underground cosmic-rays. Since the position of the bend of the energy spectrum depends on the life of pi-mesons, the latter can be determined from the position of the intensity-depth curve. In this way the life of pi-mesons was estimated in the previous note, but, as this calculation was carried out only roughly in that the energy spectra of pi- and mu-mesons were not properly considered, we now improve the calculation taking into account also the energy loss of mu-mesons passing through the earth crust.

We now carry through the calculation in the following way. We assume that the energy spectrum of pi-mesons when they are produced be of the form

$$\text{const.}E^{-\gamma-1}dE. \tag{1}$$

These pi-mesons are produced in proportion to the intensity of the primary particles which decrease with increasing depth from the top of the atmosphere as

$$I = I_0 \exp(-l/\Lambda), \tag{2}$$

l being the atmospheric depth and Λ the mean free path of the primaries. In calculating the spectrum of mu-mesons produced by these pi-mesons we first neglect the absorption of pi-mesons in the atmosphere. Of course, this does not correspond to the actual case but we can thus obtain in some way an extreme value for the intensity of mu-mesons. The calculation was then performed assuming that the absorption of pi-mesons in the atmosphere is just as large as that of the primary particles.

The energy spectrum of mu-mesons having been obtained, we can calculate the intensity-depth curve if the range-energy relation of the mu-meson is known. In order to determine this relation we estimate the energy loss of mu-mesons passing through the matter on the plausible assumption that the energy loss is caused by ionization, emission of bremsstrahlung and creation of pairs. Then the required intensity-depth curve is obtained by substituting the energy expressed as the function of range into the integral energy spectrum of mu-mesons at sea level.

The calculated intensity-depth relation is given in Fig. 1. The theoretical curves in the figure have been calculated by assuming the following numerical values for various constants: the mass of pi- and mu-mesons are 286 and 217 times electron mass, the mean life of pi-mesons is 1×10^{-8} sec.,[3] $\gamma = 1.8$, and $\Lambda = 125$ g cm^{-2}. The upper curve is the result by neglecting the absorption in the atmosphere and the lower is that obtained by assuming that this absorption is just as large as the absorption of the primary particles. The broken lines indicate the corresponding curves neglecting the radiation and pair-creation processes of mu-mesons. The experimental results of Wilson[4] and Nishina *et al.*,[5] marked by $\times$ and 0, are found to lie between the two theoretical curves. In view of our imperfect knowledge about the production and absorption of pi-mesons as well as of the statistical inaccuracy in their mean lifetime, we cannot yet draw any decisive conclusion from our result, but our result seems to show that the life of pi-mesons measured at Berkeley for the artificial mesons does not contradict the cosmic-ray data so that we may identify the cosmic-ray mesons with mesons produced at Berkeley. Our result seems further to show that no extra process other than the known electromagnetic ones is needed to explain the behavior of deep rays so far as the present status of our experimental knowledge is concerned.

A detailed account will be published in the Progress of Theoretical Physics.

[1] S. Hayakawa, Prog. Theor. Phys. **3**, 199 (1948).
[2] K. I. Greisen, Phys. Rev. **73**, 521 (1948).
[3] J. R. Richardson, Phys. Rev. **74**, 1720 (1948).
[4] V. C. Wilson, Phys. Rev. **53**, 337 (1938).
[5] Nishina, Sekido, Miyazaki, and Masuda, Phys. Rev. **59**, 401 (1941); Y. Miyazaki, to be published.

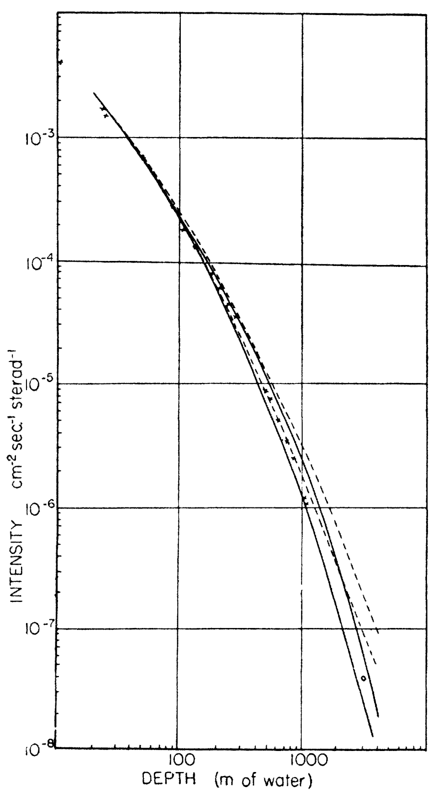

FIG. 1 Intensity-depth relation.

Crystal Interactions in Ferromagnetic Resonance

WILLIAM FULLER BROWN, JR.
Sun Physical Laboratory, Newtown Square, Pennsylvania
March 30, 1949

RADO[1] has recently attempted to correct for magnetic interactions between crystal grains in the theory of ferromagnetic resonance. His correction does not contain the anisotropy constant—a fatal defect, since for crystals of zero magnetic anisotropy the correction must vanish.

Rado's method was to replace the applied magnetic field by a "local" field, the formula for which he took from a calculation by Néel[2] on the approach to saturation. A calculation identical with Néel's was published earlier by Holstein and Primakoff;[3] their Eq. (25) contains the formula quoted (incorrectly[4]) in Rado's letter. Even earlier, I had used a similar method in a related problem.[5] Each such problem requires separate investigation; no single local field formula can claim general validity. In the work cited, I derived several such formulas, each valid for a different type of deviating force. Rado's problem requires direct treatment.

Consider an ellipsoidal specimen with principal axes Ox, Oy, Oz; demagnetizing factors L, M, N; large constant applied field (O, O, H_o); small alternating applied field (h_{ox}, h_{oy}, O); magnetization of constant magnitude J_s in variable direction (α, β, γ) with $\alpha \ll 1$, $\beta \ll 1$, $\gamma \doteqdot 1-\frac{1}{2}(\alpha^2+\beta^2)$; anisotropy and stress energy density (to the second order of small quantities)

$$w(x, y, z, \alpha, \beta) = g_1\alpha + g_2\beta + \tfrac{1}{2}(g_{11}\alpha^2 + 2g_{12}\alpha\beta + g_{22}\beta^2), \tag{1}$$

where the g's are functions of (x, y, z).

The dynamic generalization of Eqs. (32) of reference 5 is

$$\begin{aligned} C\nabla^2\alpha - (J_sH+g_{11})\alpha - g_{12}\beta + J_sH_x' - J_s\dot\beta/\gamma_o &= g_1 - J_sh_{ox}, \\ C\nabla^2\beta - g_{12}\alpha - (J_sH+g_{22})\beta + J_sH_y' + J_s\dot\alpha/\gamma_o &= g_2 - J_sh_{oy}. \end{aligned} \tag{2}$$

Here γ_o is Kittel's[6] γ, the ratio of magnetic moment to angular momentum; $H=H_o-NJ_s$; $\mathbf{H}'$ is the magnetizing force due to the transverse magnetization and to eddy currents and is related to α and β by the usual electromagnetic equations. When $h_{ox}=h_{oy}=0$ and $\alpha=\beta=0$, there are no eddy currents, and $\mathbf{H}'$ may be derived from a scalar potential U; then Eq. (2) reduces to the previous equations if, as before, the second-order terms in w are neglected. When there is no magnetic anisotropy or internal stress, all the g's vanish; then the solution for a polycrystalline aggregate is identical with that for a single crystal, provided eddy current fields are negligible.

Since the equations are linear, the contributions of (g_1, g_2) and of (h_{ox}, h_{oy}) to α, β, and $\mathbf{H}'$ may be found separately and superposed. The former is time-independent; this is the static problem treated in references 2, 3, 5. The terms in g_{11} etc. are important only for a crystal with Oz a direction of easy magnetization; in the calculation of Holstein and Primakoff, these and the terms in C are neglected. The object is to evaluate the mean squares of α and β over the specimen volume: then $\gamma_{av}=1-\frac{1}{2}[(\alpha^2)_{av}+(\beta^2)_{av}]$. It is in the integrations incidental to this mean-square evaluation that the function quoted by Rado emerges.

The complexity of this calculation is due to the variation of g_1 and g_2 with position. In the evaluation of the $\mathbf{h}_o$ term such complexity is lacking, since h_{ox} and h_{oy} are independent of (x, y, z). If the terms in g_{11} etc. are again neglected, if the time factor is $e^{j\omega t}$, and if eddy current fields are negligible, this problem is solved by a uniform α and β that produce uniform transverse magnetizing forces $H_x'=-J_sL\alpha$, $H_y'=-J_sM\beta$. For the alternating part of (α, β), Eqs. (2) therefore reduce to

$$\begin{aligned}(H+LJ_s)\alpha+(j\omega/\gamma_o)\beta&=h_{ox},\\ -(j\omega/\gamma_o)\alpha+(H+MJ_s)\beta&=h_{oy};\end{aligned} \tag{3}$$

and if the determinant of this system is set equal to zero, Kittel's resonance condition is obtained. To this approximation, therefore, magnetic interactions between the crystals have no effect on the resonance condition.

If the terms in g_{11} etc. are appreciable, interaction enters; but there is no obvious reason why the function that occurs in the resonance condition in this problem should have any simple relation to the function that occurs in the mean squares in the static problem. The order of magnitude of the effect could perhaps be estimated by carrying out the calculation for a few special cases, such as plane parallel crystal boundaries or isolated spherical inclusions. If the effect is not negligible, the simplest procedure is to make it negligible by increasing H_o.

[1] George T. Rado, Phys. Rev. **75**, 893 (1949).
[2] L. Néel, C. R. Acad. Sci. Paris **220**, 814 (1945); J. de Physique **9**, 193 (1948).
[3] T. Holstein and H. Primakoff, Phys. Rev. **59**, 388 (1941).
[4] See correction by Rado, Phys. Rev. **75**, 1451 (1949).
[5] William Fuller Brown, Jr., Phys. Rev. **58**, 736 (1940). See also Phys. Rev. **59**, 528 (1941); **60**, 139 (1941).
[6] Charles Kittel, Phys. Rev. **71**, 270 (1947); **73**, 155 (1948).

Radiations from Lu^{177}

DONALD G. DOUGLAS
Radiation Laboratory, McGill University, Montreal, Canada

May 4, 1949

SAMPLES of "Specpure"* Lu_2O_3 irradiated in the Chalk River pile, have been studied with a thin lens spectrometer. Secondary electrons from lead, gold and tin radiators reveal the presence of three gamma-rays of energies 112.2 ± 0.6, 206.3 ± 1.0, and 317.3 ± 1.5 kev, the former two of comparable intensity, the latter much weaker. X-radiation agreeing well in energy with characteristic K radiation of Hf has also been observed.

A Kurie plot of the beta-spectrum intersecting the abscissa at 495 ± 5 kev is straight for only the upper 100 kev. Regarding the departure from straightness as an indication of complexity, two lower end points at 169 ± 10, and 366 ± 25 kev, are obtained. These end points taken with the three gamma-rays are consistent with the decay scheme shown in Fig. 1.

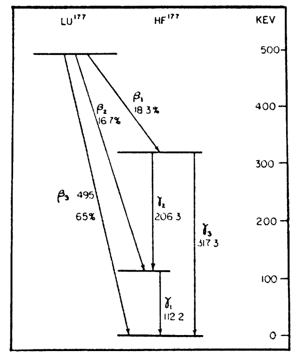

FIG. 1. Proposed decay scheme for Lu^{177}.

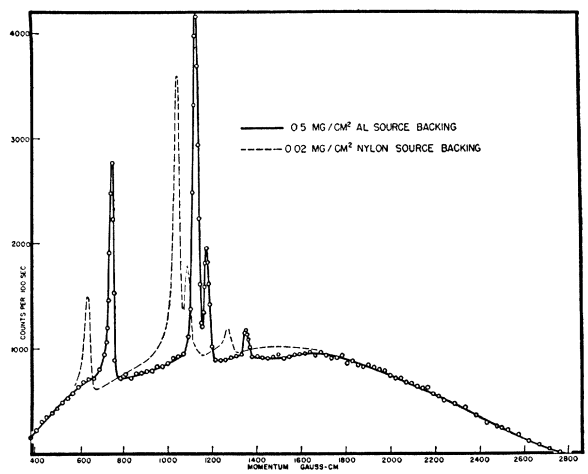

FIG. 2. Beta-spectrum of Lu^{177}. The dotted curve shows the effect of source charging.

Four conversion lines have been found at energies 46.9 ± 0.5, 101.8 ± 0.6, 109.9 ± 0.6, and 141.4 ± 0.8 kev (Fig. 2). The first three arise respectively from K, L, and M, conversion of the 112.2 kev gamma-ray, the last from K conversion of the 206.3 kev gamma-ray. Since addition of the electron binding energies of Hf gives better agreement with the gamma-energies found from the photoelectron spectrum than does addition of Lu binding energies it can be stated that the gamma-rays follow the beta-rays.

The end point 495 kev is to be compared with values 440, 520, 470 kev obtained by other workers[1] using absorption and cloud chamber methods. A gamma-ray energy 0.2 Mev, obtained by absorption,[1] is in agreement with that of 206.3 kev reported here. The weak 1.3 Mev gamma-ray found by Wilkinson and Hicks probably arises from an impurity. Values 6.8, 6.6 and 6.9 days have been reported for the half life.[1] A source was observed for 400 hours and the value 6.98 ±0.10 days obtained. There was no evidence of any contaminating activity differing appreciably in half life.

The great importance of using a well grounded source was demonstrated.[2] A typical run using an ungrounded Nylon backed source is shown by the dotted curve in Fig. 2. All the