Letters to the Editor

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On the Mesic Charge

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WHEN we adopt Dirac's hole theory, it is necessary that the electron in the negative state satisfy the same wave equation satisfied by the positron in the positive state. As a matter of fact, this is fulfilled in the case of the electromagnetic field, which is described by a vector field. In this case, the wave equation of an electron is

$$\left(\frac{i\hbar}{c}\frac{\partial}{\partial t}+\frac{e}{c}A_0+i\hbar\rho_1\boldsymbol{\sigma}\,\mathrm{grad}-\frac{e}{c}\mathbf{A}\rho_1\boldsymbol{\sigma}-\rho_3 mc\right)\psi=0. \qquad \text{(I)}$$

On the other hand, in the case of a pseudovector field, the wave equation is*

$$\left(\frac{i\hbar}{c}\frac{\partial}{\partial t}+\frac{e}{c}A_0\rho_1+i\hbar\rho_1\boldsymbol{\sigma}\,\mathrm{grad}-\frac{e}{c}\mathbf{A}\boldsymbol{\sigma}-\rho_3 mc\right)\psi=0. \qquad \text{(II)}$$

Let ψ be a solution of Eq. (II) in a positive energy state, then $\bar{\psi}$, the complex conjugate of ψ, must satisfy

$$\left(\frac{i\hbar}{c}\frac{\partial}{\partial t}-\frac{e}{c}A_0\bar{\rho}_1+i\hbar\bar{\rho}_1\bar{\boldsymbol{\sigma}}\,\mathrm{grad}+\frac{e}{c}\mathbf{A}\bar{\boldsymbol{\sigma}}+{}_3\bar{\rho}mc\right)\bar{\psi}=0.$$

Now, putting $\psi^*=\rho_2\sigma_y\bar{\psi}$, the equation satisfied by ψ^* becomes

$$\left(\frac{i\hbar}{c}\frac{\partial}{\partial t}+\frac{e}{c}A_0\rho_1+i\hbar\rho_1\boldsymbol{\sigma}\,\mathrm{grad}-\frac{e}{c}\mathbf{A}\boldsymbol{\sigma}-\rho_3 mc\right)\psi^*=0$$

which is the *same* equation as (II) itself.

As the time factor $e^{i\omega t/\hbar}$ is contained in the ψ^*, ψ^* is considered as the wave function of $-\omega$, the negative energy state. Since the solution in the positive energy state corresponds exactly to the solution in the negative one, a hole in the negative state must be considered as an electron in the positive state. The hole theory proper, however, cannot be applied in this case because there is no charge conservation. Thus the possibility of the application of Dirac's hole theory depends critically upon the form of the interaction.

TABLE I. Definition of interaction constants.*

Type	Interaction terms
Scalar	$-(\tau^+/K)f_1\rho_3U-4\pi cf_2\tau^+U^+-(1/K^2)f_2\tau^+\rho_1\boldsymbol{\sigma}\,\mathrm{grad}U$ +Hermit conj.
Vector	$-(\tau^+/K)g_1\rho_1\boldsymbol{\sigma}\mathbf{U}-4\pi cg_2\tau^+\rho_2\boldsymbol{\sigma}\mathbf{U}^+ +(4\pi cg_1/K)\tau^+\,\mathrm{div}\mathbf{U}+(\tau^+/K^2)g_2\rho_3\boldsymbol{\sigma}\,\mathrm{rot}\mathbf{U}$ +Hermit conj.
Pseudoscalar	$-(\tau^+/K)f_1\rho_2U-4\pi f_2c\tau^+\rho_1U^+-(f_2/K^2)\tau^+\boldsymbol{\sigma}\,\mathrm{grad}U$ +Hermit conj.
Pseudovector	$-(\tau^+/K)g_1\boldsymbol{\sigma}\mathbf{U}+4\pi cg_2\tau^+\rho_3\boldsymbol{\sigma}\mathbf{U}^++(\tau^+/K)4\pi cg_1\rho_1\,\mathrm{div}\mathbf{U}^+ +(\tau^+/K^2)g_2\rho_2\boldsymbol{\sigma}\,\mathrm{rot}\mathbf{U}$ +Hermit conj.

* U; field variable; U^+: canonical conj. of U.

This idea can be applied immediately to the meson field. Considering only the nuclear field between nucleons, we wish to investigate under what conditions a hole in the negative energy state of the nucleon can be considered as a nucleon with the inverse mesic charge in the positive energy state. As we shall discuss in the following, the various forms of the interaction behave differently in this respect. In doing the calculations we must, in the complex wave functions, distinguish the imaginary number for the electric charge from that for the mesic charge, denoting them, for instance, by i and j respectively. ($i^2=-1$, $j^2=-1$.)

Case 1. *Scalar Mesic Field.* The solution of the equation of a nucleon with mesic charge (f_1, f_2) in the negative energy state corresponds to the solution of a nucleon with mesic charge (f_1, $-f_2$) in the positive energy state, where f_1, f_2 are the two interaction constants defined in Table I.

If we take account of conservation of mesic charge, the hole theory cannot be held but a case $f_1=0$.

Case 2. *Pseudoscalar Mesic Field.* The solution of the wave equation of a nucleon in the negative energy state corresponds to the solution of a nucleon with unchanged mesic charge in the positive energy state. Therefore, the hole theory cannot hold because there is no conservation of mesic charge.

Case 3. *Vector Mesic Field.* A hole in the negative energy state of a nucleon with mesic charge (g_1, g_2) can be regarded as a nucleon with inverse mesic charge ($-g_1$, $-g_2$) in the positive energy state, so that there is conservation of mesic charges. Thus the hole theory may be used for both interactions.

Case 4. *Pseudovector Mesic Field.* The solution of the wave equation for a nucleon with mesic charge (g_1, g_2) in the negative state corresponds to the solution for a nucleon with mesic charge (g_1, $-g_2$) in the positive energy state. Requiring conservation of mesic charge, the hole theory can only hold if we put $g_1=0$.

Only future experiments may decide whether there are really mesic charges, positive and negative. For this purpose, we must investigate phenomena proportional to the first power of the mesic charge; for instance the mesic-magnetic force. Since it is unlikely that a negative proton exists, it is rather probable that a hole in the negative state of a nucleon corresponds to a nucleon with ***inverse*** mesic charge in the positive state. If there are really mesic charges with a charge conservation theorem, then many types of interaction between nucleons and mesic field can be excluded. Further, one may expect the existence of cascade showers of the hard component of the cosmic rays in the very high energy region. Further research on these will be reported elsewhere.

* We assume the pseudovector type interaction.