equation for the nucleons and of the Klein-Gordon equation for the mesons (both with an external homogeneous magnetic field present), we calculate the contribution of $\mathcal{H}_{\text{int}}$ to the x energy, for the special state in which the nucleon has simply the energy M, and no mesons are present.[1] This choice of the state was essential in avoiding the divergencies in the calculation of the magnetic moment of the electron, and is also essential here for the proton. However, for the case of the neutron any state would do, and we choose this special state only for simplicity of calculation. Since the mean value of $\mathcal{H}_{\text{int}}$ is zero, we must resort to second-order perturbation theory. The calculation is straightforward, proceeding in a very similar fashion to that for electrons. We quote only the results:

Neutral theory

$$\mu_N=0,$$

$$\mu_P=-\frac{4}{\pi}(f\mu)^2\frac{1}{\delta^2}\left(\frac{1}{4}+\frac{\delta^2}{2}-\frac{\delta^3}{2(4-\delta^2)^{\frac{1}{2}}}\cos^{-1}\frac{\delta}{2}-\frac{\delta^2(1-\delta^2)}{2}\log\frac{1}{\delta}\right).$$

Charged theory

$$\mu_N=\frac{-4}{\pi}(f\mu)^2\frac{1}{\delta^2}\left(1-\frac{\delta(2-\delta^2)}{(4-\delta^2)^{\frac{1}{2}}}\cos^{-1}\frac{\delta}{2}-\delta^2\log\frac{1}{\delta}\right),$$

$$\mu_P=\frac{4}{\pi}(f\mu)^2\frac{1}{\delta^2}\left(\frac{1}{2}-\delta^2-\frac{\delta}{(4-\delta^2)^{\frac{1}{2}}}(2-4\delta^2+\delta^4)\cos^{-1}\frac{\delta}{2}+\delta^2(2-\delta^2)\log\frac{1}{\delta}\right).$$

Here μ_N, μ_P are the additions (resulting from the mesons) to the magnetic moments of the neutron and proton, respectively, in units of the nuclear magneton, δ the ratio of meson to proton (or neutron) mass, and μ is the meson mass. By simply adding the results of the neutral and charged theories one may obtain the results for the "charge symmetrical" theory.

The numerical calculation for reasonable values[3] of the meson mass and coupling constant give results which are in disagreement with experiment, both in the charged and in the symmetrical theory. Whether the difficulty here lies in the model (pseudoscalar mesons with a pseudovector type of coupling) or in the use of perturbation theory can only be decided by an exact calculation with this model. Table I shows the results which should be compared with

TABLE I. Values of μ_N and μ_P.

Meson mass	Neutral theory	Charged theory	Symmetrical theory
325, electron masses	$\mu_N=0$ $\mu_P=-2.8$	$\mu_N=-8.0$ $\mu_P=2.5$	$\mu_N=-8.0$ $\mu_P=-0.3$
200, electron masses	$\mu_N=0$ $\mu_P=-2.8$	$\mu_N=-9.3$ $\mu_P=3.6$	$\mu_N=-9.3$ $\mu_P=0.8$