

Here γ_0 is Kittel's⁶ γ , the ratio of magnetic moment to angular momentum; $H=H_0-NJ_s$; \mathbf{H}' is the magnetizing force due to the transverse magnetization and to eddy currents and is related to α and β by the usual electromagnetic equations. When $h_{ox}=h_{oy}=0$ and $\alpha=\beta=0$, there are no eddy currents, and \mathbf{H}' may be derived from a scalar potential U ; then Eq. (2) reduces to the previous equations if, as before, the second-order terms in w are neglected. When there is no magnetic anisotropy or internal stress, all the g 's vanish; then the solution for a polycrystalline aggregate is identical with that for a single crystal, provided eddy current fields are negligible.

Since the equations are linear, the contributions of (g_1, g_2) and of (h_{ox}, h_{oy}) to α, β , and \mathbf{H}' may be found separately and superposed. The former is time-independent; this is the static problem treated in references 2, 3, 5. The terms in g_{11} etc. are important only for a crystal with Oz a direction of easy magnetization; in the calculation of Holstein and Primakoff, these and the terms in C are neglected. The object is to evaluate the mean squares of α and β over the specimen volume: then $\gamma_{av}=1-\frac{1}{2}[(\alpha^2)_{av}+(\beta^2)_{av}]$. It is in the integrations incidental to this mean-square evaluation that the function quoted by Rado emerges.

The complexity of this calculation is due to the variation of g_1 and g_2 with position. In the evaluation of the \mathbf{h}_0 term such complexity is lacking, since h_{ox} and h_{oy} are independent of (x, y, z) . If the terms in g_{11} etc. are again neglected, this problem is solved by a uniform α and β that produce uniform transverse magnetizing forces $H_x'=-J_s L\alpha$, $H_y'=-J_s M\beta$. For the alternating part of (α, β) , Eqs. (2) therefore reduce to

$$\begin{aligned} (H+LJ_s)\alpha+(j\omega/\gamma_0)\beta &= h_{ox}, \\ -(j\omega/\gamma_0)\alpha+(H+MJ_s)\beta &= h_{oy}; \end{aligned} \quad (3)$$

and if the determinant of this system is set equal to zero, Kittel's resonance condition is obtained. To this approximation, therefore, magnetic interactions between the crystals have no effect on the resonance condition.

If the terms in g_{11} etc. are appreciable, interaction enters; but there is no obvious reason why the function that occurs in the resonance condition in this problem should have any simple relation to the function that occurs in the mean squares in the static problem. The order of magnitude of the effect could perhaps be estimated by carrying out the calculation for a few special cases, such as plane parallel crystal boundaries or isolated spherical inclusions. If the effect is not negligible, the simplest procedure is to make it negligible by increasing H_0 .

¹ George T. Rado, Phys. Rev. **75**, 893 (1949).

² L. Néel, C. R. Acad. Sci. Paris **220**, 814 (1945); J. de Physique **9**, 193 (1948).

³ T. Holstein and H. Primakoff, Phys. Rev. **59**, 388 (1941).

⁴ See correction by Rado, Phys. Rev. **75**, 1451 (1949).

⁵ William Fuller Brown, Jr., Phys. Rev. **58**, 736 (1940). See also Phys. Rev. **59**, 528 (1941); **60**, 139 (1941).

⁶ Charles Kittel, Phys. Rev. **71**, 270 (1947); **73**, 155 (1948).

Radiations from Lu¹⁷⁷

DONALD G. DOUGLAS

Radiation Laboratory, McGill University, Montreal, Canada

May 4, 1949

SAMPLES of "Specpure"* Lu₂O₃ irradiated in the Chalk River pile, have been studied with a thin lens spectrometer. Secondary electrons from lead, gold and tin radiators reveal the presence of three gamma-rays of energies 112.2±0.6, 206.3±1.0, and 317.3±1.5 kev, the former two of comparable intensity, the latter much weaker. X-radiation agreeing well in energy with characteristic *K* radiation of Hf has also been observed.

A Kurie plot of the beta-spectrum intersecting the abscissa at 495±5 kev is straight for only the upper 100 kev. Regarding the departure from straightness as an indication of complexity, two lower end points at 169±10, and 366±25 kev, are obtained. These end points taken with the three gamma-rays are consistent with the decay scheme shown in Fig. 1.

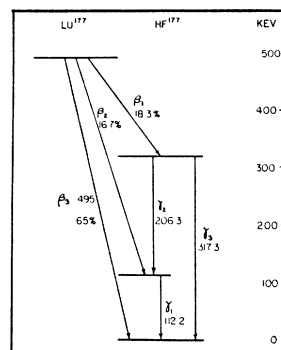


FIG. 1. Proposed decay scheme for Lu¹⁷⁷.

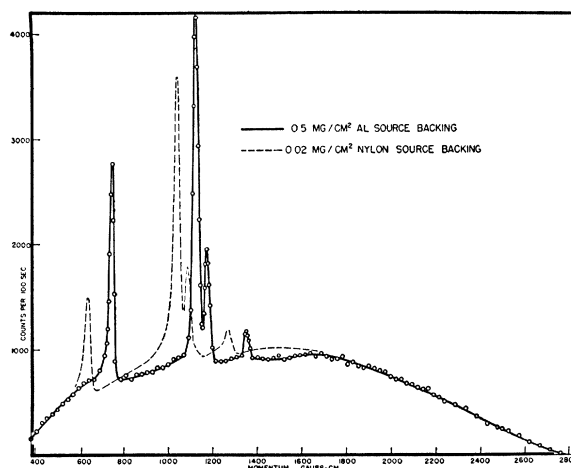


FIG. 2. Beta-spectrum of Lu¹⁷⁷. The dotted curve shows the effect of source charging.

Four conversion lines have been found at energies 46.9±0.5, 101.8±0.6, 109.9±0.6, and 141.4±0.8 kev (Fig. 2). The first three arise respectively from *K*, *L*, and *M*, conversion of the 112.2 kev gamma-ray, the last from *K* conversion of the 206.3 kev gamma-ray. Since addition of the electron binding energies of Hf gives better agreement with the gamma-energies found from the photoelectron spectrum than does addition of Lu binding energies it can be stated that the gamma-rays follow the beta-rays.

The end point 495 kev is to be compared with values 440, 520, 470 kev obtained by other workers¹ using absorption and cloud chamber methods. A gamma-ray energy 0.2 Mev, obtained by absorption,¹ is in agreement with that of 206.3 kev reported here. The weak 1.3 Mev gamma-ray found by Wilkinson and Hicks probably arises from an impurity. Values 6.8, 6.6 and 6.9 days have been reported for the half life.¹ A source was observed for 400 hours and the value 6.98±0.10 days obtained. There was no evidence of any contaminating activity differing appreciably in half life.

The great importance of using a well grounded source was demonstrated.² A typical run using an ungrounded Nylon backed source is shown by the dotted curve in Fig. 2. All the

lines are shifted toward lower energies. Later runs revealed even greater shifts, as much as 19 kev for the 46.9 kev line. Line positions were always closely reproducible when thin aluminum backing was used.

Source thicknesses for the beta-spectrum were estimated to be less than 0.1 mg/cm². Nylon windows 0.08 mg/cm² were used to close the argon-ethylene filled counter used as detector. No correction was made for window absorption.

The author is indebted to Professor J. S. Foster, Director of this Laboratory, for his keen interest in this work, and to Mr. J. S. Fraser for many valuable discussions and for assistance in taking spectrometer data.

* Obtained from Johnson Matthey, London, England.

¹ G. T. Seaborg and I. Perlman, *Rev. Mod. Phys.* **20**, 585 (1948).

² C. H. Braden *et al.*, *Phys. Rev.* **74**, 1539 (1948).

Anomalies in the Microwave Spectrum of Methyl Cyanide and Methyl Iso-Cyanide

HARALD H. NIELSEN

*Mendenhall Laboratory of Physics, The Ohio State University,
Columbus, Ohio*

April 25, 1949

THE microwave spectra of methyl cyanide and methyl isocyanide have been studied by Gordy¹ who has observed three lines in the former case and four lines in the latter case which can be shown to originate with rotational transitions in an excited vibration state. The electric moment lies along the axis of symmetry so that $\Delta K = 0$. From the general positions of the lines in the spectrum Gordy has identified them with transitions $J = 1 \rightarrow J = 2$.

The lines observed by Gordy occur at the frequencies 36870.94 mc, 36903.40 mc and 36942.15 mc in the spectrum of methyl cyanide and at 40313.37 mc, 40364.07 mc, 40366.55 mc and 40424.49 mc in the spectrum of methyl iso-cyanide. The general pattern in the former case consists of two lines separated by a frequency interval of about 72 mc with a third line almost in the center of these. In the latter example the two extreme lines are separated by an interval of about 111 mc with two lines near the center of these separated by an interval of 1.5 mc. These patterns cannot be explained on the basis of a rigid symmetric rotator, but require that certain degeneracies be removed.

It seems reasonable to suppose that the excited vibration states which here are involved are the frequencies ω_8 in the two molecules, namely 380 cm⁻¹ and 290 cm⁻¹. These are doubly degenerated perpendicular vibrations and associated with each of them there exists one unit of internal angular momentum of vibration. A Coriolis interaction between vibration and rotation splits the levels where $K \neq 0$ into two components, the separation between components being $Kh/2\pi^2 I_{zz}c$. Examination of the Hamiltonian for a symmetric molecule reveals that the component state corresponding to the situation where the molecular framework remains at rest (i.e., $K = l$) may further be split by an l -type doubling.²

The magnitude of the l -type doubling may be calculated accurately only when the normal coordinate problem has been solved and the shape of the molecule is known. An estimate may nevertheless, be made which may be regarded as a fair approximation. Taking B to be 0.335 cm⁻¹ and $\omega_8 = 290$ cm⁻¹ for methyl cyanide these splittings may be estimated to be about 0.0047 cm⁻¹ and 0.0015 cm⁻¹ respectively for the states $J = 1$ and $J = 2$. The appropriate selection rule yields a pattern like the one observed with the two extreme lines separated by about 0.0031 cm⁻¹ (93 mc) and two lines near the center separated by a much smaller interval of the order of 3×10^{-5} cm⁻¹ (1.0 mc). For methyl cyanide the splittings for $J = 1$ and $J = 2$ are respectively predicted to be 0.003 cm⁻¹ and 0.001 cm⁻¹ and the line pattern will be similar to the

former, the extreme lines in the pattern being here separated by about 0.002 cm⁻¹ (i.e. 60 mc) with two lines near the center, their separation being about 1.3×10^{-5} cm⁻¹ (i.e. 0.4 mc). Since the two central lines would have the same frequency except for centrifugal distortion it is proposed that the single line observed by Gordy in the latter case is in reality two unresolved lines. Because of the roughness of the approximation the agreement with experiment is regarded as satisfactory. A further check is the comparison of the interval between the extreme lines in the two cases. Experimentally this ratio is about 0.65 and the theory indicates a ratio of 0.64.

The theory of l -type doubling in polyatomic molecules will be discussed in detail later with specific applications to the above two molecules.

The author wishes to express his gratefulness to Professor Walter Gordy for having made his measurements available to him.

¹ Walter Gordy, private communication.

² It is found that in the original work of Nielsen and Shaffer (*J. Chem. Phys.* **11**, 140 (1943)) an error in sign was introduced. Two terms of equal magnitude which were cancelled against each other should have been added together. When this correction is made the coefficient of the matrix element in the case of a linear molecule becomes $(Be^2/2\omega_8) [1 + 4 \sum_s \xi_{ss} \lambda_s / (\lambda_{s'} - \lambda_s)]$, where $\lambda_s = 4\pi^2 c^2 \omega_s^2$, instead of $(Be^2/2\omega_8)$. This change appears to bring the theoretical relations for l -type splitting in linear molecules into substantial agreement with experiment.

Polarization of the Vacuum

JERZY RAYSKI*

N. Copernicus University, Torun, Poland

April 7, 1949

THE problem of the vacuum polarization and the closely connected question of the photon self-energy has recently caused much confusion. As is well known, Heisenberg¹ obtained for the photon self-energy a logarithmic divergence while Wentzel² found a finite but non-zero value. On the other hand Schwinger³ believes that there is no doubt in the gauge invariance of the formalism and demonstrates that the photon self-energy is strictly zero.

The reason for this discrepancy is as follows: The proofs of the gauge invariance are based on the assumption that the Schrödinger equation possesses a solution. But in frames of the quantized field theory (at least in case of two coupled fields), this assumption is obviously not true. The quantized field equations possess no solution and, after the unitary transformations, the gauge invariance must be restored again (similarly to the mass and charge constants which must be restored by renormalization).

The basic expressions for the vacuum polarization containing products of $\bar{\Delta}$ and $\Delta^{(l)}$ functions and their derivatives are mathematically meaningless at the light cone⁴ so that by formal operations on non-existing integrals one may obtain any result one likes. The most straightforward evaluation yields for the photon self-energy a quadratically divergent result. Heisenberg obtained a logarithmic divergence only because he arbitrarily subtracted a part of the effect.

In order to give a mathematical meaning to such expressions as e.g. products of $\bar{\Delta}$ and $\Delta^{(l)}$ functions on the light cone a special regularizing procedure is needed. This may be achieved by replacing $\bar{\Delta} \Delta^{(l)}$ by a (finite) sum

$$\bar{\Delta}(m_0) \cdot \Delta^{(l)}(m_0) \rightarrow \sum_i C_i \bar{\Delta}(m_i) \Delta^{(l)}(m_0).$$

m_0 is the mass of the electron while m_i , $i = 1, 2, \dots$ play only an auxiliary role and are assumed to tend to infinity. C_i are constants ($C_0 = 1$) such that

$$\sum C_i = 0, \quad \sum C_i m_i^2 = 0. \quad (I)$$