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On the Experimental Value of the Fine Structure Constant

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TAUB and Kusch[1] have recently measured the ratio of the magnetic moments of electron and proton, with accuracy hitherto unattained, by measuring the ratio of the "flop-frequencies" of the electron and proton spins in a strong magnetic field. The result is

$$\mu_P/\mu_e=(1.51927\pm0.00010)\times10^{-3}. \tag{1}$$

On the other hand, Nafe and Nelson[2] have measured with great precision the hyperfine structure separation of the ground state of the H-atom. Their result for the frequency ν corresponding to the separation is

$$\nu=1420.410\pm0.006\text{ Mc.} \tag{2}$$

Taub and Kusch have combined the results (1) and (2) to give an accurate determination of the fine-structure constant, $\alpha=e^2/\hbar c$.

The Fermi hyperfine-structure formula,[3] when modified to include (*a*) Breit's relativistic correction,[4] (*b*) the effect of the reduced mass,[5] and (*c*) the electromagnetic correction to the electron magnetic moment,[6] may be written, for hydrogen,

$$\nu=\alpha^2\left[\frac{16}{3}\left(\frac{\mu_P}{\mu_e}\right)cR_\infty(1+\tfrac{3}{2}\alpha^2)\times\left(1+\frac{m}{M}\right)^{-3}\left(1+\frac{\alpha}{2\pi}\right)^2\right]. \tag{3}$$

Here the last three terms in the square bracket are, in the order of their appearance, the corrections (*a*), (*b*), and (*c*) above. R_∞ is the Rudberg constant, and c is the velocity of light. Equation (3) is regarded as an equation for the unknown.

Experimentally, the least accurately known quantity in (3) is the ratio of moments. However, there are also theoretical uncertainties in the derivation of (3), *viz.*:

(1) It has been assumed that the proton has a point magnetic dipole, whereas meson theory would indicate a current distribution over the range of the nuclear forces. If this is taken as $\hbar/\mu c$(μ = meson mass = 285 m), and if only the extra moment $\mu_P-\mu_D$ (Dirac moment) is so distributed, the right-hand side of (3) should be corrected by an amount of the relative order of magnitude

$$-\frac{\mu_P-\mu_D}{\mu_P}\alpha\frac{m}{\mu}\approx-1.6\times10^{-5}. \tag{4}$$

(2) The electron has also been assumed to have a point magnetic dipole, in diagreement with Schwinger's result that the electromagnetic correction represents a "smeared" dipole distributed over a region of the order of the Compton wave-length. This may give a correction to Eq. (3) of the relative order

$$-\frac{\alpha^2}{2\pi}\approx-0.8\times10^{-5}. \tag{5}$$

(3) The ratio of the electron moment to the Bohr magneton has been assumed to have the theoretical value, $1+\alpha/2\pi=1.00116$. The average of the experimental values is slightly higher (1.00119); if it were taken, the theoretical value of ν would be increased by a relative amount $+6\times10^{-5}$.

The result for $1/\alpha$ would, in each case, be changed by one-half of these fractions.

Equations (1), (2), and (3) give the result

$$1/\alpha=137.041\pm0.005. \tag{6}$$

This value is in disagreement with the value given by Dumond and Cohen,[7] namely,

$$1/\alpha=137.021\pm0.007 \tag{7}$$

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[1] We are indebted to Drs. Taub and Kusch for information concerning their results before publication.

[2] John E. Nafe and Edward B. Nelson, Phys. Rev. **73**, 718 (1948).

[3] E. Fermi, Zeits. f. Physik **60**, 320 (1930).

[4] G. Breit, Phys. Rev. **35**, 1447 (1930).

[5] G. Breit and E. R. Meyerott, Phys. Rev. **72**, 1023 (1947).

[6] P. Kusch and H. M. Foley, Phys. Rev. **72**, 1256 (1947); J. Schwinger, Phys. Rev. **73**, 416 (1948).

[7] J. W. M. Dumond and E. R. Cohen, Rev. Mod. Phys. **20**, 82 (1948).

(but it does not disagree with the earlier value 137.030±0.015 of Birge).[8]

In order to check whether the discrepancy with the value of Dumond and Cohen is real, we have carried out least-square calculations similar to those of Dumond and Cohen, but with some differences in input data. Referring to Table VII of Dumond and Cohen, we have made the following changes:

(1) The value of e^2/m, from the refractive index for x-rays (item 4 of their table) must be corrected for scattering by the atomic nuclei. The electrons give a contribution proportional to Ze^2/m, and the nucleus adds $(Ze)^2/MA$ to this (A = atomic weight, M = proton mass), so that the effect of the electrons is multiplied by $1+mZ/MA$. This correction changes the input value of e^2/m by a relative amount of -2.7×10^{-4}.

(2) We have omitted the direct determination of hc/e^2 from the x-ray fine structure (item 7) since this input datum should, according to recent theory, be corrected upwards by an (absolute) amount $1/2\pi$, because of the extra magnetic moment of the electron. (If it is so corrected, it becomes higher than our result for hc/e^2, *viz.*, 137.11, so that its inclusion among the input data would further increase our result for $1/\alpha$.)

(3) We have omitted items 8, 9, and 10 entirely because their probable errors are large. This was done merely to simplify the numerical work. This omission of a small part of the input data *ought* not to influence the result by an amount comparable to or greater than the probable error.

(4) The remaining items 1, 2, 3, 5, and 6 were used without change.

The least-squares method of combining these data (using the Faraday, Avogadro's number, and Planck's constant as unknowns) gives

$$1/\alpha = 137.033 \pm 0.007. \qquad (8)$$

The error was determined, as in the article of Dumond and Cohen, by projecting the error ellipsoid on the α-direction.

The large difference between the values (7) and (8) compared to the errors quoted is not entirely due to the correction of item 4. About one-half (0.006) of the change is the result of neglecting items 7, 8, 9, and 10, and shows an inconsistency between these items and the first six. There are other inconsistencies in the input data; one is the long-recognized smallness of the experimental value of h/e (item 6, as determined from the Duane-Hunt limit). We find further that neglecting item 4 in addition to items 7, 8, 9, and 10 leads to the value $1/\alpha = 137.040$; hence item 4 is inconsistent with the others. Fluctuations in the result due to neglect of a *small* part of the input data are, of course, to be expected, but the probable error assigned to the result ought to cover these fluctuations. The errors assigned in (7) and (8) are seen to be too small. The error-ellipsoid method of computing errors, which is based on the multiplication of the Gaussian distributions of individual experiments to obtain a resultant Gaussian, does not adequately take into account inconsistencies in the input data.

We conclude that there is no definite discrepancy between the new value (6) of $1/\alpha$ and the value computed from other experiments. The new value is probably the more reliable.

The largest inconsistency in the input data seems to exist (still) for the Duane-Hunt limit (item 6). The least-squares solution leading to (8) also leads to a value of h/e which is larger that the experimental value by a relative amount of 4.7×10^{-4}, a discrepancy of 1.6 times the experimental error given. However, this discrepancy happens to have practically no effect on the value of hc/e^2 (although it has larger effects on e and h separately). If item 6 is neglected entirely (this is equivalent to making item 6 agree with the other data), $1/\alpha$ is changed by an absolute amount of only -0.001. Therefore, if there were any essential disagreement between the new value (6) of $1/\alpha$ and the value from other experiments, it would not be because of the Duane-Hunt limit.

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[8] Raymond T. Birge, Rev. Mod. Phys. **13**, 233 (1941).