On Spins, Moments, and Shells in Nuclei*

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Known values of nuclear spins and magnetic moments are used to assign orbital quantum numbers to the last odd particle. A consistent scheme is developed that permits the correlation of a wide variety of nuclear data. An interpretation is obtained of the general trends of nuclear spins, for the particular stability of nuclei containing 20, 50, or 82 neutrons or protons, for the occurrence of isomeric states, and for the allowed or forbidden character of β-decays for nuclei with odd mass numbers.

I. INTRODUCTION—NUCLEAR SHELLS

SINCE the early days of nuclear physics, attempts have been made to detect regularities in the structure of nuclei[1] in analogy to the construction principle and electron catalogue for atomic electrons. There is convincing evidence, as recently re-emphasized by Maria G. Mayer[2] for the particular stability of nuclei that contain neutrons or protons in numbers given in Table I. The differences should, of course, give the numbers of particles in subsequent closed shell configurations.

In the past, several attempts[3] have been made to explain these numbers on the basis of a one-particle picture. It is assumed that a reasonable first approximation can be obtained by having each particle moving in an average potential caused by the others. The most natural form for such a potential is in first approximation a simple well since the nuclear matter has essentially constant density and the range of the forces is smaller than the nuclear radius, at least for not too light nuclei. The order of terms in such a well with infinitely high walls, as worked out by Elsasser,[3] is given in Table II, together with the number of particle places. The principal feature of this order is that for a given total quantum number the state with the smallest number of radial nodes, i.e., the highest angular momentum, is energetically the lowest, which is just the opposite as in an atomic field. The order of states is not changed in a well with finite walls,[4] though, of course, there will be only a finite number of places.

A comparison between the numbers of Tables I and II fails to give a correlation above 20 particles. This negative result is not surprising since the one-particle picture is certainly a very crude approximation. There are so many degeneracies and the interaction between configurations is so strong that there will be an extensive mixing of many states which may change greatly the order in which new orbits are added and the character of the ground state of a particular nucleus may be largely accidental.

II. ASSIGNMENT OF ORBITS ON BASIS OF SPIN AND MAGNETIC MOMENT

There is however one more type of information available that throws light on the character of the ground states of nuclei, their spins and moments.[5] It is well known that the magnetic moments as functions of spin lie between two limits as pointed out first by Schmidt.[6] They are given by the assumption that the spin is due solely to the extra odd nucleon and that for the one limit its intrinsic spin and orbital momentum are parallel and for the

TABLE I. Nuclear configurations of particular stability.

No. of particles	2		8		20		50		82		126
Differences		6		12		30		32		44	
Shell	*I*		*II*		*III*		*IV*		*V*		*VI*

TABLE II. Order of states and numbers of places n in a spherical potential well.

Orbit	1s	2p	3d	2s	4f	3p	5g	4d	6h	3s	5f	7i	4p	8j	6g	5d	4s
n	2	6	10	2	14	6	18	10	22	2	14	26	6	30	18	10	2

TABLE III. Schmidt limits for the magnetic moments of odd A nuclei in dependence on spin.

	I	1/2	3/2	5/2	7/2	9/2
Odd Z	$I=l+\frac{1}{2}$	+2.79	3.79	4.79	5.79	6.79
	$I=l-\frac{1}{2}$	−0.24	+0.17	+0.88	+1.71	+2.52
Odd N	$I=l+\frac{1}{2}$	−1.91	−1.91	−1.91	−1.91	−1.91
	$I=l-\frac{1}{2}$	+0.64	1.15	1.36	1.49	1.57

* Please see comments on this and the preceding paper in a Letter to the Editor by E. Feenberg, K. C. Hammack, and L. W. Nordheim, this issue.

[1] For a review and discussion of these attempts see H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. **8**, 168 *ff.* (1936) and E. P. Wigner and E. Feenberg, Reports on Progress in Phys. **8**, 274, London (1942).

[2] Maria G. Mayer, Phys. Rev. **74**, 235 (1948).

[3] See the articles in reference 1 and particularly W. Elsasser, J. de phys. et Rad. **5**, 389, 625 (1934).

[4] See Henry Margenau, Phys. Rev. **46**, 613 (1934).

[5] The most recent and complete tabulation of nuclear spins and moments is due H. H. Goldsmith and D. R. Inglis, issued by the Brookhaven National Laboratory as report BNL-1-5, October 1, 1948. All data used here have been taken from this work.

[6] T. Schmidt, Zeits. f. Physik. **106**, 358 (1937).

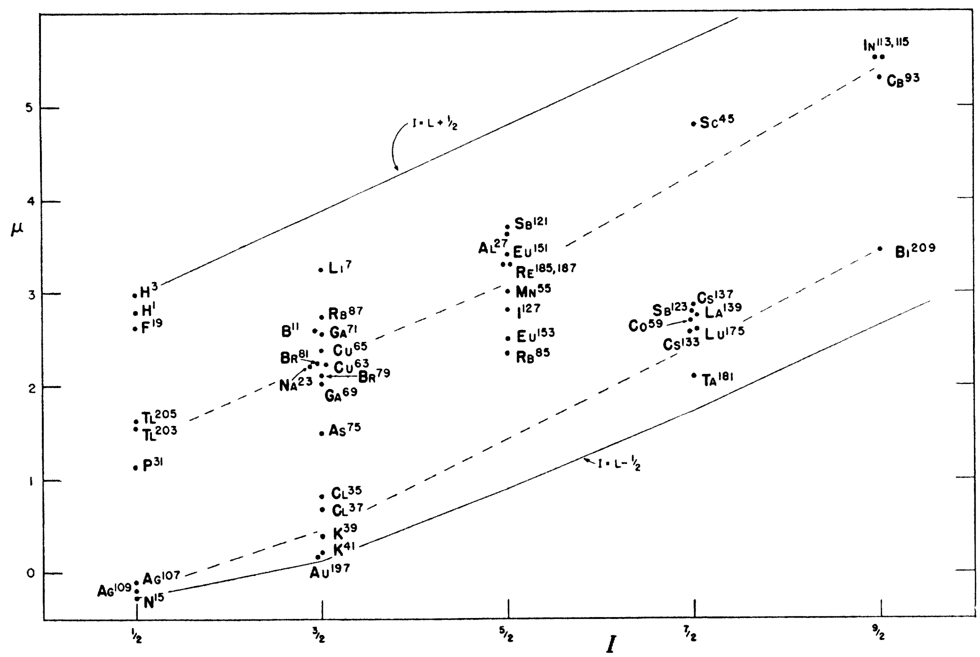

FIG. 1a. Magnetic moments and Schmidt limits as function of spin in nuclei of odd Z.

other antiparallel. The g formulas are

$$\mu_l = g_l l + g_s \qquad \text{for } I = l + 1/2,$$

$$\mu_l = g_l \frac{(l+1)(2l-1)}{2l+1} - g_s \cdot \frac{2l-1}{2l+1} \quad \text{for } I = l - 1/2.$$

With an orbital $g_l = 1$ for protons and 0 for neutrons and the intrinsic moments $g_s = 2.79$ for protons and $g_s = -1.91$ for neutrons the following values for the Schmidt limits are obtained. (See Table III.)

The actual values of the magnetic moments μ lie between these limits. One has thus to conclude that the actual wave function of the odd particle is a mixture of the two possibilities, combined with states of different parity for the residual nucleus with spin zero.

Figure 1 gives a representation of the distribution of magnetic moments and of the spins of the isotopes with odd mass numbers. The Schmidt limits are indicated by the solid lines. Inspection of Fig. 1 shows that the moments fall definitely into two groups,[7] each of which lies nearer to one of the limits, though there are some nuclei which are about halfway in between. The groups are indicated by the dotted lines. The fact that these lines do not coincide with the Schmidt limits indicates, of course, that the one-particle picture is not entirely adequate. However, it seems to be worth while to investigate the pattern which results from the assignment of orbital momenta according to the groupings in Fig. 1.

In Tables IV and VI the apparent orbital moments for all nuclei of odd atomic weight with known spins are listed separately for odd protons and odd neutrons. We assign also a principal quantum number in accordance with the exclusion principle, following as far as possible the sequence of orbits from Table II. The superscripts $\pm$ indicate addition or subtraction of angular and spin momentum to the total nuclear spin. The cases of strong mixtures are indicated, and the places, where shells are filled according to Table I, are also marked. Several magnetic moments have not been measured and the assignments in these cases are subject to doubt. The choice for the two Ir isotopes has been made on the basis of the information that their moments have opposite signs, which can be reconciled with the spin values only by the orbits given in the table.

We discuss first Table IV for protons. It is very striking that, in the main, new orbits appear in a definite order, which is very similar but not quite the same as in Table II. There are frequent shifts between orbits which are energetically near to each other. It is remarkable, however, how new orbits appear after passing through the closed shell numbers of Table I. ${}_9F^{19}$ shows that here the $2s$ orbit is lower than $3d$. ${}_{21}Sc^{45}$ brings the first $4f$. The $5g$'s appear fairly solidly with ${}_{51}Sb^{123}$, and ${}_{83}Bi^{209}$ gives beautiful evidence for a $6h$ orbit.

In any attempt to interpret nuclear structure, one has to be aware of the following limitations:

[7] This has been noticed by David R. Inglis, Phys. Rev. **53**, 470 (1938).

a one-particle picture cannot be as clearcut as in the atomic or molecular case. The actual energy difference between states of different symmetry are much smaller than would result from a Hartree-Fock approximation (compare the discussion by Wigner and Feenberg, reference 1). There are always many configurations which are energetically near each other, and which may thus interact strongly. The situation is in some respects analogous to the case of the rare earths for atomic structure where also a number of competing orbits are available. It is a matter of accident which new orbit out of several possible ones will be added, and there may be re-arrangements in the sequence as heavier nuclei are built up. Also, an orbit may not be favored for single occupancy and thus may not appear explicitly as an observable last orbit, while it may be filled when two particles are available. It is thus to be expected that possible shells will have real significance only when they are just completed. It is also not unlikely that re-arrangements will not respect shell boundaries. Thus, orbits may appear before they are expected, and even a completed shell may loosen up again when a re-arrangement is accidentally favored.

The material presented in Table IV suggests strongly and quite naturally a grouping of orbits which accounts for the magic numbers of Table I up to and inclusive of shell V, when the above limitations are taken into account. This scheme, which we will adopt as a working hyopthesis, is given in Table V.

The shells are indicated by brackets. The shell numbers correspond also to the highest total

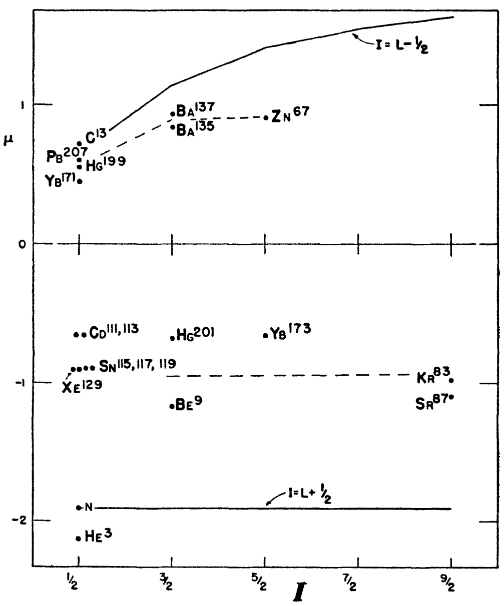

FIG. 1b. Magnetic moments and Schmidt limits as function of spin in nuclei of odd N.

TABLE IV. Spins I, magnetic moments μ, and assignment of orbital quantum numbers to the last proton in elements with odd Z.

Nucleus	I	μ	Orbit
$_{3}Li^{7}$	3/2	3.25	$2p^+$
$_{5}B^{11}$	3/2	2.69	$2p^+$
$_{7}N^{15}$	1/2	−0.28	$2p^-$
$_{9}F^{19}$	1/2	2.63	$2s$
$_{11}Na^{23}$	3/2	2.2	$2p^+ +3d^-$
$_{13}Al^{27}$	5/2	3.6	$3d^+$
$_{15}P^{31}$	1/2	1.13	$2p^- +2s$
$_{17}Cl^{35}$	3/2	0.82	$3d^-$
$_{17}Cl^{37}$	3/2	0.68	$3d^-$
$_{19}K^{39}$	3/2	0.39	$3d^-$
$_{19}K^{41}$	3/2	0.22	$3d^-$
$_{21}Sc^{45}$	7/2	4.8	$4f^+$
$_{23}V^{51}$	7/2		$4f^+$
$_{25}Mn^{55}$	5/2	3.0	$4d^+ +4f^-$
$_{27}Co^{59}$	7/2	2.7	$5g^-$
$_{29}Cu^{63}$	3/2	2.22	$3p^+ +4d^-$
$_{29}Cu^{65}$	3/2	2.38	$3p^+ +4d^-$
$_{31}Ga^{59}$	3/2	2.02	$3p^+ +4d^-$
$_{31}Ga^{71}$	3/2	2.56	$3p^+ +4d^-$
$_{33}As^{75}$	3/2	1.5	$3p^+ +4d^-$
$_{35}Br^{79}$	3/2	2.1	$3p^+ +4d^-$
$_{35}Br^{81}$	3/2	2.27	$3p^+ +4d^-$
$_{37}Rb^{85}$	5/2	1.35	$4f^-$
$_{37}Rb^{87}$	3/2	2.74	$3p^+$
$_{39}Y^{89}$	1/2		$3p^-$
$_{41}Cb^{93}$	9/2	5.3	$5g^+$
$_{47}Ag^{107}$	1/2	−0.10	$3p^-$
$_{47}Ag^{109}$	1/2	−0.19	$3p^-$
$_{49}In^{113}$	9/2	5.5	$5g^+$
$_{49}In^{115}$	9/2	5.5	$5g^+$
$_{51}Sb^{121}$	5/2	3.7	$4d^+$
$_{51}Sb^{123}$	7/2	2.8	$5g^-$
$_{53}I^{127}$	5/2	2.8	$5f^- +4d^+$
$_{55}Cs^{133}$	7/2	2.58	$5g^-$
$_{55}Cs^{137}$	7/2	2.86	$5g^-$
$_{57}La^{139}$	7/2	2.76	$5g^-$
$_{59}Pr^{141}$	5/2		$5f^-$
$_{63}Eu^{151}$	5/2	3.4	$5d^+ +5f^-$
$_{63}Eu^{153}$	5/2	1.5	$5f^-$
$_{65}Tb^{159}$	3/2		$5d^-$?
$_{67}Ho^{165}$	7/2		$5g^-$
$_{69}Tm^{169}$	1/2		$3s$?
$_{71}Lu^{175}$	7/2	2.6	$5g^-$
$_{73}Ta^{181}$	7/2	2.1	$5g^-$
$_{75}Re^{185}$	5/2	3.3	$5d^+ +5f^-$
$_{75}Re^{187}$	5/2	3.3	$5d^+ +5f^-$
$_{77}Ir^{191}$	1/2		$4p^-$
$_{77}Ir^{193}$	3/2		$5d^-$
$_{79}Au^{197}$	3/2	0.195	$5d^-$
$_{81}Tl^{203}$	1/2	1.55	$3s +4p^-$
$_{81}Tl^{205}$	1/2	1.63	$3s +4p^-$
$_{83}Bi^{209}$	9/2	3.45	$6h^-$

TABLE V. Average energetic order of one particle levels and shell structure for nuclei.

Orbits	(1s)	(2p)	(2s, 3d)	(4f, 3p, 4d)	(5g, 5f)	(6h, 3s ⋯)
No of places	2	6	12	30	32	44
Shell	I	II	III	IV	V	VI

quantum number occurring in it. There is no indication in heavier nuclei of sub-shells comprised of orbits of one kind only.** This means that new orbits are always added before the filling out of the preceding levels, except at the places given by the magic numbers. The latter are thus the more or less accidental positions where a clean break happens to be possible.

The order of levels in Table V differs only slightly from the level scheme for a potential well given in Table II. Thus one gains the impression that the one-particle picture is not quite devoid of all significance. It should be said, however, that it does seem neither necessary nor warranted to assume that one-particle wave functions can be ascribed to all individual particles in nuclei. However, such an assignment makes sense according to the evidence for the last odd particle. The exclusion principle will then still determine the nature of its wave function, though those for the inner particles may have lost all semblance to one-particle wave functions.

** There is a possibility that a shell may be completed at N or Z equal to 40 , which could be interpreted as a filling of only the $4f$ and $3p$ levels of shell IV. The evidence consists in a good isotopic and isotonic spread, the frequent occurrence of new type orbits for N and $Z>40$, and the fairly frequent occurrence of isomers with particle number 39. However, there is no necessity for this assumption.

The above interpretation of the structure of nuclear shells is strengthened by consideration of the nature of the occurring deviations.

The only difficulty in shell *III* comes from $_{11}$Na^{23} and the similar case of $_{15}$P^{31}, which have strongly mixed orbits and have magnetic moments which show evidence of a considerable contribution of a p orbit which regularly should not occur in this group. The explanation may be that in these cases the residual nucleus has an angular momentum different from zero.

In shell *IV* there is a group of nuclei from $_{29}$Cu^{63} to $_{35}$Br^{79} which have to be explained as mixtures of $3p^+$ and $4d^-$ orbits, or again, as due to the presence of angular momentum in the residual nucleus. The most remarkable nucleus in this shell is $_{27}$Co^{59} which has definitely a $5g$ orbit. The early appearance of this orbit must be due to an accident. This is borne out by the β-decay properties of other nuclei with 27 particle groups, which demand different assignments (compare Section IV). It is not unlikely that there is an angular momentum contribution from the residual nucleus in the 27 particle configuration, which would obviate the necessity of introducing a $5g$ orbit at this early position.

The $5g$ orbits belonging regularly to shell *V* appear with fair frequency toward the end of shell *IV* ($_{41}$Cb^{93}, $_{49}$In^{113}, and $_{49}$In^{115}). It will be seen later (Section III) that this premature coming in of a new orbit from a higher shell is intimately connected with the occurrence of isomeric states. For instance, both In isotopes have such isomeric states which can be interpreted as belonging to $3p^-$ orbits as in $_{47}$Ag.

In shell *V* one finds many $5g$ and some $5f$ levels, though others occur also. The most surprising case is $_{51}$Sb^{121} which has to be interpreted as an accidental loosening up of the preceding closed shell. Mixtures occur frequently in the middle of the shell. The case of the $_{63}$Eu isotopes with same spin but large difference in magnetic moment is remarkable. Towards the end of this shell many levels appear corresponding to states which are higher in the sequence of Table II.

We now take up the case of odd neutron nuclei. The available material is collected in Table VI, which gives also the number of neutrons for each isotope.

The rather meager material for $N<50$ fits well within the same scheme as that for protons. We find that all odd neutron nuclei with $N<50$ have exactly the same configuration, also with respect to relative orientation of particle spin and orbital momentum, as the corresponding proton numbers. The one exception $_{36}$Kr$_{47}^{83}$ has an isomer which should be the configuration corresponding to $_{47}$Ag^{107}.

However, there is a striking difference between the neutron and proton cases for nuclei with larger neutron numbers. In place of $5g$ or $5f$ orbits, only orbits with low angular momentum appear. The assignment of total quantum numbers becomes thus rather doubtful since we cannot know whether higher l states are added in between, or whether the total quantum number increases. We are inclined toward the first assumption in view of the apparent closed shell of 82 neutrons. Higher angular momentum neutrons must then always be added in pairs.

There are two effects which may help to explain the difference between neutrons and protons. The electric charge in nuclei is, according to available evidence, nearly uniformly distributed over the nuclear volume. It will thus produce a quasi-elastic repulsive force which will tend to favor orbits with high angular moment. Secondly, protons have always neutron counterparts with which they can interact, while the last neutrons have no proton partners. Thus they will not move in exactly the same "average field."*** Looking at the combined

TABLE VI. Neutron numbers N, spins I, magnetic moments μ and assignment of orbital quantum numbers to the last neutron in elements with odd N.

Nucleus	N	I	μ	Orbit	Nucleus	N	I	μ	Orbit
$_4$Be^9	5	3/2	−1.18	$2p^+$	$_{70}$Yb^{171}	101	1/2	0.45	$4p^-$
$_6$C^{13}	7	1/2	0.70	$2p^-$	$_{70}$Yb^{173}	103	5/2	−0.65	$5d^+$
					$_{72}$Hf^{177}	105	$\leq 3/2$		
$_{16}$S^{33}	17	3/2		$3d^-$	$_{72}$Hf^{179}	107	$\leq 3/2$		
$_{30}$Zn^{67}	37	5/2	0.9	$4f^-$	$_{74}$W^{183}	109	1/2		
$_{34}$Se^{77}	43	1/2		$3p^-$	$_{76}$Os^{187}	113	1/2		s or p
$_{36}$Kr^{83}	47	9/2	−0.97	$5g^+$	$_{78}$Pt^{195}	117	1/2		s or p
$_{38}$Sr^{87}	49	9/2	−1.1	$5g^+$	$_{80}$Hg^{199}	119	1/2	0.55	$4p^-$
					$_{80}$Hg^{201}	121	3/2	−0.61	$4p^+$
$_{42}$Mo^{95}	53	1/2		$3s$	$_{82}$Pb^{207}	125	1/2	0.6	$4p^-$
$_{48}$Cd^{111}	63	1/2	−0.65	$3s$					
$_{48}$Cd^{113}	65	1/2	−0.65	$3s$					
$_{50}$Sn^{115}	65	1/2	−0.9	$3s$					
$_{50}$Sn^{117}	67	1/2	−0.89	$3s$					
$_{50}$Sn^{119}	69	1/2	−0.89	$3s$					
$_{54}$Xe^{129}	75	1/2	−0.90	$3s$					
$_{54}$Xe^{131}	77	3/2	+0.8	$5d^-$					
$_{56}$Ba^{135}	79	3/2	+0.84	$5d^-$					
$_{56}$Ba^{137}	81	3/2	+0.94	$5d^-$					

TABLE VII. Nuclei with isomeric states.

Odd proton	Odd neutron	Odd neutron	Odd-odd	Odd-odd
	$_{22}$Ti$_{23}$	$_{52}$Te$_{79}$	$_{21}$Sc$_{23}$	$_{65}$Tb$_{95}$
$_{33}$As$_{40}$	$_{20}$Ca$_{29}$	$_{54}$Xe$_{73}$	$_{25}$Mn$_{27}$	$_{71}$Lu$_{105}$
$_{39}$Y$_{48}$	$_{22}$Ti$_{29}$	$_{54}$Xe$_{81}$	$_{27}$Co$_{33}$	$_{73}$Ta$_{107}$
$_{39}$Y$_{52}$	$_{30}$Zn$_{39}$	$_{56}$Ba$_{77}$	$_{35}$Br$_{45}$	$_{73}$Ta$_{109}$
$_{41}$Cb$_{54}$	$_{32}$Ge$_{39}$	$_{66}$Dy$_{99}$	$_{41}$Cb$_{51}$	$_{77}$Ir$_{115}$
$_{43}$Tc$_{54}$	$_{32}$Ge$_{45}$	$_{68}$Er$_{101}$	$_{41}$Cb$_{53}$	$_{79}$Au$_{117}$
$_{43}$Tc$_{56}$	$_{34}$Se$_{49}$	$_{68}$Er$_{103}$	$_{45}$Rh$_{59}$	$_{81}$Tl$_{119}$
$_{45}$Rh$_{58}$	$_{36}$Kr$_{43}$	$_{70}$Yb$_{105}$	$_{47}$Ag$_{63}$	$_{91}$Pa$_{143}$
$_{47}$Ag$_{62}$	$_{36}$Kr$_{47}$	$_{78}$Pt$_{119}$	$_{49}$In$_{63}$	
$_{49}$In$_{64}$	$_{36}$Kr$_{49}$	$_{80}$Hg$_{117}$	$_{49}$In$_{65}$	even-even
$_{49}$In$_{66}$	$_{38}$Sr$_{49}$	$_{80}$Hg$_{119}$	$_{49}$In$_{67}$	$_{32}$Ge$_{40}$
$_{59}$Pr$_{82}$	$_{40}$Zr$_{49}$		$_{51}$Sb$_{71}$	$_{52}$Te$_{72}$
$_{69}$Tm$_{100}$	$_{42}$Mo$_{51}$		$_{51}$Sb$_{73}$	$_{58}$Ce$_{82}$
$_{69}$Tm$_{102}$	$_{48}$Cd$_{67}$		$_{55}$Cs$_{79}$	$_{78}$Pt$_{118}$
$_{73}$Ta$_{108}$	$_{50}$Sn$_{73}$		$_{55}$Cs$_{85}$	$_{82}$Pb$_{122}$
$_{75}$Re$_{112}$	$_{52}$Te$_{75}$		$_{57}$La$_{81}$	
$_{79}$Au$_{118}$	$_{52}$Te$_{77}$		$_{63}$Eu$_{89}$	

*** Compare the discussion in Bethe and Bacher, reference 1, §37.

TABLE VIII. Spin, orbit and parity assignments for nuclei in β-transitions.

(a) Allowed transitions

A and class	Initial nucleus	Orbit	Spin	Final nucleus	Orbit	Spin	γ	Δ*I*	Δ*p*
35*a*	${}_{16}S_{19}$	$3d^-$	3/2	${}_{17}Cl_{18}$	$3d^-$	3/2		0	no
43*a*	${}_{21}Sc_{22}$	$4f^+$	7/2	${}_{20}Ca_{23}$	$4f^+$	7/2	γ	0	no
45*a*	${}_{22}Ti_{23}$ I.T.	$4f^+$	7/2	${}_{21}Sc_{24}$	$4f^+$	7/2		0	no
49*b*	${}_{23}V_{26}$	$4f^+$	7/2	${}_{22}Ti_{27}$	$5g^-$	7/2		1	yes
									no
51*a*	${}_{25}Mn_{26}$	$4d^++4f^-$	5/2	${}_{24}Cr_{27}$	$5g^-$	7/2		1	
					$4d^++4f^-$	5/2		0	no
57*a*	${}_{28}Ni_{29}$	$3p^++4d^-$	3/2	${}_{27}Co_{30}$	$5g^-$	7/2		2	
					$3p^++4d^-$	3/2		0	no
61*b*	${}_{29}Cu_{32}$	$3p^++4d^-$	3/2	${}_{28}Ni_{33}$	$3p^++4d^-$	3/2		0	no
63*a*	${}_{30}Zn_{33}$	$3p^++4d^-$	3/2	${}_{29}Cu_{34}$	$3p^++4d^-$	3/2		0	no
69*a*	${}_{30}Zn_{39}$	$3p^-$	1/2	${}_{31}Ga_{38}$	$3p^++4d^-$	3/2		1	no
75*a*	${}_{32}Ge_{43}$	$3p^-$	1/2	${}_{33}As_{42}$	$3p^++4d^-$	3/2	γ	1	no
71*c*	${}_{33}As_{38}$	$3p^++4d^-$	3/2	${}_{32}Ge_{39}$	$3p^-$	1/2		1	no
81*c*	${}_{34}Se_{47}$ I.T.	$3p^-$	1/2	${}_{35}Br_{46}$	$3p^++4d^-$	3/2		1	no
83*a*	${}_{35}Br_{48}$	$3p^++4d^-$	3/2	${}_{36}Kr_{47}$ I.T.	$5g^+$	9/2		3	
					$3p^-$	1/2			no
79*c*	${}_{36}Kr_{43}$	$3p^-$	1/2	${}_{35}Br_{44}$	$3p^++4d^-$	3/2		1	no
97*a*	${}_{41}Cb_{56}$	$5g^+$	9/2	${}_{42}Mo_{55}$	$3s$	1/2	γ	4	
		$3p^-$?	1/2					1	yes
93*c*	${}_{42}Mo_{51}$ I.T.	$3s$	1/2	${}_{41}Cb_{52}$	$5g^+$	9/2		4	
		$5g^-$	7/2					1	no
101*a*	${}_{43}Tc_{58}$	$3p^-$	1/2	${}_{44}Ru_{57}$	$3s$	1/2	γ	1	yes
117*a*	${}_{49}In_{68}$	$5g^+$	9/2	${}_{50}Sn_{67}$	$3s$	1/2		4	
		$3p^-$	1/2					0	yes
139*a*	${}_{56}Ba_{83}$	$6h^-$	9/2	${}_{57}La_{82}$	$5g^-$	7/2	γ	1	yes

proton and neutron evidence, we are inclined to conclude that in nuclei higher numbers of radial nodes are not as sharply penalized as in a simple potential well, particularly for neutrons. This effect is compensated partly but not entirely by the Coulomb repulsion for protons. The non-appearance of $6h$ orbits in shell *V* as contrasted to the early appearance of $5g$ states in shell *IV* seems also to indicate a discrimination against too high angular momentum values. However, the discussion of the next section indicates that the $6h$ orbits are occurring in isomeric states.

There is definitely not enough evidence to substantiate any assignment to the magic number 44 for shell *VI*. If one wishes to speculate and to play with numbers, one can make the observation that a ($6h3s6g4s$) configuration gives 44 places and that ($4p5d$) gives 16 additional ones which would bring us to ${}_{90}Th^{232}$ by taking all the orbits from Table II up to $4s$ except the very high angular momentum states $7i$ and $8j$.

There is no definite trend discernible as to the relative alignment of intrinsic spin and orbital momentum. One might perhaps say that there is in general a tendency towards antiparallelism (i.e., the minus sign has the edge) except for just added new type orbits.

III. OCCURRENCE OF ISOMERIC STATES

An interesting illustration of the way, in which the construction principle for nuclei operates, is furnished by the statistics of isomeric states. Table VII gives all nuclei with isomeric states listed in the isotope chart of the General Electric Research Laboratory.[8] The different types, odd proton, odd neutron, odd-odd, even-even, are separated, and both proton as well as neutron numbers are given.

We consider first the odd *A* nuclei. No isomeric states are connected with particle numbers below 20, since no high orbital momentum states are present in such nuclei. A few isomeric states occur in the twenties, where the first high spins are observed. At larger numbers the definite rule is established, with very few exceptions, that isomeric states appear at odd particle numbers which are not too far below those for completed shells. These cases are indicated in Table VII by vertical lines on the side (neutron or proton) to which the rule applies. The scarcity of isomeric states for proton

[8] Prepared by G. Friedlander and M. L. Perlman, 1948. *Note added in proof.*—The Table of Isotopes by G. T. Seaborg and I. Perlman, Rev. Mod. Phys. **20**, 585 (1948), and the new Segrè chart AECD-2111 lead to a still more convincing statistics of isomers.

TABLE VIII. (*Cont.*)

(b) First forbidden

A and class	Initial nucleus	Orbit	Spin	Final nucleus	Orbit	Spin	γ	ΔI	Δp
23*a*	${}_{10}\mathrm{Ne}_{23}$	$3d^+$	5/2	${}_{11}\mathrm{Na}_{12}$	$2p^+ + 3d^-$	3/2		1	yes
27*a*	${}_{12}\mathrm{Mg}_{15}$	$2p^- + 2s$	1/2	${}_{13}\mathrm{Al}_{14}$	$3d^+$	5/2	γ	2	yes
29*a*	${}_{13}\mathrm{Al}_{16}$	$3d^+$	5/2	${}_{14}\mathrm{Si}_{15}$	$2p^- + 2s$	1/2		2	yes
31*a*	${}_{14}\mathrm{Si}_{17}$	$3d^-$	3/2	${}_{15}\mathrm{P}_{16}$	$2p^- + 2s$	1/2		1	yes
41*a*	${}_{18}\mathrm{A}_{23}$	$4f^+$	7/2	${}_{19}\mathrm{K}_{22}$	$3d^-$	3/2	γ	2	yes
45*a*	${}_{20}\mathrm{Ca}_{25}$	$4d^+ + 4f^-$	5/2	${}_{21}\mathrm{Sc}_{24}$	$4f^+$	7/2	γ	1	yes
49*d*	${}_{20}\mathrm{Ca}_{29}$ I.T.	$3p^+ + 4d^-$	3/2	${}_{21}\mathrm{Sc}_{28}$	$4f^+$	7/2	γ	2	no
51*a*	${}_{22}\mathrm{Ti}_{29}$ I.T.	$3p^+ + 4d^-$	3/2	${}_{23}\mathrm{V}_{28}$	$4f^+$	7/2	γ	2	no
59*a*	${}_{26}\mathrm{Fe}_{33}$	$3p^+ + 4d^-$	3/2	${}_{27}\mathrm{Co}_{32}$	$5g^-$	7/2	γ	2	no?
65*a*	${}_{28}\mathrm{Ni}_{37}$	$4f^-$	5/2	${}_{29}\mathrm{Cu}_{36}$	$3p^+ + 4d^-$	3/2	γ	1	no?
71*a*	${}_{32}\mathrm{Ge}_{39}$ I.T.	$3p^-$	1/2	${}_{31}\mathrm{Ga}_{40}$	$3p^+ + 4d^-$	3/2		1	no?
89*a*	${}_{37}\mathrm{Rb}_{52}$	$4f^-$	5/2	${}_{38}\mathrm{Sr}_{51}$	$3s$	1/2	γ	2	yes
89*a*	${}_{40}\mathrm{Zr}_{49}$ I.T.	$5g^+$	9/2	${}_{39}\mathrm{Y}_{50}$	$3p^-$	1/2			
		$3s$?	1/2					0	yes
101*b*	${}_{42}\mathrm{Mo}_{59}$	$3s$	1/2	${}_{43}\mathrm{Tc}_{58}$	$3p^-$	1/2	γ	0	yes
109*a*	${}_{46}\mathrm{Pd}_{63}$	$3s$	1/2	${}_{47}\mathrm{Ag}_{62}$ I.T.	$3p^-$	1/2	γ	0	yes
115*a*	${}_{48}\mathrm{Cd}_{67}$ I.T.	$3s$	1/2	${}_{49}\mathrm{In}_{66}$ I.T.	$5g^+$	9/2	γ	4	no
131*a*	${}_{53}\mathrm{I}_{78}$	$5f^- + 4d^+$	5/2	${}_{54}\mathrm{Xe}_{77}$	$5d^-$	3/2	γ	1	yes
141*a*	${}_{58}\mathrm{Ce}_{83}$	$6h^-$	9/2	${}_{59}\mathrm{Pr}_{82}$	$5f^-$	5/2	γ	2	no
165*a*	${}_{66}\mathrm{Dy}_{99}$	$4p^+$	3/2	${}_{67}\mathrm{Ho}_{98}$	$5g^-$	7/2	γ	2	yes
187b	${}_{74}\mathrm{W}_{113}$	$4p^-$	1/2	${}_{75}\mathrm{Re}_{112}$	$5f^- + 5d^+$	5/2	γ	2	no

(c) Second and higher forbidden

A and class	Initial nucleus	Orbit	Spin	Final nucleus	Orbit	Spin	γ	ΔI	Δp
59*a*	${}_{26}\mathrm{Fe}_{33}$	$3p^+ + 4d^-$	3/2	${}_{27}\mathrm{Co}_{32}$	$5g^-$	7/2	γ	2	no?
87*a*	${}_{37}\mathrm{Rb}_{50}$	$4f^-$	3/2	${}_{38}\mathrm{Sr}_{49}$ I.T.	$5g^+$	9/2	γ	3	yes
89*a*	${}_{38}\mathrm{Sr}_{51}$	s	1/2	${}_{39}\mathrm{Y}_{50}$	$3p^-$	1/2			
		$5g^+$	9/2					4	yes
99*a*	${}_{42}\mathrm{Mo}_{57}$	$3s$	1/2	${}_{43}\mathrm{Tc}_{56}$ I.T.	$3p^-$	1/2	γ		
					$5g^+$	9/2		4	yes
185*b*	${}_{74}\mathrm{W}_{111}$	p^-	1/2	${}_{75}\mathrm{Re}_{110}$	$5d^+ + 5f^-$	5/2	γ	2	no

and neutron numbers between 50 and 70 and around 82 to 117 is very striking.

The interpretation of this phenomenon can be found in the observation that new orbits appear not infrequently before the completion of the preceding shell. Thus, there will be a state in which the last particle goes into the new orbit with high spin and a competing state in which the incomplete shell is added to. These two states are likely to have widely different spins, since the lowest orbits of new shells have high angular momentum coupled with a tendency for parallelism of spin and orbital momentum, while the upper states in a shell have low spins and a tendency towards antiparallelism. It cannot be predicted, of course, which of these states is the lower one.

The spread of isomeric states is fairly large, from 39 to 49 for shell *IV*; from ~69 to 81 in shell *V*; and from 117 on in shell *VI*. Neutron and proton shells behave very much alike. It is quite noticeable that certain numbers seem to favor isomerism both for neutrons and protons. This is most pronounced for 49 and to a lesser degree for 39, 47 and 79.

The reverse case of a competing state with loosening up of a shell after its completion seems practically not to occur. A once reached shell configuration seems thus to remain stable.

Our rule applies rather surprisingly also to odd-odd nuclei as is evident from the corresponding section of Table VII. This is a further indication for a marked independence of neutron and proton configurations, which emerges as an outstanding feature of the whole picture.

The cases of isomers in even-even nuclei are too few to permit any conclusions.

The above rule does not imply, of course, that isomeric states have to occur in all nuclei with not quite completed shells or that they could not occur in others. However, the preference is very pronounced.

IV. INTERPRETATION OF β-TRANSITIONS

Another group of phenomena that can be connected with the assignment of spins and orbits to nuclei is the allowed or forbidden character of β-transitions.[9]

The possibility of making assignments of spins to radio-active nuclei is based on the following observations:

(1) The spins of odd neutron nuclei with $N<50$ are generally the same as those with the same number of protons.

(2) Most spins of odd neutron nuclei with $N>50$ are low and follow a fairly definite pattern.

(3) The spin of isotopes differing by two neutrons is the same with very few exceptions.

These rules make it possible with the help of Table IV and Table VI to assign spin values and orbital quantum numbers to most nuclei of odd mass number. It is not expected, of course, that this primitive procedure will be true in all cases, and some corrections will be made later. We give the resulting assignments in Table VIII for all odd A nuclei contained in Konopinski's[9] classification of β-transitions. The transitions listed in his table have been checked with the G.E. isotope chart[8] and some isotope identifications have been corrected.

We have not included the 14 known conjugate nuclear pairs, where the number of neutrons in the product equals the number of protons in the initial state. All these nuclei should show no spin change, and all such β-transitions are allowed.

Table VIII contains, besides the nuclei with the numbers of protons (left) and neutrons (right) for the initial and final states, the spin assignments from rules 1, 2, 3. The measured spins are underlined. Different assignments are indicated below those from rules 1 to 3 in the cases where we believe that changes have to be made. All those cases will be discussed in the text. The table contains the symbol I.T. behind every isotope, which is known to possess an isomeric state, whether it is necessary for the interpretation or not. The symbol γ means, that γ-radiation is emitted in connection with the transition. The parity column, to be discussed later, indicates whether one should assume that the parity is different (yes) or the same (no) in the initial and final nucleus. The preferred parity in mixtures is always the one of the orbit given first. For instance, $2p^-+2s$ means that the parity is odd. The class symbols have the usual meaning

a isotope certain,
b mass number probable, element certain,
c one of a few mass numbers, element certain,
d element certain.

Table VIII shows at once striking correlations. The allowed transitions are in most cases connected with $\Delta I=0$ with some $\Delta I=1$. There are practically no γ-rays, if both N and Z are below 50. There are γ-rays, however, if $Z<50$ but $N>50$, that is, the neutrons and protons end in different shells. There are only a few such transitions altogether, and no allowed cases for odd A are known beyond Ba^{139}. The explanation for this rule lies in the difference of the behavior of the last proton and neutron for $Z>50$.****

Among the forbidden transitions, one finds frequently $\Delta I=2$ with some $\Delta I=1$, and γ-radiations in the majority of cases. Among the higher forbidden cases some larger spin changes are encountered. There is again a marked difference between the cases with N smaller or larger than 50.

A closer interpretation demands a comparison with theoretical selection rules, which are still rather uncertain. It can be said, however, that the material presented here is in good accord with the Gamow-Teller selection rules. In these, parity plays the essential role.

With no change in parity $\Delta I=0$, ± 1 lead to allowed transitions, and $\Delta I=\pm 2$, ± 3 to second-order forbidden ones. With change in parity $\Delta I=0$, $\pm 1, \pm 2$ are all first-order forbidden and higher ΔI of third order. These rules are rather flexible, particularly in presence of γ-radiation. For the latter, there are the two possibilities that they either come from a side path, in which case only a small fraction of γ-radiation is expected, or they may also come from a more allowed path than the direct transition, in which case there should be at least one γ-ray per disintegration.

Since we ascribed orbital momenta to the last odd number, we obtain also its contribution to the parity of the ground state. It would seem likely that the residual nucleus, with spin zero, would give an even contribution to parity. However, as mentioned before, the actual magnetic moments lie between and not on the Schmidt limits, and cores of different parity have to be added to the two orbits to make a mixture of definite parity possible. Thus, no really definite statement on parity can be made, particularly in case of strong mixtures, as they occur frequently in half-filled shells. It is, however, possible to make reasonable assignments in practically all cases, though they are not always unique and may have to be revised later.

We discuss now how the parities of mixtures have been determined.

$2p^+ + 3d^-$ for 11 particles is odd according to the first forbidden character of ${}_{10}Ne_{23}$;

[9] For a complete review of the theory of β-decay, see Emil J. Konopinski, Rev. Mod. Phys. **15**, 209 (1943).

**** We have left out in general the consideration of the heavy naturally radioactive elements, since no spin and moments are known here. It seems that some allowed transitions occur in this group. This may be due to the circumstance that low orbital momentum states are possible here for both protons and neutrons.

$2p^-+2s$ for 15 particles is odd from the first forbidden character of $_{12}Mg_{15}$ and $_{14}Si_{17}$;

$4d^++4p^-$ for 25 particles is even from the first forbidden character of $_{20}Ca_{25}$;

$5f^-+4d^+$ in $_{53}I_{78}$ should be odd from the first forbidden character of its decay;

$3p^++4d^-$ for particle numbers between 29 and 35 is odd from the allowed character of $_{30}Zn_{39}$. Strong support for this assignment is furnished by $_{20}Ca_{29}$ and $_{22}Ti_{29}$ whose decays are first forbidden with a γ-ray in series,[10] which indicates that the transition to the ground state of the final nucleus should be second forbidden.

It is now possible to make a prediction as to the last orbit for nuclei with N or Z equal to 43. A $3p^-$ orbit as in $_{47}Ag$ accounts for $_{32}Ge_{93}$ and $_{36}Kr_{43}$ (allowed) and for $_{42}Mo_{59}$ (first forbidden).

A reversal of the energetic order with a known isomeric state can be invoked in the cases of $_{42}Mo_{51}$, and $_{49}In_{68}$ (allowed) and in $_{40}Zr_{49}$ (first forbidden). $_{35}Br_{98}$ (allowed) goes into an isomeric state of $_{36}Kr_{47}$.

A similar explanation should hold for $_{41}Cb_{56}$, (allowed) which has to be assumed to have one of the regular shell *IV* orbits ($3p$ or $4d$) and not the $5g$ configuration of $_{41}Cb_{52}$.

The case of $_{56}Ba_{83}$ (allowed) is interesting. It has 83 neutrons and the assignment of a $6h^-$ orbit as in $_{83}Bi$ seems to be justified, in place of the $5d^-$ of $_{56}Ba_{81}$. Similarly, $_{38}Sr_{51}$ (highly forbidden) corresponds rather to the proton case with a $5g$ orbit for $Z=51$, in place of the $3s$ orbits found in shell V for neutrons.

The nuclei with N or Z equal to 27 show definitely an anomalous behavior, which we believe to be connected with the anomalous spin and moment of $_{27}Co_{32}{}^{59}$. The isotope $_{26}Fe_{33}{}^{59}$ goes over a second forbidden transition into $_{27}Co^{59}$, accompanied by a first forbidden one with a series γ-ray. This behavior is in order, if it is assumed that the $5g^-$ configurations of Co^{59} has odd parity. This assumption can explain the allowed character of the decay of $_{23}V_{26}$. It is, however, necessary to assume that $_{24}Cr_{27}$ and $_{27}Co_{30}$ have different orbits, corresponding to the 25 and 29 configurations, in order to be in agreement with the allowed character of the decays of $_{25}Mn_{26}$ and $_{28}Ni_{29}$.

It is necessary to suppose in several cases that the decay goes into an excited state of the product nucleus with a subsequent γ-emission. This applies to most of the allowed transitions with $N>50$ (that is, protons and neutrons ending in different shells). The existence of a series γ-ray is known[11] for $_{43}Tc_{58}$. It is also proven[10] for the cases in the first forbidden group, that is, $_{20}Ca_{29}$, $_{22}Ti_{29}$, and $_{26}Fe_{33}$. We are inclined to explain the large spin change of $_{48}Cd_{67}$ (first forbidden) by series γ-rays, in place of ascribing it to the isomeric state, which would lead to an allowed transition.

The assignments for some heavy nuclei with odd N ($_{66}Dy_{99}$, $_{79}W_{111}$, and $_{74}W_{113}$) are not uniquely determined, owing to lack of information. The entries in the table are compatible with the β-decay properties. It has also to be admitted that $_{28}Ni_{37}$ and $_{32}Ge_{39}$ would be classified as allowed in place of the empirical assignment of first forbidden.

The material presented in Table VIII, after incorporation of the above considerations, looks, however, very encouraging. It seems that much useful information can be obtained by the comparison of β-decay data and orbital state assignment based on observed spins and moments. Further work on these lines is in progress.

V. CONCLUDING REMARKS

The principal aim of this investigation has been to correlate the information on nuclear states from spins and moments with other nuclear properties. There can be little doubt that such a correlation exists and that the magic numbers 20, 50, and 82 have real significance. This result is strengthened further by the discovery of interesting regularities in the distribution of quadrupole moments by W. Gordy and of neutron cross sections by H. Newson, which will be published separately.

It should be emphasized that all data used here have been taken from existing compilations, without further scrutiny. It is thus to be expected that some of the assignments will prove to be in error. One may hope, however, that corrections necessitated by new and better data, will tend more to clarify than to confuse the situation.

Most of the evidence collected here does not depend on the interpretation of shell numbers in terms of definite combinations of single particle states. The proposed level scheme is more or less a by-product and may have to be revised later. There is, for instance, the possibility that a far reaching rearrangement of levels occurs in the building up of a larger and larger nuclei, so that the sequence of states in heavy nuclei may be quite different from that in light nuclei. However, such an assumption does not seem to be necessary at the present time, since the proposed level scheme appears to be natural and reasonable.

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[10] Walke, Thompson, and Holt, Phys. Rev. **57**, 177 (1940); Walke, Williams, and Evans, Proc. Roy. Soc. **A171**, 360 (1939).

[11] W. Maurer and F. W. Ramm, Zeits. f. Physik **119**, 334 (1942).