Letters to the Editor

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Non-Radiating Maxwell Waves

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SINCE Bohr's suggestion of steady states, many attempts have been made to describe, mathematically, non-radiating electrodynamic oscillations, including non-radiating solutions of Maxwell's field equations. However, no one seems to have pointed out that a single restriction, consistent with those equations and equivalent to zero permeability, puts them in a form for which *all solutions* are non-radiating.

It is assumed that Maxwell's field equations are correct, with their original physical interpretations. It is also assumed that a component portion of the field, particularly that associated with transitions between states, can be described by the usual Maxwell-Lorentz equations. There is abundant evidence, however, of non-radiating oscillation within a steady state, and for brevity only the non-radiating oscillatory portion of the field will be here described. In Eqs. (1) and (2) Maxwell's $\mathbf{D}$ is replaced by $\mathbf{E}+4\pi\mathbf{P}$ and $\mathbf{B}$ by $\mathbf{H}+4\pi\mathbf{M}$; $\mathbf{u}$ and ρ are considered to be zero for this portion of the field.

$$\text{curl}\mathbf{H}=\dot{\mathbf{E}}/c+4\pi\dot{\mathbf{P}}/c \tag{1}$$

$$\text{curl}\mathbf{E}=-\dot{\mathbf{H}}/c-4\pi\dot{\mathbf{M}}/c \tag{2}$$

$$\mathbf{H}=\text{curl}\mathbf{A}-\text{grad}\Omega \tag{3}$$

$$4\pi\mathbf{M}=\text{curl}\mathbf{Q}+\text{grad}\Omega \tag{4}$$

$$\mathbf{E}=-\text{grad}\phi-\dot{\mathbf{A}}/c-\dot{\mathbf{Q}}/c \tag{5}$$

$$\text{div}\mathbf{A}+\dot{\phi}/c=0. \tag{6}$$

The potentials are unusual, but (3), (4) and (5) are obviously consistent with (1) and (2). Equation (6) is also easily proved to be consistent by standard methods. From the above equations the following inhomogeneous wave equations are obtained:

$$\nabla^2\phi-\ddot{\phi}/c^2=4\pi\,\text{div}\mathbf{P}-\text{div}\dot{\mathbf{Q}}/c \tag{7}$$

$$\nabla^2\mathbf{A}-\ddot{\mathbf{A}}/c^2=-4\pi\dot{\mathbf{P}}/c+\ddot{\mathbf{Q}}/c^2. \tag{8}$$

Equations (7) and (8) are as general as (1) and (2), but for the oscillation under discussion we make the particular restriction, compatible with (1)–(8):

$$\mathbf{Q}=-\mathbf{A}. \tag{9}$$

Equations (7) and (8) reduce to:

$$\nabla^2\phi=4\pi\,\text{div}\mathbf{P} \tag{10}$$

$$\nabla^2\mathbf{A}=-4\pi\dot{\mathbf{P}}/c. \tag{11}$$

These equations are easily solved for any given oscillation of $\dot{\mathbf{P}}$ and div$\mathbf{P}$, and the solutions are non-radiating because there is no wave equation. The non-radiative character of similar equations is well established in classical theory. Presumably Ω vanishes, but even if it does not, it cannot make Eqs. (10) and (11) radiative.

The assumption, here, of the correctness of classical theory does not include that portion of classical theory specifically applying to the electron, such as the Lorentz electron theory. On the contrary, one may consider the time average of charge density, ρ, to be distributed according to Schrödinger's equation, with a superposed charge density $-$div$\mathbf{P}$, which oscillates with twice the de Broglie frequency, without radiation. This is entirely consistent with classical (pre-electron) electromagnetic theory, and accordingly the total instantaneous charge density at any point is $\rho-$div$\mathbf{P}$, which may be considered to vary in the range between 0 and 2ρ. Any motion of the electron as a whole gives rise to the convection current density, $\mathbf{u}$, which is connected with radiation according to the Maxwell-Lorentz equations.

Present study indicates that a comprehensive theory can be formulated along these lines, the time-dependence of the charge density removing the chief difficulties encountered by such theories in the past.

Neutron-Hydrogen Mass Difference from the D–D Reactions*

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THE energy release in the reactions

$$D^2+D^2\rightarrow H^3+H^1+Q_1 \tag{1}$$

$$D^2+D^2\rightarrow He^3+n^1+Q_2 \tag{2}$$

has been determined by measurement of the energy of the singly charged H^3 and He^3 ions produced in a heavy ice target at a known angle with a deuteron beam of well-defined energy. The results have been used to evaluate the neutron-hydrogen mass difference as follows:

$$n^1-H^1=(Q_1-Q_2)+(H^3-He^3).$$

The mass difference of H^3 and He^3 is taken from measurements on the maximum energy of the β-rays from tritium. Two recent determinations[1,2] give a weighted mean of 18.3 ± 0.3 kev. In the second reference it is shown that the neutrino mass is less than 1.0 kev and hence can be disregarded in these calculations.

Deuterons mono-energetic to better than 0.1 percent were obtained from an electrostatic accelerator and analyzer previously described.[3] The analyzer was calibrated by measurements on the strong gamma-ray resonance in $F^{19}(p\alpha',\gamma)O^{16}$ which has been carefully standardized by Herb *et al.*[4] at 873.5 kev. The target was produced by allowing D_2O vapor to enter the target chamber adjacent to a flat face on a copper rod which was kept at liquid air temperature. The D_2O vapor condensed on the rod continuously thus insuring a fresh target surface at all times. Oil vapors were trapped out of the system by means of a liquid air trap at the diffusion pump.

The H^3 and He^3 ions were detected with a scintillation counter. The ZnS screen[5] was located in the vacuum system. Fluorescent light from the scintillations was collected by a hemispherical mirror, brought out through a Lucite window, and focused on the cathode of a 931-A electron multiplier. The pulses from the electron multiplier were amplified $\sim10^4$ times. A discriminator was used to eliminate the dark current pulses.

The energies of the H^3 and He^3 ions emerging from the target in the angular interval 134.5°–141.1° were measured by means of a double-focusing magnetic spectrometer.[6] In one set of measurements the energy of the doubly charged He^3 ions was also measured. For accurate comparison of the results for reactions (1) and (2), the majority of the observations were made on singly charged He^3 ions. The magnetic field in the spectrometer, which was determined by means of

a null reading fluxmeter,[7] could be held fixed to 0.05 percent and reproduced to 0.3 percent. Since the energies of the H^3 and He^3 ions are very dependent upon angle, the angular opening of 6.6° was the major factor in determining the experimental resolution (4 percent in energy).

The spectrograph was calibrated by observing the maximum energy protons and deuterons scattered into the spectrograph from a copper target and from the oxygen of the D_2O target. The energy of protons scattered from copper is almost independent of angle, whereas the energy of deuterons scattered from oxygen is very dependent upon angle. Thus it is possible to determine the fluxmeter calibration and also to check the direct measurement of the angle of observation.

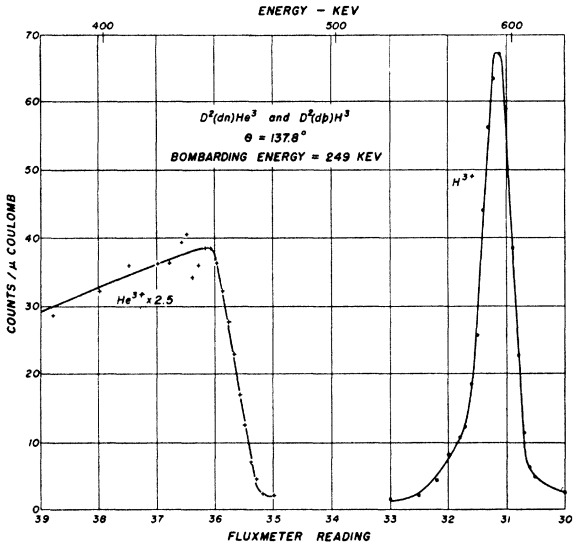

FIG. 1. Energies of H^3 and He^3 ions.

Figure 1 shows curves obtained with deuterons at a bombarding energy of 249 kev. The target was about 30 kev thick for the deuterons. This figure was ascertained by comparing the neutron intensity with that from a thick target. All runs were made while the neutron count was slowly increasing, which indicated that fresh ice was being deposited. The target was very thick for the He^3 ions due to the large stopping cross section of the target atoms for helium ions. The curve for the H^3 ions is similar to a thin target curve even though the target was ~30 kev thick for these ions. This apparent contradiction can be explained by the fact that at the mean angle of 137.8°, the energy of the H^3 particles increases with decreasing incident deuteron energy. Thus H^3 particles formed deep in the target receive more energy than those formed at the surface. However, those formed deep in the target must expend energy to reach the surface of the target. For thin smooth targets, calculations indicate that the H^3 particles entering the spectrograph from the surface have slightly greater energy than those from deeper in the target. This is contingent on the target being turned at such an angle that the paths for penetration of the deuterons and reemergence of the H^3 are nearly equal. A careful study of the results at different target angles confirmed this conclusion. Because of irregularities in the target surface, presumably arising from its crystalline nature, a few particles from deep in the target escape from the target with energies greater than those from the surface. This accounts for the "fillet" on the leading edge of the H^3 peak. The effect was more pronounced for thick targets, as was expected. For this reason only thin targets were employed, and the extrapolation of the front edge of the H^3 curve was taken as corresponding to the energy of particles entering at the minimum acceptance angle of the spectrograph, 134.5°. The slope of the front edge is consistent with the resolution expected from the spectrograph entrance and exit windows and from the dependence of the energy of the H^3 on angle. A small correction gave the energy for those entering at the mean acceptance angle of 137.8°. The corrected values agreed with the energy at the peak of the curve. An additional correction for one-half the energy equivalent of the target thickness was made. The point of half-maximum intensity on the "thick" target curves for He^3 was taken as corresponding to the energy of those He^3 ions leaving the surface of the target at 137.8°. The extrapolated end points of the He^3 curves properly corrected gave the same results. Fourteen separate curves were obtained at five different bombarding voltages. The weighted averages are $Q_1=4.036\pm0.022$ Mev, $Q_2=3.265\pm0.018$ Mev, $Q_1-Q_2=771\pm6$ kev.

The probable errors were computed by combining the observed statistical errors, which were very small, with estimated systematic errors. The systematic probable errors assumed were 0.3° in angle of observation, 0.6 percent in observed energy, and 0.2 percent in energy of deuterons. The small error in Q_1-Q_2 is obtained because only the difference between the H^3 and He^3 energies is significant in determining this number. The results for Q_2 are substantially in agreement with other recent determinations.[8] The value for Q_1 is significantly higher than the value 3.98±.02 previously determined.[9] The above data yield $n^1-H^1=789\pm6$ kev. This is in satisfactory agreement with the recent determinations from (p, n) reaction thresholds of 776 ± 10[10] and 782 ± 2 kev.[11] Combined with the mass spectroscopic[12] value for $2H^1-H^2=1.432\pm.002$ Mev, one obtains the binding energy of the deuteron as $2.221\pm.006$ Mev which is somewhat lower than the value $2.235\pm.009$ recently determined by Bell and Elliott.[13]

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** While on leave from the University of California.
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Note on the Use of the Shock Tube as an Intermittent Supersonic Wind Tunnel*

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THE use of a shock tube as an intermittent supersonic wind tunnel was discussed in a recent letter.[1] An expression was given for the ratio of the pressures on the two sides of the shock which would just make the flow behind the shock supersonic with respect to the tube.

In two notes published more recently elsewhere,[2,3] it was shown that there exists a maximum Mach number which can be produced behind a shock wave advancing into a gas at rest. For an infinite pressure ratio across the shock this limiting Mach number was shown to be 1.89 for air.

This is considerably lower than the value of 2.42 shown in the sample photograph of reference 1 and this apparent incompatibility will be discussed in the following.