On the Hot-Wire Length Correction

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TWO of the conclusions reached by F. N. Frenkiel[1] concerning the effect of finite length of a hot-wire anemometer upon measurement of turbulence intensity appear to be in error:

(1) Although $1/I_b^2(\equiv\langle e^2\rangle_{Av}/\langle e_0^2\rangle_{Av})$, the ratio of mean square measured voltage to ideal mean square voltage (for $R_y(s)=1$) approaches zero as $1/l$ when the wire length l approaches infinity, still the longitudinal turbulent energy measured by this hot-wire does not approach zero. The measured energy is proportional to $\langle e^2\rangle_{Av}=\langle e_0^2\rangle_{Av}/I_b^2$, and since $\langle e_0^2\rangle_{Av}\sim l^2$, $\langle e^2\rangle_{Av}$ increases proportionally to l as $l\to\infty$.

(2) A Gaussian longitudinal correlation function does not lead to $I_b=1$ when the turbulence is isotropic. If we substitute $R_x=\exp\{-\pi x^2/4L_x^2\}$ into the special form for isotropic turbulence, i.e.

$$1/I_b^2=\frac{1}{l}\int_0^l R_x(s)ds$$

we find

$$\frac{1}{I_b^2}=\frac{L_x}{l}\operatorname{erf}\left\{\frac{(\pi)^{\frac{1}{2}}}{2}\frac{l}{L_x}\right\}$$

which is less than unity when $l>0$. In fact, it is physically obvious that for any turbulent flow $1/I_b^2<1$ whenever the fluctuations are not perfectly correlated all along the hot-wire. This can also be seen immediately from a sketch of the general length-correction integrand, $[(l-s)R_y(s)]$.

It should furthermore be noted that in the two expressions for measured correlation as a function of true correlation (either the general relation for $R_{b,y}$ or the additional relation for isotropic turbulence), there is zero error due to finite wire length when the true correlation function is Gaussian.

[1] F. N. Frenkiel, Phys. Rev. **75**, 1263 (1949).

Elastic Constants and Internal Loss of Single Nickel Crystals

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SINCE the elastic constants of single nickel crystals have never been measured and are of some interest in magnetic studies, the velocities and approximate attenuations have been measured for two single nickel crystals. In preparing the crystals, 750 grams of electrolytic nickel of high purity (0.04 percent Co, 0.002 percent Fe) were melted in an alumina crucible (dextron bond, Norton ATM409) in a molybdenum wound resistance furnace in pure dry hydrogen. The bottom of the crucible was placed in the hottest part of the furnace and the charge melted. The furnace was cooled at the rate of 8°C/hour until the metal was well below the melting point. Several single crystals of 50 to 75 grams each were obtained and were carefully cut from the ingot with a thin carborundum wheel. The surfaces were etched, and the orientation determined approximately by reflection of light from the etch pits, which were of the (111) form. Surfaces were cut parallel to the (100) and (110) planes as nearly as possible, and were tested by x-rays using back reflection, then corrected by careful grinding and etching to within a few minutes of arc of the desired plane.

The crystal upon which most of the measurements were made was cut in the form of a disk about 2.5 cm in diameter and 0.472 cm thick, the circular surfaces being parallel to the (110) planes. The other crystal used for check purposes had its two major surfaces parallel to the (110) plane and 3.13 cm apart, and two other surfaces parallel to (001) and 1.35 cm apart. Other surfaces were irregular.

The velocities and attenuations were measured by pulsing methods. The most satisfactory arrangement, shown by Fig. 1,

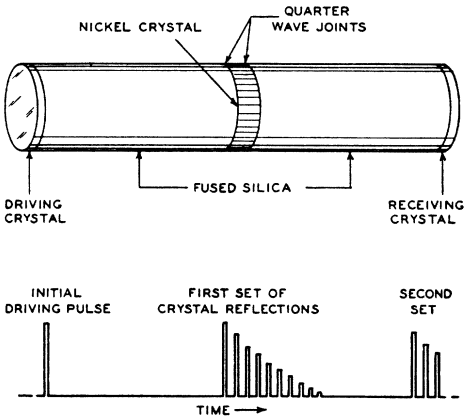

FIG. 1.

consisted in soldering longitudinal or shear wave crystals to the ends of two one-inch fused silica rods and connecting the nickel crystal to the polished ends of the rods by an approximate quarter wave-length of a poly-α-methyl styrene liquid and film of polystyrene. This provided a low mechanical impedance on either side of the nickel crystal, and a longitudinal or shear wave in the crystal was reflected back and forth giving a series of pulses in time and amplitude, from which the velocity and attenuation could be measured. The silica rods were long enough so that reflections from the ends did not arrive in time to interfere with the multiple pulses in the crystal. The most satisfactory way to determine the velocity was by making the pulses considerably longer than the time between reflections and adjusting the frequency until all the pulses added in phase. A number of such frequencies were determined, using the wide frequency range possible with the soldered crystals, and the number of wavelengths corresponding to each frequency determined. By this method a velocity determination good to better than one percent was possible with a crystal thickness of 0.5 cm.

TABLE I. Orientations and elastic constants.

Direction of propagation	Direction of particle motion	Type of mode	Equation for velocity	Measured velocity	Elastic constants (dynes/cm²)
110	110	Long.	$v=[(c_{11}+c_{12}+2c_{44})/2\rho]^{\frac{1}{2}}$	6.06×10^5 cm/sec.	$c_{11}+c_{12}+2c_{44}=6.53\times10^{12}$
110	1$\bar{1}$0	Shear	$v=[(c_{11}-c_{12})/2\rho]^{\frac{1}{2}}$	2.26×10^5	$c_{11}-c_{12}=0.90\times10^{12}$
110	001	Shear	$v=[c_{44}/\rho]^{\frac{1}{2}}$	3.65×10^5	$c_{44}=1.185\times10^{12}$
100	100	Long.	$v=[c_{11}/\rho]^{\frac{1}{2}}$	5.30×10^5	$c_{11}=2.50\times10^{12}$
100	010	Shear	$v=[c_{44}/\rho]^{\frac{1}{2}}$	3.65×10^5	$c_{44}=1.185\times10^{12}$

All three constants were derived from measurements made along the single [110] direction. Calculations show that a longitudinal and two shear waves can be generated, all having different velocities. There is no coupling between the longitudinal and shear modes so that shear motions are at right angles to the direction of propagation whereas the longitudinal motion is along the direction of motion. Table I shows the orientations, the associated elastic constants, the velocities determined and the calculated elastic constants, with $\rho=8.90$. From it one has the following values of elastic constants accurate to within a percent.

$c_{11}=2.50\times10^{12}$ dynes/cm^2; $c_{12}=1.60\times10^{12}$; $c_{44}=1.185\times10^{12}$.

The anisotropy factor $2c_{44}/(c_{11}-c_{12})$, is equal to 2.63.

The absorption is very high for a single metal crystal, indicating damping of the magnetic type. The two crystals gave somewhat different results but agreed in showing a considerably higher damping for shear waves than for longitudinal. Within the experimental error the attenuation measured was proportional to the square of the frequency, indicating a damping due to micro-eddy currents. At 10 megacycles the Q's for the first three orientations of the table were

$$Q_1=\frac{\pi}{\delta_1}=385;\quad Q_2=90;\quad Q_3=85.$$

A New Type of Focusing in a Magnetic Lens Field

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SINCE some time ago we have been using a β-lens spectrometer (see Fig. 1) which operates according to a new

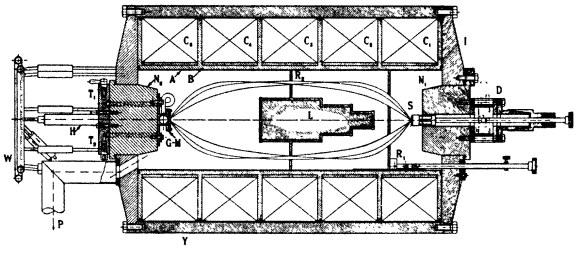

FIG. 1. Section of the β-spectrometer.

focusing principle. This was actually found when we were investigating the possibilities of decreasing the spherical abberation error by changing the field form along the axis of the lens. Such investigations have been made in our laboratory before.[1] Besides ordinary measurements of line profiles with G-M counters we also used a photographic tracing technique to investigate the behavior of the different electron paths in the spectrometer. This method was previously used here in another connection to study electron paths in a spectrometer.[2] We have found that if the magnetic gradient in the spectrometer is made sufficiently strong (the magnetic field having a minimum halfway between source and counter) a first or intermediate ring-formed "image" of the source is obtained midway between source and detector. If a ring-formed shutter is placed here, which transmits the beam, a second *point* image is formed at the G-M slit (see Fig. 2). It is found that quite a substantial solid angle of the total radiation can be utilized at a good resolving power. Thus at a resolving power of 4 percent a transmission of ~8 percent can be obtained. Furthermore, another essential advantage is that an ordinary small counter still can be used since we get essentially a point image. If wanted the resolution can, of course, be increased.

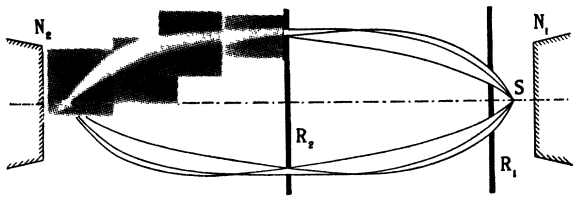

FIG. 2. A photographic record of the focused electron beam.

Thus we have also adjusted the spectrometer for a resolving power of 1.7 percent.

Some general features of this focusing system may briefly be mentioned here. To a certain radius of the central ring baffle a certain magnetic field gradient is related. In our case this gradient was found simply by changing the ratio of the current $I_{2,3,4}$ through the coils 2, 3, 4, to the current I through all coils at a given radius of central shutter. A series of measurements of this kind is given in Fig. 3. As can be seen, there

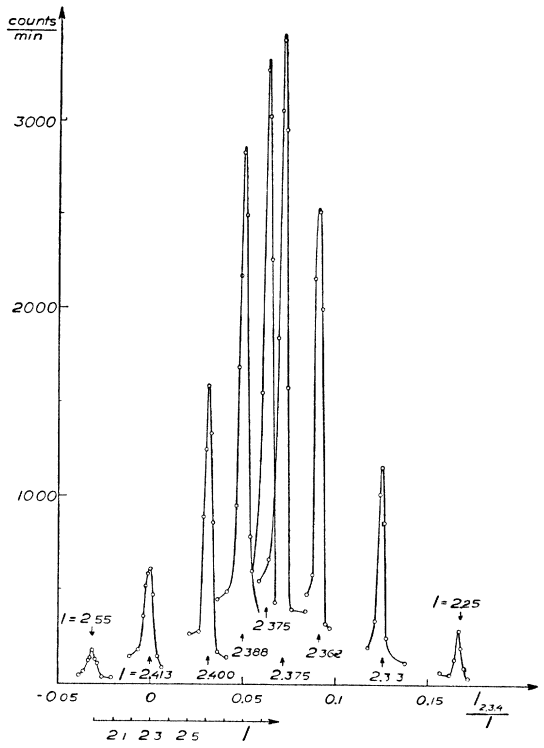

FIG. 3. Adjustment of the magnetic gradient for a given radius of the central shutter.

is a very pronounced maximum at a ratio of these two currents equal to 1:15 for the radius used. Another interesting feature is the strong dependence of the number of counts in the detector on the position of the source. Actually, if the source is moved only a few mm from its correct position no electrons will hit the G-M slit and be counted, no matter what the current is. This means that scattered radiation emanating from other places than the source will not contribute to the counting.

A full description of the spectrometer is given in Arkiv. für Fysik.

[1] Kai Siegbahn, Phil. Mag. **XXXVIII**, 162 (1946).
[2] Hilding Slätis, Arkiv. f. Mat., Astr. o. Fysik **32A**, No. 20 (1945).