

since a rigid or liquid nucleus as a whole would have no orbital momentum in its lowest state.

The scheme proposed by Mayer follows exactly the order in a potential well. It achieves the breaks at the correct places by the assumption of a very strong spin-orbit coupling at high angular momentum values.

A summary of the three schemes is given in Table I. All three schemes give, of course, the empirical shell numbers and a statistical correlation with observed spins and moments. A decision between the schemes may be hoped for through discussion of new data which may tend to tip the scales in a definite direction, or by more theoretical work. Among the latter would be a refined calculation of the effects of the Coulomb forces on the density distribution in a nucleus, improved treatment of the many body problem, and better understanding of the spin-orbit coupling in nuclei.

It should be emphasized that the existence and the characteristics of nuclear shell structure have become now much more clearly established than formerly in spite of the ambiguities in their interpretation. Particularly there is a definite correlation between spin and shell structure. This does not mean necessarily that the individual particle model is better than hitherto assumed. The shell structure in nuclei, is, however, so pronounced an effect that one may hope to obtain an interpretation even on basis of such a crude approximation as the individual particle model.

\* This letter has been written on request by the editor of the *Physica Review*, who received the papers, reference 1 and 2, by the same mail.

<sup>1</sup> Eugene Feenberg and Kenyon C. Hammack, *Phys. Rev.* **75**, 1877 (1949).

<sup>2</sup> L. W. Nordheim, *Phys. Rev.* **75**, 1894 (1949).

<sup>3</sup> Maria G. Mayer, *Phys. Rev.* **75**, 1969 (1949).

## On Closed Shells in Nuclei. II

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THE spins and magnetic moments of the even-odd nuclei have been used by Feenberg<sup>1,2</sup> and Nordheim<sup>3</sup> to determine the angular momentum of the eigenfunction of the odd particle. The tabulations given by them indicate that spin orbit coupling favors the state of higher total angular momentum. If strong spin-orbit coupling, increasing with angular momentum, is assumed, a level assignment different from either Feenberg or Nordheim is obtained. This assignment encounters a very few contradictions with experimental facts and requires no major crossing of the levels from those of a square well potential. The magic numbers 50, 82, and 126 occur at the place of the spin-orbit splitting of levels of high angular momentum.

Table I contains in column two, in order of decreasing binding energy, the levels of the square well potential. The quantum number gives the number of radial nodes. Two levels of the same quantum number cannot cross for any type of potential well, except due to spin-orbit splitting. No evidence of any crossing is found. Column three contains the usual spectroscopic designation of the levels, as used by Nordheim and Feenberg. Column one groups together those levels which are degenerate for a three-dimensional isotropic oscillator potential. A well with rounded corners will have a behavior in between these two potentials. The shell grouping is given in column five, with the numbers of particles per shell and the total number of particles up to and including each shell in column six and seven, respectively.

Within each shell the levels may be expected to be close in energy, and not necessarily in the order of the table, although the order of levels of the same orbital angular momentum and different spin should be maintained. Two exceptions,  ${}_{11}\text{Na}^{23}$

with spin 3/2 in stead of the expected  $d_{5/2}$ , and  ${}_{25}\text{Mn}^{55}$  with 5/2 instead of the expected  $f_{7/2}$ , are the only violations.

Table II lists the known spins and orbital assignments from magnetic moments<sup>4</sup> when these are known and unambiguous, for the even-odd nuclei up to 83. Beyond 83 the data is limited and no exceptions to the assignment appear.

Up to  $Z$  or  $N=20$ , the assignment is the same as that of Feenberg and Nordheim. At the beginning of the next shell,  $f_{7/2}$  levels occur at 21 and 23, as they should. At 28 the  $f_{7/2}$  levels should be filled, and no spins of 7/2 are encountered any more in this shell. This subshell may contribute to the stability of  $\text{Ca}^{48}$ . If the  $g_{3/2}$  level did not cross the  $p_{1/2}$  or  $f_{5/2}$

TABLE I.

Osc. No.	Square well	Spect. term	Spin term	No. of states	Shells	Total No.		
0	1s	1s	1s <sub>1/2</sub>	2	2	2		
1	1p	2p	1p <sub>1/2</sub>	4	6	8		
			1p <sub>3/2</sub>	2				
2	1d	3d	1d <sub>5/2</sub>	6	12	20		
			1d <sub>3/2</sub>	4				
			2s	2s			2s <sub>1/2</sub>	2
3	1f	4f	1f <sub>7/2</sub>	8	22	28?		
			1f <sub>5/2</sub>	6				
			2p <sub>3/2</sub>	4				
			2p <sub>1/2</sub>	2				
4	2p	3p	1g <sub>9/2</sub>	10	32	50		
			1g <sub>7/2</sub>	8				
			2d <sub>5/2</sub>	6				
			2d <sub>3/2</sub>	4				
			3s	3s			3s <sub>1/2</sub>	2
5	1h	6h	1h <sub>11/2</sub>	12	44	82		
			1h <sub>9/2</sub>	10				
			2f <sub>7/2</sub>	8				
			2f <sub>5/2</sub>	6				
			3p <sub>3/2</sub>	4				
			3p <sub>1/2</sub>	2				
6	2g	6g	1i <sub>13/2</sub>	14	126	126		
			1i <sub>11/2</sub>	12				
			3d	5d			3d <sub>5/2</sub>	6
			4s	4s			4s	4

levels, the first spin of 9/2 should occur at 41, which is indeed the case. Three nuclei with  $N$  or  $Z=49$  have  $g_{9/2}$  orbits. No  $s$  or  $d$  levels should occur in this shell and there is no evidence for any.

The only exception to the proposed assignment in this shell is the spin 5/2 instead of 7/2 for  $\text{Mn}^{55}$ , and the fact that the magnetic moment of  ${}_{27}\text{Co}^{59}$  indicates a  $g_{7/2}$  orbit instead of the expected  $f_{7/2}$ .

In the next shell two exceptions to the assignment occur. The spin of 1/2 for  $\text{Mo}^{95}$  with 53 would be a violation, but is experimentally doubtful. The magnetic moment of  $\text{Eu}^{158}$  indicates  $f_{5/2}$  instead of the predicted  $d_{5/2}$ . No  $h_{11/2}$  levels appear. It seems that these levels are filled in pairs only,

which does not seem a serious drawback of the theory as this tendency already shows up at the filling of the  $g_{9/2}$  levels. Otherwise, the agreement is satisfactory. The shell begins with  ${}_{51}\text{Sb}$ , which has two isotopes with  $d_{5/2}$  and  $g_{7/2}$  levels, respectively, as it should. The thallium isotopes with 81 neutrons and a spin of 1/2 indicate a crossing of the  $h_{11/2}$  and 3s levels. This is not surprising, since the energies of these levels are close together in the square well. The assignment

TABLE II. Spins of even-odd nuclei.

No. of neutrons or protons	Element	Odd protons		Odd neutron		$O = \text{neutron}$ $X = \text{proton}$						No. odd $p$ or $n$	Levels	
		Mass No. orbit	Mass No. orbit	Element	Mass No. orbit	spin								
						$s$	$p$	$d$	$f$	$g$	$h$			
1	H	1 $s$	3 $s$	He	3 $s$	⊗							1	1s
3	Li	7 $p$					X						3	$p_{3/2}$
5	B	11 $p$		Be	9 $p$	⊗							5	
7	N	15 $p$		C	13 $p$	⊗							7	$p_{1/2}$
9	F	19 $s$					X						9	
11	Na	23					X						11	$d_{3/2}$
13	Al	27 $d$						X					13	
15	P	31					X						15	$s_{1/2}$
17	Cl	35 $d$	37 $d$	S	33	⊗							17	$d_{3/2}$
19	K	39 $d$	41 $d$				X						19	
21	Sc	45 $f$								X			21	$f_{7/2}$
23	V	51								X			23	
25	Mn	55						X					25	
27	Co	59 $g$								X			27	
29	Cu	63	65				X						29	$p_{3/2}$
31	Ga	69					X						31	
33	As	75					X						33	
35	Br	79	81	Zn	67 $f$		X						35	$f_{5/2}$
37	Rb	85 $f(5/2)$	87 $p(3/2)$				X	⊗					37	
39	Y	89					X						39	$p_{1/2}$
41	Cb	93 $g$								X			41	$g_{9/2}$
43	Tc			Se	77	O							43	
45	Rh												45	
47	Ag	107 $p$	109 $p$	Kr	83 $g$	X				O			47	
49	In	113 $g$	115 $g$	Sr	87 $g$					⊗			49	
51	Sb	121 $d(5/2)$	123 $g(7/2)$				X	X					51	$g_{7/2}$
53	I	127		Mo	95	O	X						53	$d_{5/2}$
55	Cs	133 $g$	137 $g$					X					55	
57	La	139 $g$						X					57	
59	Pr	141						X					59	
61	Pm												61	
63	Eu	151	153 $f$	Cd	111 $s$	O		X					63	
65	Tb	159		Cd, Sn	8, 8	O	X						65	$h_{11/2}$
67	Ho	165		Sn	117 $s$	⊗				X			67	
69	Tm			Sn	119 $s$	⊗				X			69	
71	Lu	175 $g$								X			71	
73	Ta	181 $g$								X			73	
75	Re	185	187	Xe	129 $s$	O		X					75	
77	Ir	191 (1/2)	193 (3/2)	Xe	131 $d$	X	⊗						77	$d_{3/2}$
79	Au	197 $d$		Ba	135 $d$	⊗							79	
81	Tl	203	205	Ba	137 $d$	X	⊗						81	$s_{1/2}$
83	Bi	209 $h$								X			83	$h_{9/2}$

demands that there be no spins of 9/2 in this shell, and none have been found. No  $f$  or  $p$  levels should occur and, except for  $\text{Eu}^{153}$ , there is no indication of any.

The spin and magnetic moment of  ${}_{83}\text{Bi}$ , indicating an  $h_{9/2}$  state, is a beautiful confirmation of the correct beginning of the next shell. Here information begins to be scarce. The spin and magnetic moment of  $\text{Pb}^{207}$  with 125 neutrons interpret as  $p_{1/2}$ . This is the expected end of the shell since  $7i$  and  $4p$  have practically the same energy in the square well model. No spins of 11/2 and no  $s$ ,  $d$ , or  $g$  orbits should occur in this shell, and the data indicates none.

The prevalence of isomerism towards the end of a shell, noticed by Feenberg and Nordheim, is easily understood by this assignment. These are the regions where levels with very different spins are adjacent. These ground and isomeric states should also have different parity.

Thanks are due to Enrico Fermi for the remark, "Is there any indication of spin-orbit coupling?" which was the origin of this paper.

<sup>1</sup> Eugene Feenberg, Phys. Rev. **75**, 320 (1949).

<sup>2</sup> Eugene Feenberg and K. C. Hammack, Phys. Rev. **75**, 1877 (1949).

<sup>3</sup> Lothar Nordheim, Phys. Rev. **75**, 1894 (1949). The author is indebted to these authors for having obtained copies of both references 2 and 3 before publication.

<sup>4</sup> H. H. Goldsmith and D. R. Inglis, *The Properties of Atomic Nuclei. I.* (Information and Publications Division, Brookhaven National Laboratory.)

### Erratum: Charged Particles Emitted by Carbon Bombarded by 90-Mev Neutrons

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THROUGH inadvertence, the curve for the energy distribution of protons leaving the carbon target was omitted. It is presented here.

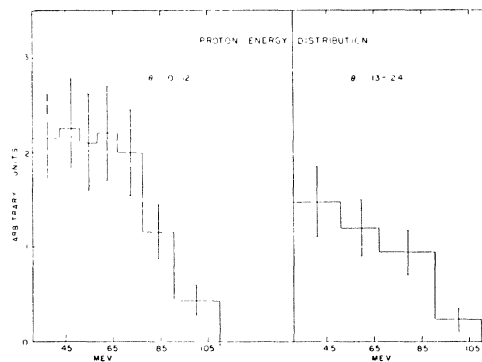


FIG. 2 (a) Energy distribution of the protons leaving carbon within 12° of the neutron beam. (b) Energy distribution of the protons leaving carbon with angles between 13° and 24° from the neutron beam. The errors indicated are standard deviations based only on the number of tracks used to determine the values.