Nuclear Shell Structure* **

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The existence of closed shells in nuclei is indicated by the particular stability and abundance of nuclear systems with certain numbers of neutrons and of protons. An interesting correspondence exists between these numbers and the degeneracy of energy levels in the model of free particles in a simple rectangular potential well. The empirical relations suggest the addition of a central depression to the well for light nuclei (N or $Z\leqq 20$) and a central elevation (Elsasser's wine bottle potential) for heavy nuclei. The qualitative physical explanation of these modifications is that for light nuclei, the particle density, and thus the nuclear interaction energy, is greatest at the center of the nucleus; for heavy nuclei, the Coulomb repulsion between protons produces a particle density varying from a minimum at the center to a maximum near the boundary, and therefore a similarly varying interaction energy. In the free particle model, levels with small angular momentum and nodes within the nucleus are displaced upward by the central elevation and crossing of levels in the desired direction occurs, though not exactly as required to explain the complete empirical list of closed shell numbers. It is apparent that a single particle model of the nucleus is an insufficient approximation; however qualitative arguments based on configuration interaction enable one to formulate, with little ambiguity, a shell model in agreement with the empirical facts.

Experimentally known spins, magnetic moments and quadrupole moments of nuclei with a small number of particles outside of closed shells, or missing from closed shells, tend to corroborate the model. The agreement is particularly good for odd nuclei with one kind of nucleon forming a closed shell±one particle (and still better if the even group of nucleons constitutes a closed shell). Under these conditions, the orbit of the odd particle, as given by the shell model, usually determines the state of the nucleus, which is checked for consistency with the known spins and moments. In some cases, the reasoning is reversed as a means of fixing the crossover points between energy levels in the theory.

Islands of isomerism appear in a table of odd nuclei arranged according to the odd member of the pair N, Z. Since isomerism occurs when large spin differences exist between the ground state and the excited state, the correlation of isomerism with the crossing or close spacing of energy levels in the shell model is readily seen. Possible paired configurations representing ground state and isomeric state are assigned to various isomeric transitions. Spins and moments (where known), and parity relationships given by Wiedenbeck's plot of half-life *vs.* energy, help to decide the preferred configurations and to clarify the details of the shell model.

Beta-decay transitions are classified empirically into allowed and forbidden categories on the basis of calculated ft values. The configurations of the parent and daughter nuclei, as given by the shell model, determine the parity and range of possible spin values for each. Using the Gamow-Teller selection rules, the consistency of the empirical allowed or forbidden classification with the possibilities of the shell model are investigated. In many cases the ambiguity of spin in the parent or daughter nucleus is lessened. The empirical transition classification breaks down in several cases, chiefly those in which transitions that are theoretically first forbidden ($\Delta I=\pm 2$, change in parity) have ft values that place them in the empirical second forbidden category, indicating a very small nuclear matrix element. An extensive table of allowed (unfavored) transitions shows no trend of ft with atomic mass over the range $19\leqq A\leqq 140$.

I. INTRODUCTION

THE proton and neutron numbers 2, 8, 10, 20, 50, and 82 and the neutron number 126 are associated with particularly stable nuclear systems.[1–4] These numbers correlate in an interesting fashion with the degeneracy of the levels in the model of free particles in a potential well. Known results[5] for the oscillator potential and the rectangular well of infinite depth are listed in Tables I and II to facilitate the present discussion.

With a well of finite depth the levels are pushed closer together against the bottom of the well, but no crossovers occur.[6,7]

The two potentials possess the closed shell numbers $\Sigma\nu=2$, 8, 20 and 40 in common. Neither shape of well yields the complete set of numbers $\Sigma\nu=2$, 8, 10, 20 at the beginning of the empirical series. However there exist reasonable modifications of the potentials which put the $2s$ level below the $3d$ in agreement with the empirical order. In a light nucleus the inner region of constant particle density is lacking and the optimum single particle potential may be expected to resemble an inverted error

TABLE I. *Oscillator potential.* ϵ_{nl}-energy in arbitrary units measured from the bottom of the well. ν-degeneracy (statistical weight) of the level ($=2(2l+1)$).

Level	$1s$	$2p$	$2s+3d$	$3p+4f$	$3s+4d+5g$	$4p+5f+6h$
ϵ_{nl}	3	5	7	9	11	13
ν	2	6	12	20	30	42
$\Sigma\nu$	2	8	20	40	70	112

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** Please see comments on this and the succeeding paper in a Letter to the Editor by Eugene Feenberg, Kenyon C. Hammack, and L. W. Nordheim, this issue.
1 W. Elsasser, J. de phys. et rad. **5**, 625 (1934).
2 Eugene Wigner, Phys. Rev. **51**, 947 (1937).
3 William H. Barkas, Phys. Rev. **55**, 691 (1939).
4 Maria G. Mayer, Phys. Rev. **74**, 235 (1948).
5 H. A. Bethe and R. F. Bacher, Rev. Mod. Phys. **8**, 82 (1936). *Added in proof.*—The idea of a nuclear shell structure analogous to atomic shell structure was proposed by J. H. Bartlett, Nature **130**, 165 (1932). In this connection the early papers of W. D. Harkins (J. Frank. Inst. **194**, 645 (1923)) should be consulted.
6 Henry Margenau, Phys. Rev. **46**, 613 (1934).
7 L. Motz and E. Feenberg, Phys. Rev. **54**, 1055 (1938).

TABLE II. Rectangular potential of infinite depth and radius R. ϵ_{nl} in units $\hbar^2/MR^2 \sim 19.3/A^{2/3}$ (Mev) measured from the bottom of the well ($R \sim 1.47 \times 10^{-13} A^{1/3}$ cm).

Level	1s	2p	3d	2s	4f	3p	5g	4d	6h	3s	5f	7i	4p	6g
ϵ_{nl}	4.93	10.1	16.6	19.7	24.4	29.8	33.5	41.3	43.8	44.4	54.2	55.3	59.4	68.5
ν	2	6	10	2	14	6	18	10	22	2	14	26	6	18
$\Sigma\nu$	2	8	18	20	34	40	58	68	90	92	106	132	138	156
$\Sigma^{I}\nu$				///////		///////	50		82					
$\Sigma^{II}\nu$				///////		///////				///////	///////		///////	126
$\Sigma^{III}\nu$						///////						126		
$\Sigma^{IV}\nu$								///////		///////			126	

function. Such a function can be approximated, after a fashion, by a rectangular well with a central depression. It is clear that a narrow central depression pulls down the 2s level relative to the 3d. Thus there is an adequate physical basis for the observed sequence of closed shells in light nuclei (N or $Z \leqq 20$). In somewhat heavier systems the optimum single particle potential begins to approximate to a rectangular well of finite depth paralleling the tendency for the particle density to become constant within the nucleus as the 4f shell fills up (because of the small magnitude of the 4f wave function in the central region). Elsasser[1] proposed omitting the 2s, 3p, and 3s levels. One would now add that the omission of 2s should not apply to light nuclei. The numbers 50 and 82 then occur among the possible values of $\Sigma\nu$ (fifth row of Table II). The number 126 occurs if the levels 2s, 3p, 3s, 5f, and 4p are omitted (sixth row of Table II) or if the 3p level only is omitted (seventh row of Table II) or if the 4d and 3s levels are omitted (last row of Table II).

Elsasser[1] also considers the possibility of a wine bottle shape of well to account for the displacement of the 2s and 3p levels above those which must be occupied to give the numbers 50 and 82. His numerical calculations show that a considerable displacement (although not enough) is produced by a central potential elevation of the form

$$(30\hbar^2/MR^2)\exp\{-25(r/R)^2\} \tag{1}$$

rising from the floor of the well. This central elevation inverts the order of 2s and 4f and of 3p and 5g and brings 4d and 6h almost into coincidence. Clearly a displacement approaching or perhaps meeting the empirical requirements can be realized by increasing the range of the central elevation and simultaneously modifying the altitude.

Physical Basis for the Wine Bottle Potential

The explanation of the wine bottle potential is to be sought in the Coulomb repulsion between protons acting in opposition to the uniformizing tendency of the specifically nuclear forces. Because of the Coulomb force, the proton density within a heavy nucleus varies from a minimum value at the center to a maximum near the boundary. In addition a non-uniform proton density creates forces which distort the neutron distribution and tend to make the two particle densities vary in the same manner. This effect has been investigated independently by E. Wigner[7a] and one of the present writers.[8] A simple estimate yields the following results for the ratio of maximum to minimum particle density:

$$\begin{aligned}\{\rho_{\max}/\rho_{\min}\}_{\text{proton}} &\sim 1+0.0022A,\\ \{\rho_{\max}/\rho_{\min}\}_{\text{neutron}} &\sim 1+0.0010A.\end{aligned} \tag{2}$$

The existence of a central elevation growing rapidly with increasing atomic number follows at once from the behavior of the particle densities. A simple generalization of the theory leading to Eq. (2) should permit the numerical determination of the single particle potential within reasonable limits of uncertainty.

As the central elevation grows, levels with small angular momentum and nodes within the nucleus ($n > l+1$) move up on the energy diagram. In coordinate space, the corresponding wave functions are pushed out of the central region thus contributing to the decrease in particle density from which the central elevation arises. Simultaneously the filling of the 4f, 5g, and 6h orbits ($n = l+1$, no nodes within

[7a] Eugene Wigner, Bicentennial Symposium University of Pennsylvania (1940).

[8] Eugene Feenberg, Phys. Rev. **59**, 593 (1941); Richard D. Present, Phys. Rev. **72**, 1 (1947), has applied these results to explain the division of nuclear charge between light and heavy fragments in fission.

Equation (2) is based on the semi-empirical energy formula, in particular on the symmetry term $u_\tau(N-Z)^2/A$ with u_τ treated as a constant. However u_τ is not constant, but varies in an oscillatory manner (Eugene Feenberg, Rev. Mod. Phys. **19**, 239 (1947)). The inclusion of the first and second derivatives of u_τ with respect to A in the calculation of the non-uniform particle density should result in the addition of an oscillatory component to the right-hand member of Eq. (2). The writers hope to discuss the relation between these oscillatory effects and shell structure in a future paper.

the nucleus) reinforce the trend toward the formation of a semihollow nucleus.

Section V contains two numerical calculations on the behavior of energy levels in a wine bottle shape of potential well. The results show that the central elevation does indeed produce a notable relative shift of certain levels in the desired direction, but does not encourage the view that an exact reproduction of the empirical situation can be achieved in the single particle approximation. It is satisfactory that extremely simple models based on general physical ideas do go far in the desired direction.

For orientation one may consider the extreme limiting situation of a particle confined to a spherical shell with $R-\Delta R \leqq r \leqq R$ and $\Delta R \ll R$. The energy eigenvalues are

$$\epsilon_{nl}=\frac{\hbar^2\pi^2(n-l)^2}{2M(\Delta R)^2}+\frac{\hbar^2\pi^2}{2MR^2}\left(1+\frac{\Delta R}{R}\right)l(l+1). \quad (3)$$

In this case all states with nodes within the shell $(n>l+1)$ are pushed far up on the energy diagram and the levels with $n=l+1$ occur in the order of increasing angular momentum.

The Shell Model

The shell model can now be formulated as follows:

(*a*) There exist closed shells of neutrons and protons having the structures

$$\begin{array}{l}(1s)^2\\ (1s)^2(2p)^6\\ (1s)^2(2p)^6(2s)^2\\ (1s)^2(2p)^6(2s)^2\,(3d)^{10}\\ (1s)^2(2p)^6(3d)^{10}(4f)^{14}(5g)^{18}\\ (1s)^2(2p)^6(3d)^{10}(4f)^{14}(5g)^{18}(6h)^{22}(4d)^{10}\\ (1s)^2(2p)^6(3d)^{10}(4f)^{14}(5g)^{18}(6h)^{22}(7i)^{26}\end{array}\left\{\begin{array}{l}(4d)^{10}(6g)^{18}\\ (4d)^{10}(5f)^{14}(2s)^2(3s)^2\\ (5f)^{14}(3p)^6\,(4p)^6(2s)^2\end{array}\right\}. \quad (4)$$

In the "82" and "126" structures, the order in which the "outer" shells are filled still requires discussion.

(*b*) Certain levels ($2s$, $3p$, $4d$ and others) drift upward with increasing N and Z.

(*c*) In odd nuclei (A odd) the shell structure of the odd group of particles is most pronounced and the predictions of the model most reliable when the even group of particles forms a closed shell.

Statements *a*, *b*, and *c* may be accepted as working hypotheses and utilized in the interpretation of spins, parities, magnetic moments, quadrupole moments, radioactivity, and isomerism. The model is particularly useful for studying systems containing one particle outside of closed shells or with one particle missing from a closed shell.

II. SPINS AND MOMENTS

The spins and magnetic moments of nuclei with a small number of particles outside of closed shells or missing from closed shells provide valuable corroborative evidence on the angular momentum of the single particle levels. Experimental results[9] and interpretations are collected in Table III(a).

The configuration $(3d)^1(3d)^2$ contains completely symmetrical states ^2I, ^2G, ^2F, ^2D, and ^2S; the absence of ^2P is a difficulty in the interpretation of the predominantly $^2P_{3/2}$ ground state of $_{11}Na^{23}$.

The interpretation of the $_7N^{15}$, $_9F^{19}$, $_{19}K^{39}$, and $_{19}K^{41}$ spins and moments in terms of shell structure is not new.[3,10,11] To account for the spin of $_{41}Cb^{93}$ it is supposed that the $3p$ and $4f$ levels fill up before any particle enters a $5g$ orbit; the first $5g$ proton then occurs at $Z=41$ and as the $5g$ shell fills up the $2s$ and $3p$ levels are pushed above the $5g$. Hypothesis *a* is therefore extended to include the closed shell structures

$$\begin{array}{l}(1s)^2(2p)^6(3d)^{10}(4f)^{14}(3p)^6\\ (1s)^2(2p)^6(3d)^{10}(4f)^{14}(3p)^6(2s)^2\\ (1s)^2(2p)^6(3d)^{10}(4f)^{14}(2s)^2(3p)^6.\end{array} \quad (5)$$

The instability of all known nuclei containing 39 neutrons can be understood most simply in terms of an energetically favored closed shell at $N=40$ A similar situation exists at $N=19$ and 20, there being no stable systems with $N=19$. The magnetic moment of $_{39}Y^{89}$ favors the closing of a $3p$ shell at $Z=40$. If this interpretation is correct, the $2s$ and $3p$ levels have not yet crossed at $Z=39$ and only the third structure listed in Eq. (5) actually occurs. Independent evidence for the proposed shell structure is provided by the distribution of isomerism as a function of the odd member of the pair N, Z in

[9] H. H. Goldsmith and D. R. Inglis, *The Properties of Atomic Nuclei I* (Information and Publications Division, Brookhaven National Laboratory, Upton, New York, October 1, 1948), L. Rosenfeld, *Nuclear Forces II* (Interscience Publishers, New York, 1949), p. 393, R. Pound, Phys. Rev. **73**, 112 (1948).

[10] M. E. Rose and H. A. Bethe, Phys. Rev. **51**, 205 and 993 (1937).

[11] David R. Inglis, Phys. Rev. **53**, 470 (1938).

odd nuclei (next section). Table III(b) is helpful in locating the $2s$, $4f$ crossover.

Here the configurations are based on a $2s$, $4f$ crossover between $Z=31$ and 33. An earlier crossover would replace the $(4f)^{-3}$ configuration by $(4f)^{-1}$. However, the even group of neutrons in ${}_{31}Ga^{71}$ forms a closed "40" shell and should contribute no additional angular momentum. Thus a $(4f)^{-1}$ configuration at $Z=31$ is unlikely and the crossover should occur at $Z \geqq 31$. The existence of one or two nuclei with isomeric levels at $N=29$ (next section) suggests a crossover as close to $Z=29$ as possible consistent with other evidence. The jump from $(4f)^{-3}$ at $Z=31$ to $(4f)^{14}(2s)^{1}$ at $Z=33$ illustrates a characteristic feature of the shell model. A similar jump marks the closing of the $6h$ shell (Section III).

Note added in proof.—None of the immediate possibilities of the shell model fits ${}_{33}As^{75}$. Here the state contains approximately equal parts of ${}^2P_{3/2}$ and ${}^2D_{3/2}$. The shell model offers $(4f)^{14}(2s)^{1}$ (crossover at $Z=31$) or $(4f)^{-1}$ (crossover just beyond $Z=33$). In either case one or more units of additional angular momentum are required. Support for the latter assignment may be derived from the ${}_{27}Co^{61}(g) \rightarrow {}_{28}Ni^{61}(g)$ radioactivity (Parmley, Moyer, and Lilly, Phys. Rev. **75**, 619 (1949)) with $W_0 = 3.6$ mc^2 and half-life 6.3×10^3 sec. From $ft=2.6\times10^5$, the assignment to the allowed (unfavored) type of transition is unambiguous. Hence, the parities of both ground states are identical and consequently both are odd since the $(4f)^7$ configuration is assigned to ${}_{27}Co^{61}$ without ambiguity. In the absence of additional angular momentum supplied by the even group of particles the model requires a ${}^2F$ ground state for ${}_{28}Ni^{61}$.

TABLE III. (a) Nuclear spins and magnetic moments.*

Nucleus	Spin	Mag. moment	Configuration	Additional ang. moment	State
${}_{7}N^{15}$	1/2	$\pm$(?)0.280	$(2p)^{-1}$	0	${}^2P_{1/2}+({}^2S_{1/2})$
${}_{9}F^{19}$	1/2	2.63	$(2s)^{-1}$	0	${}^2S_{1/2}+({}^2P_{1/2})$
${}_{11}Na^{22}$	3	1.8	$(3d)^1(3d)^1$	—	${}^3D_3+{}^3G_3$
${}_{11}Na^{23}$	3/2	2.22	$(3d)^1(3d)^2$	—	$({}^2P_{1/2})+{}^2D_{3/2}$
${}_{19}K^{39}$	3/2	0.39	$(3d)^{-1}$	0	${}^2D_{3/2}+({}^2P_{3/2})$
${}_{19}K^{41}$	3/2	0.22	$(3d)^{-1}$	0	${}^2D_{3/2}+({}^2P_{3/2})$
${}_{21}Sc^{45}$	7/2	4.8	$(4f)^1$	0	${}^2F_{7/2}+({}^2G_{7/2})$
${}_{39}Y^{89}$	1/2(?)	<0.1	$(2s)^{-1}$	0	${}^2S_{1/2}+({}^2P_{1/2})$
${}_{39}Y^{89}$	1/2(?)	<0.1	$(3p)^{-1}$	0	${}^2P_{1/2}+({}^2S_{1/2})$
${}_{41}Cb^{93}$	9/2	5.3	$(5g)^1$	0	${}^2G_{9/2}+({}^2H_{9/2})$
${}_{38}Sr^{87}$	9/2	−1.1	$(5g)^{-1}$	0	${}^2G_{9/2}+({}^2H_{9/2})$
${}_{49}In^{113}$	9/2	5.5	$(5g)^{-1}$	0	${}^2G_{9/2}+({}^2H_{9/2})$
${}_{49}In^{115}$	9/2	5.5	$(5g)^{-1}$	0	${}^2G_{9/2}+({}^2H_{9/2})$
${}_{51}Sb^{121}$	5/2	3.7	$(4d)^1$	0	${}^2D_{9/2}+({}^2F_{9/2})$
${}_{51}Sb^{123}$	7/2	2.8	$(4d)^1$	$\geqq 1$	${}^2F_{7/2}+{}^2G_{7/2}$
${}_{56}Ba^{137}$	3/2	0.94	$(4d)^{-1}$	0	${}^2D_{3/2}+({}^2P_{3/2})$
${}_{81}Tl^{203}$	1/2	1.61	$(4d)^{-1}$	$\geqq 1$	$({}^2S_{1/2})+({}^2P_{1/2})$
${}_{81}Tl^{205}$	1/2	1.63	$(4d)^{-1}$	$\geqq 1$	$({}^2S_{1/2})+({}^2P_{1/2})$
${}_{83}Bi^{209}$	9/2	3.45	?	?	$({}^2G_{9/2})+({}^2H_{9/2})$
${}_{82}Pb^{207}$	1/2	0.6	$(4p)^{-1}$	0	${}^2P_{1/2}+({}^2S_{1/2})$

* The moments may be compared with the following formulas based on a definite value of orbital angular momentum and g_s (proton) $=5.58$, g_l (proton) $=1$, g_s (neutron) $=-3.82$ and g_l (neutron) $=0$:

I	μ odd proton	μ odd neutron
$l+1/2$	$I+2.29$	-1.91
$l-1/2$	$I-2.29\,I/(I+1)$	$1.91\,I/(I+1)$

The configuration symbol in Table III(a) usually denotes the orbit of the odd particle or hole; as a rule the configurations of the even groups are omitted. Both bracketed and unbracketed state symbols are usually required for the quantitative interpretation of the magnetic moments [T. Schmidt, Zeits. f. Physik **106**, 358 (1937); H. Margenau and E. Wigner, Phys. Rev. **58**, 103 (1939) but only the latter occur in the shell model. The second reference contains a partial list of spins and moments and references to the experimental papers. The authors list four nuclei ${}_{21}Sc^{45}$, ${}_{49}In^{115}$, ${}_{38}Sr^{87}$, and ${}_{36}Kr^{83}$ as falling outside the limits derived from an assumption of uniformity at the opposite extreme from the shell model. It is interesting that three of the exceptional cases involve single particle or single hole configurations ($Z=21$, 49 and $N=49$)] but only the latter occur in the shell model. With one exception, the states in Table III(a) are predominantly of the unbracketed character (in accord with the shell model) whenever a single particle or hole determines the state. The exception is ${}_{83}Bi^{209}$ where the shell model fails to provide a suitable orbit for the odd particle. Also ${}_{82}Pb^{207}$ does not lend support to the model since the $4p$ orbit is only one among several possibilities.

Although the ground states of the bromine isotopes are predominantly ${}^2P_{3/2}$ there is a large admixture of ${}^2D_{3/2}$. An explanation may be sought in the proximity of the $(3p)^3$ configuration. The small positive quadrupole moments (${}_{35}Br^{79}$: 0.28×10^{-24} cm^2, ${}_{35}Br^{81}$: 0.23×10^{-24} cm^2) are consistent with a large admixture of ${}^2D_{3/2}$.

Three nuclei fit satisfactorily with the closing of the $5g$ shell at $Z=50$ and the beginning of the $4d$ shell at $Z=51$. However, ${}_{51}Sb^{123}$ presents a difficulty since its spin suggests that an f or g shell begins to fill at $Z=51$. It is probable, of course, that the even group of neutrons supplies one or more units of orbital angular momentum. A similar situation with the addition of two neutrons changing the spin by one unit when N is even occurs at $Z=37$ and 77. More often the addition of two neutrons when N is even produces no change in spin and very little change in magnetic moment ($Z=1$, 17, 19, 29, 31, 47, 49, 55, 63, 75, and 81). The latter statement also holds for the addition of two protons ($N=65$) when Z is even.

The next item ${}_{56}Ba^{137}$ indicates the completion of the $4d$ shell at $N=82$. It appears that between $A=121$ and $A=137$ the $4d$ neutron level is displaced above the $6h$ and thus $4d$ orbits show up at $N=81$ as well as at $Z=51$. Since the $4d$ and $6h$ levels are quite close in the rectangular well the displacement required is not large and could be produced by a sufficiently rapid development of the central elevation with increasing atomic number.

The two thallium isotopes conform to the proposed shell structure only if the neutrons are permitted to contribute one or more units of orbital

TABLE III(b). Spins and moments at $Z=29$, 31, 33, and 35.

Nucleus	Spin	Mag. moment	Configuration	Additional ang. mom.	State*
${}_{29}Cu^{63}$	3/2	2.23	$(4f)^{-5}$	—	${}^2P_{3/2}+{}^2D_{3/2}$
${}_{29}Cu^{65}$	3/2	2.38	$(4f)^{-5}$	—	${}^2P_{3/2}+{}^2D_{3/2}$
${}_{31}Ga^{69}$	3/2	2.02	$(4f)^{-3}$	—	${}^2P_{3/2}+{}^2D_{3/2}$
${}_{31}Ga^{71}$	3/2	2.56	$(4f)^{-3}$	0	${}^2P_{3/2}+{}^2D_{3/2}$
${}_{33}As^{75}$	3/2	1.5	$(4f)^{14}(2s)^1$	$\geqq 1$	${}^2P_{3/2}+{}^2D_{3/2}$
${}_{35}Br^{79}$	3/2	2.11	$(2s)^2(3p)^1$	0	${}^2P_{3/2}+({}^2D_{3/2})$
${}_{35}Br^{81}$	3/2	2.27	$(2s)^2(3p)^1$	0	${}^2P_{3/2}+({}^2D_{3/2})$

* $(4f)^{-3}$ contains one ${}^2P$ and two linearly independent ${}^2D$ states; $(4f)^{-5}$ contains four linearly independent ${}^2P$ and five linearly independent ${}^2D$ states; see Gibbs, Wilbur, and White, Phys. Rev. **29**, 790 (1927); Pauling and S. Goudsmit, *The Structure of Line Spectra* (McGraw-Hill Book Company, Inc., New York, 1930), p. 156.

angular momentum. In $_{83}Bi^{209}$ the neutrons form a closed shell of 126 particles, hence should contribute no angular momentum. The only available orbit for the eighty-third proton yielding spin 9/2 is 6*g*. Granting, the remote possibility that the 2*s*, 3*p*, 3*s*, 4*p*, and even the 5*f* levels are all displaced above 6*g* by the large central elevation in heavy nuclei, it is not clear why 6*g* should lie lower than 7*i*. The situation is not improved by the fact that the magnetic moment favors a predominantly $^2H_{9/2}$ state. In view of these difficulties the possibility of starting the 6*g* shell at $Z=83$ must be dismissed. *Note added in proof.*—The absence of a distinction between neutron and proton shells may be the source of the difficulty. The possibility of a different structure for the proton shells beyond $Z=60$ deserves careful study.

Three possible structures for a shell of 126 particles are listed under hypothesis *a*. Of these the first puts 6*g* below 2*s*, 3*p*, 3*s*, 5*f*, and possibly also 4*d*. The extent of level crossing is somewhat extreme. The second structure places 3*p* above 3*s*. Here the difficulty is an extremely unlikely inversion or crossover. Not only is 3*p* below 3*s* in a rectangular well, but 3*s* rises more rapidly when perturbed by a central elevation. The third structure puts 4*d* and 3*s* above 4*p*. Again the inversion is unlikely. The spin and magnetic moment of $_{82}Pb^{207}$ favor the third structure.

Considering the difficulties at $Z=83$ and $N=126$ it is possible that the validity of the shell model is so far reduced in heavy nuclei that only the stability of the actual closed shell remains to reveal the structure. With one particle more or one less nearly all trace of structure may be lost in the unpredictable interaction of a large number of configurations. However, a less drastic conclusion is indicated by the fact that the quadrupole moments of $_{49}In^{115}$ and of $_{83}Bi^{209}$ are, respectively, positive and negative (0.84×10^{-24} cm^2 and -0.4×10^{-24} cm^2) in agreement with expectations for a hole in a proton shell and for a proton outside of closed shells. The negative value for $_{83}Bi^{209}$ is particularly interesting because it is the only known negative quadrupole moment in a heavy nucleus ($A>127$).

Three nuclei with N or $Z=37$ require separate discussion because of the uncertainty in the order of the 2*s* and 3*p* levels just before the beginning of the 5*g* shell. Experimental information and alternative interpretations are collected in Table III(c).

Only $_{37}Rb^{87}$ fits the shell model without additional angular momentum if the 3*p* shell is filled before 2*s*. The alternative $(3p)^{-3}$ configuration generates ^2P and ^2D levels belonging to the [21] irreducible representation of the symmetric group.[12] Again the agreement with $_{37}Rb^{87}$ is satisfactory, but the fit

TABLE III(c). Spins and moments at N or $Z=37$.

Nucleus	Spin	Mag. moment	Configuration	Additional ang. mom.	State
$_{30}Zn^{67}$	5/2	0.9	$(3p)^{-1}$	$\geqq1$	$(^2F_{5/2})+(^2D_{5/2})$
$_{30}Zn^{67}$	5/2	0.9	$(3p)^{-3}$	0	$^2D_{5/2}+(^2F_{5/2})$
$_{37}Rb^{85}$	5/2	1.35	$(3p)^{-1}$	$\geqq1$	$(^2F_{5/2})+(^2D_{5/2})$
$_{37}Rb^{85}$	5/2	1.35	$(3p)^{-3}$	0	$^2D_{5/2}+(^2F_{5/2})$
$_{37}Rb^{87}$	3/2	2.75	$(3p)^{-1}$	0	$^2P_{3/2}+(^2D_{3/2})$
$_{37}Rb^{87}$	3/2	2.75	$(3p)^{-3}$	0	$^2P_{3/2}+^2D_{3/2}$

with Zn^{67} and Rb^{85} is still poor (although improved) since the moments of these nuclei favor predominantly $^2F_{5/2}$ states. It may be argued that the shell model is unreliable at N or $Z=37$ and 39 because the $(3p)^{-1}$ and $(3p)^{-3}$ configurations have odd parity and are far removed from other odd configurations. On the other hand a large number of low configurations have even parity (2*s*, 5*g*, and 4*d* orbits all even); hence, configuration interaction may produce a ground state of even parity and unpredictable spin. It is perhaps significant that $_{63}Eu^{153}$ departs from theory in exactly the same manner as $_{30}Zn^{67}$ and $_{37}Rb^{85}$. Here also strong configuration interaction may be invoked to account for the failure of the $(3p)^3$ configuration to determine the ground state. The reduced number of low even configurations when the neutrons form a stable closed shell is perhaps responsible for the success of the shell model at $_{37}Rb^{87}$ and $_{39}Y^{89}$.

Experimental information regarding spins, magnetic moments and quadrupole moments is lacking

TABLE IV. Correlation of isomerism with the odd member of the pair N, Z. $(A)-N$ and Z certain; $(B)-Z$ certain, N probable.

N or Z	Number of known stable and radioactive nuclei	Number of cases of isomerism
1–28	—	0
29	7–8	1(*A*)
31–37	22–26	0
39	6	3(*A*), 1(*B*)
41	4–6	1(*A*)
43	3–5	3(*A*), 1(*B*)
45	3	1(*A*), 1(*B*)
47	5	4(*A*), 1(*BB*)
49	9	5(*A*), 1(*B*)
51	8–9	1(*BB*)
53–61	29–39	0
63	5–6	1(*A*)
65	4	1(*A*)
67	3	1(*A*)
69	3	1(*A*), 1(*B*), 1(*BB*)
71	3	0
73	2–3	2(*A*), 1(*BB*)
75	4–5	2(*A*)
77	4	2(*A*)
79	5	2(*A*)
81	6–7	2(*A*)
83–97	36–38	0
99	2	1(*A*)

[12] E. Feenberg and M. Phillips, Phys. Rev. **51**, 597 (1937).

on the stable nuclei

$$_8O^{17},\ _{10}Ne^{21},\ _{32}Ge^{73},\ _{40}Zr^{91},\ _{46}Pd^{105},\ \text{and}\ _{60}Nd^{143}.$$

These systems are particularly interesting from the point of view of the shell model. The predicted ground states are $^2S_{1/2}$, ^2D, ^2G, ^2D, and ^2D for the first five in the same order. No prediction is possible for $_{60}Nd^{143}$ but experimental information may help to clarify the confused situation at N and $Z=83$.

A discussion of the spins and moments of $(nd)^{\pm 3}$ configurations appears in the following section.

III. ISOMERISM IN ODD NUCLEI

A preliminary report on the topic of this section was published recently.[13] Isomerism throws light on shell structure because it occurs as a rule, only when the spin of the ground state is either quite small (1/2 or 3/2) or quite large (7/2, 9/2, or 11/2) permitting the possibility of a first excited state with spin differing by three or more units from that of the ground state.[14] If a particular shell structure predicts two closely spaced levels differing by three or four units in spin isomerism becomes a likely possibility. To test for a correlation between shell structure and isomerism all known nuclei, both stable and radioactive are classified with respect to the odd member of the pair N, Z. The known cases of isomerism are classified in the same manner.

Table IV[15–17] reveals a sharply defined island of isomerism extending from N or $Z=39$ to N or $Z=49$. At N or $Z=41$ and 49 the predicted spins and parities are 7/2 or 9/2 (even) while the experimental spin is 9/2 in four cases. For N or $Z=47$, the known spin values are 1/2 (two and 9/2(one). In the range $50<N$ or $Z<60$, the configuration $(4d)^{2n+1}$ might be expected to favor intermediate values of spin (3/2 and 5/2). Actually the tendency is again to extremes, 7/2 occuring four times, 5/2 twice, 3/2 not at all, and 1/2 with some uncertainty twice. Short-lived isomers might be expected in this region. So far none have been observed. In particular, DeBenedetti and McGowan (see reference b, Table V) obtained negative results from nine odd radioactive decay products on the range $51\leqq Z\leqq 59$ in a search for isomeric transitions with half-lives between 10^{-6} and 10^{-3} sec.

From the large number of isomeric nuclei at N or $Z=39$, one may infer that one or the other of the paired configurations

$$(1s)^2(2p)^6(3d)^{10}(4f)^{14}(2p)^6\begin{bmatrix}(5g)^1\\(2s)^1\end{bmatrix}\begin{matrix}\text{even parity}\\\text{even parity}\end{matrix}\tag{6a}$$

$$(1s)^2(2p)^6(3d)^{10}(4f)^{14}(2s)^2\begin{bmatrix}(3p)^4(5g)^1\\(3p)^5\end{bmatrix}\begin{matrix}\text{even parity}\\\text{odd parity}\end{matrix}\tag{6b}$$

determines the ground state and first excited state of such systems. An occasional inversion of the spin order is not unlikely and would not prejudice the occurrence of isomerism.

The occurrence of an isomer at $N=29$ suggests that the $2s$ level overtakes the $4f$ before the completion of the $4f$ shell (see Table III (b)). Isomeric and ground states could then be generated by one or the other of the paired configurations.

$$(1s)^2(2p)^6(3d)^{10}\begin{bmatrix}(4f)^{10}(2s)^1\\(4f)^{11}\end{bmatrix}\begin{matrix}\text{even parity}\\\text{odd parity}\end{matrix}\tag{7a}$$

$$(1s)^2(2p)^6(3d)^{10}\begin{bmatrix}(2s)^1(4f)^{10}\\(2s)^2(4f)^9\end{bmatrix}\begin{matrix}\text{even parity}\\\text{odd parity.}\end{matrix}\tag{7b}$$

From the point of view of the shell model, the existence of additional isomers at N or $Z=29$ and 31 is a likely possibility.

In the interval 51–60 marking the filling of the $4d$ shell, a temporary halt in the decrease of the central particle density and the rise of the central elevation should occur, thus permitting the $4d$ shell to fill completely before the inversion of the $6h$ and $4d$ levels takes place. The reversed situation exists during the filling of the $5g$ shell, the central density falling rapidly as the number of particles in the shell is increased. The fact that there are no stable nuclei with N or $Z=61$ supports the conclusion that a closed $4d$ shell is formed at N or $Z=60$ and suggests a large spacing between the $4d$ and adjacent levels at that point. The spin value 5/2 of $_{59}Pr^{141}$ is also in agreement with the closing of the $4d$ shell at $Z=60$. Unfortunately the magnetic moment has not yet been measured. The shell model predicts $\mu>2.8$ nuclear magnetons.

A second island of isomerism stretches from N or $Z=63$ to N or $Z=81$ with some concentration (by a factor of two) on the upper half of the range. Included are systems with $N=63$, 65, 67, 69, N and

[13] E. Feenberg, Phys. Rev. **75**, 320 (1949).

[14] C. F. v. Weizsacker, Naturwiss. **24**, 813 (1936).

[15] H. N. Friedlander and I. Perlman, General Electric Nuclear Information Chart (April, 1948).

[16] E. Segrè, Nuclear Information Chart, Revised to April, 1948.

[17] G. T. Seaborg and I. Perlman, "Table of the isotopes," Rev. Mod. Phys. **20**, 585 (1948). Thanks are due to Professor Joseph W. Kennedy for the loan of a prepublication copy of this table. Additional references to the most recent publications occur throughout the text.

$Z=69$, 73, 75, 77, 79, and $Z=81$. The shell model provides an unforced interpretation of this distribution based on the upward drift of the $2s$, $3p$, and $4d$ levels. Already at N or $Z=50$, the $2s$ and $3p$ levels lie above $4d$. At N or $Z=82$ they lie above $6h$. It is natural to assume that one or both cross the $6h$ level between 50 and 82 giving rise to a range of N, Z values in which the paired configurations

$$[60 \text{ shell}]\begin{bmatrix}(6h)^{2n}(2s)^1 \\ (6h)^{2n+1}\end{bmatrix}\begin{matrix}\text{even parity} \\ \text{odd parity}\end{matrix} \qquad (8a)$$

$$[60 \text{ shell}](3p)^6\begin{bmatrix}(6h)^{2n}(2s)^1 \\ (6h)^{2n+1}\end{bmatrix}\begin{matrix}\text{even parity} \\ \text{odd parity}\end{matrix} \qquad (8b)$$

or

$$[60 \text{ shell}]\begin{bmatrix}(6h)^{2n}(2p)^1 \\ (6h)^{2n+1}\end{bmatrix}\begin{matrix}\text{odd parity} \\ \text{odd parity}\end{matrix} \qquad (9)$$

generate the ground and first excited states. Configurations containing $(2s)^2$ or $(3p)^5$ may also occur.

The nuclei $_{48}Cd^{111}$, $_{48}Cd^{113}$, $_{50}Sn^{115}$, $_{50}Sn^{117}$, $_{50}Sn^{119}$, and $_{54}Xe^{129}$ all have spin 1/2 and magnetic moments ranging from -0.65 to -0.90 nuclear magnetons. Thus the ground states are all predominantly $^2S_{1/2}$. We suggest that these states are generated by configurations containing a single $2s$ orbit in addition to an even number of $3p$ and $6h$ orbits; however it is not clear why the state of minimum angular momentum and even parity should be favored in so many cases.

The previously discussed crossover of the $4d$ and $6h$ levels also produces possibilities for the occurrence of isomerism correlated with the closely spaced paired configurations.

$$[50 \text{ shell}]\begin{bmatrix}(4d)^9\,(6h)^{2n+2} \\ (4d)^{10}(6h)^{2n+1}\end{bmatrix}\begin{matrix}\text{even parity} \\ \text{odd parity}\end{matrix} \qquad (10a)$$

$$[50 \text{ shell}]\begin{bmatrix}(6h)^{2n}(4d)^1 \\ (6h)^{2n+1}\end{bmatrix}\begin{matrix}\text{even parity} \\ \text{odd parity}\end{matrix} \qquad (10b)$$

Linear combinations of configurations in (8) and (10) may on occasion determine the ground and first excited states.

If the $4d$, $6h$ crossover occurred before or at N or $Z=72$, the $6h$ shell would close there and the $4d$ shell begin anew at N or $Z=73$. However the spin value 7/2 of $_{71}Lu^{175}$ and $_{73}Ta^{181}$ suggests a later crossover, most likely at $Z=74$ in accord with the spins and magnetic moments of $_{75}Re^{187}$ and $_{75}Re^{189}$ (5/2 and 3.3 in both cases). On this latter view the proton configuration in $_{73}Ta^{181}$ is $[50 \text{ shell}]\cdot(4d)^{10}(6h)^{13}$ and the spin 7/2 presents no difficulty (although, of course, it could not be predicted). Similarly the proton configuration at $Z=75$ is $[50 \text{ shell}](6h)^{22}(4d)^3$. Accepting these configurations there are four odd nuclei in which three protons occupy d orbits. These are listed below:

Nucleus	Spin	Magnetic moment
$_{13}Al^{27}$	5/2	3.64
$_{53}I^{127}$	5/2	2.8
$_{75}Re^{185}$	5/2	3.3
$_{75}Re^{187}$	5/2	3.3

The shell model provides a choice of states, ^2H, ^2G, ^2F, ^2D, ^2D, and ^2P in the absence of additional angular momentum supplied by the even group of neutrons. Interaction within the configurations may be invoked to account for the experimental spins and moments ($^2D_{5/2}+^2F_{5/2}$). One nucleus with 75 neutrons has spin $\frac{1}{2}$ ($_{54}Xe^{129}$, correlated with Eq. (8)). It is possible that the $4d$, $6h$ crossover in the neutron group is delayed until near the completion of the 82 shell. An example of a neutron group containing three $4d$ neutrons is supplied by

TABLE V. Odd isomers; N or $Z\leqq 81$.

N or Z	Nucleus	Half-life	Energy (Mev)	ℓ
29	$_{22}Ti^{51}$			
39	$_{30}Zn^{69}$	13.8 hr.	0.44	5
39	$_{32}Ge^{71}$			
39	$_{39}Y^{87}(B)$	14 hr.	0.5	5
39	$_{39}Y^{91}$	50 min.	0.61	5
41	$_{41}Cb^{95}$	80–90 hr.	0.24	5
43	$_{34}Se^{77}$	17.5 sec.	0.15	4
43[a]	$_{43}Tc^{93}$			
43	$_{43}Tc^{95}(B)$			
43	$_{43}Tc^{97}$	90 days	0.10	5
43	$_{43}Tc^{99}$	6 hr.	0.14–0.18	5(?)
45	$_{32}Ge^{77}(B)$			
45	$_{45}Rh^{103}$	50 min.	0.04	4
47	$_{34}Se^{81}(BB)$	59 min.	0.10	4
47	$_{36}Kr^{83}$	113 min.	0.046	4
47	$_{38}Sr^{85}$	70 min.	0.17	4–5
47	$_{47}Ag^{107}$	40 sec.	0.093	4
47	$_{47}Ag^{109}$	40 sec.	0.087	4
49	$_{34}Se^{83}$	$\gg$67 sec.	0.26	5
49	$_{36}Kr^{85}(B)$			
49	$_{38}Sr^{87}$	2.7 hr.	0.37	5
49	$_{40}Zr^{89}$	4.5 min.		
49	$_{40}In^{113}$	105 min.	0.39	5
49	$_{49}In^{115}$	4.5 hr.	0.34	5
51	$_{42}Mo^{73}(BB)$			
63	$_{48}Cd^{111}$	48.7 min.	0.230, 0.145	4–5
65	$_{48}Cd^{113}$	2.3 min.		
67	$_{48}Cd^{115}$	>43 days[b]	0.2[b]	$\geqq$5
69	$_{52}Te^{121}$	1.2×10^7, 5×10^{-8} sec.	0.05, 0.23	5,3
69[c]	$_{69}Tm^{169}(B)$	1×10^{-6} sec.	0.2	3
69[c]	$_{69}Tm^{171}(BB)$	2.5×10^{-6} sec.	0.1	3
73[d]	$_{52}Te^{125}$	60 days	0.12	5
73	$_{54}Xe^{127}(BB)$	75 sec.	0.175, 0.125	4
73[c]	$_{73}Ta^{181}$	2×10^{-5} sec.	0.13	3
75	$_{52}Te^{127}$	90 days	0.086	5
75[c]	$_{75}Re^{187}$	0.65×10^{-6} sec.	0.13	3
77	$_{52}Te^{129}$	32 days	0.102	5
77	$_{56}Ba^{133}$	38 hr.	0.276	5
79	$_{52}Te^{131}$	30 hr.	0.177	5
79	$_{79}Au^{197}$	7.5 sec.	0.25	4
81	$_{54}Xe^{135}$	10 min.	0.52	5
81	$_{56}Ba^{137}$	2.5 min.	0.75	5

[a] D. N. Kundu and M. L. Pool, Phys. Rev. **74**, 1775 (1948).
[b] R. W. Hayward and A. C. Helmholz, Phys. Rev. **75**, 1469 (1949).
[c] S. DeBenedetti and F. K. McGowan, Phys. Rev. **74**, 728 (1948).
[d] Friedlander, Goldhaber, and Sharff-Goldhaber, Phys. Rev. **74**, 981 (1948).

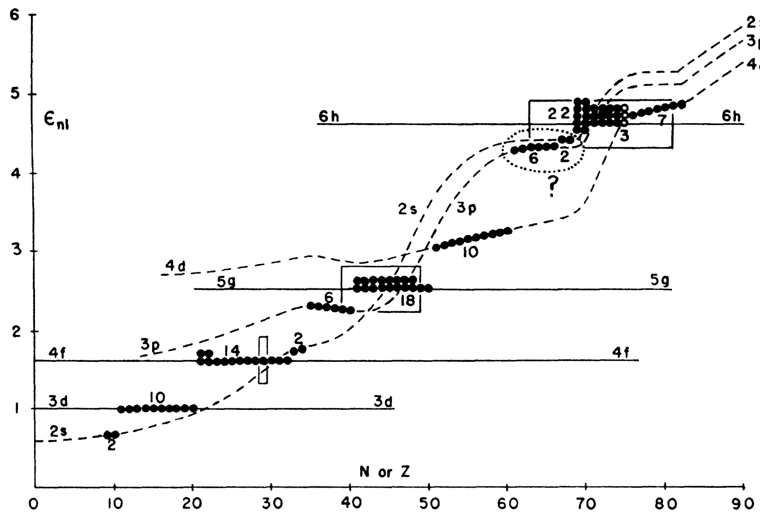

FIG. 1. Qualitative behavior of the single particle energy levels. Energy scale in arbitrary units. Solid dots denote the filling of the shells; the three open circles at N or $Z=75$ denote the three $4d$ particles that remain in the system when the $4d$ level crosses the $6h$ level. Boxes outline regions in which isomerism occurs in odd nuclei.

${}_{42}Mo^{95}$. In this case the experimental spin determination yields $\frac{1}{2}$, but is uncertain. The analysis of radioactive transitions and isomerism at $A=95$ (next section) associates spin 5/2 and even parity with the ground state of ${}_{42}Mo^{95}$. A conclusive determination of the spin and magnetic moment of this nucleus would be helpful in the development of the shell model. The same statement may be applied to ${}_{12}Mg^{25}$.

Odd nuclei with three like particles missing from d orbits constitute another interesting group, among them:

Nucleus	Spin	Magnetic moment
${}_{17}Cl^{35}$	3/2	0.82
${}_{17}Cl^{37}$	3/2	0.68
${}_{16}S^{33}$	3/2	—
${}_{56}Ba^{135}$	3/2	0.84
${}_{79}Au^{197}$	3/2	0.195

Those listed above are all in predominantly ${}^2D_{3/2}$ states excepting ${}_{16}S^{33}$ for which the magnetic moment is not known. Interaction within the $(nd)^{-3}$ configuration accounts for a ${}^2P_{3/2}$ admixture. Two other stable nuclei ${}_{44}Ru^{101}$ (spin and moment not known) and ${}_{57}La^{139}$ ($I=7/2$, $\mu=2.76$) belong in the $(4d)^{-3}$ group. No regularity is apparent in the group of odd nuclei containing half-filled d shells. Among the nuclei in the $(nd)^5$ group are two ${}_{15}P^{31}$ and ${}_{77}Ir^{191}$ with $I=\frac{1}{2}$. The first has $\mu=1.13$, hence is predominantly ${}^2P_{1/2}$ with a large admixture of ${}^2S_{1/2}$. A neighboring $(3d)^6(2s)^{-1}$ configuration may account for the ${}^2S_{1/2}$ admixture. From the spin of ${}_{77}Ir^{193}$ ($I=3/2$) and the ratio of magnetic moments $\mu^{191}/\mu^{193}=-0.92$ we conclude that ${}_{77}Ir^{191}$ is predominantly ${}^2P_{1/2}$. The fact that no nucleus in the $(nd)^5$ group is in a predominantly ${}^2S_{1/2}$ state may be correlated with the absence of ${}^2S_{1/2}$ states in the $(nd)^5$ manifold (references in Table III(b)).

It is a curious circumstance that all the known large positive quadrupole moments ($Q>2.5\times10^{-24}$ cm^2) occur in the narrow range of Z values (69–75) associated with the filling and termination of the $6h$ shell. What significance attaches to this coincidence is not clear. We note however that a configuration containing several orbits with large angular momentum generates a large number of linearly independent states with, for example, $I=5/2$. The largest eigenvalue of the quadrupole moment matrix in the subspace $I=5/2$ may then be much greater than the quadrupole moment produced by a single particle or hole.

Now it may be argued that the central particle density is so far reduced by the successive filling of the long $5g$ and $6h$ shells that a strong reaction occurs during the final filling of the $4d$ shell. The increasing central particle density and reduced (or reversed) rate of growth of the central elevation as more particles enter the 4d shell may maintain an approximate coincidence of the $4d$ and $6h$ levels between the crossover point and the completion of the "82" shell. Thus a condition may develop in which the lowest levels arising from the paired configuration,

$$[50\text{ shell}]\begin{bmatrix}(6h)^{21}(4d)^{2n+2}\\(6h)^{22}(4d)^{2n+1}\end{bmatrix}\begin{matrix}\text{odd parity}\\\text{even parity}\end{matrix}\qquad(11)$$

stay close together over the entire range $Z=75$, 77, 79, and 81. Systems in which this occurs would have small spin ($\frac{1}{2}$, 3/2, 5/2 and rarely 7/2) in the ground state and large spin (possibly 9/2 or 11/2) in a close excited state.

Radiation transitions may be classified as electric $2^{\mathfrak{L}}$ or magnetic $2^{\mathfrak{L}-1}$ pole. Odd values of $\mathfrak{L}$ are associated with a parity change, even values with no change. The spin change is

$$\Delta I=\pm\mathfrak{L}\text{ (electric }2^{\mathfrak{L}}\text{ pole)},$$
$$\Delta I=\pm(\mathfrak{L}-1)\text{ (electric }2^{\mathfrak{L}}\text{ pole, magnetic }2^{\mathfrak{L}-1}\text{ pole)}.\qquad(12)$$

The theoretical relations between half-life, energy, and multipole order do not distinguish effectively between electric $2^{\mathfrak{L}}$ and magnetic $2^{\mathfrak{L}-1}$ poles.[18]

Wiedenbeck's plot of half-life against energy conforms to theoretical expectations sufficiently well to permit the identification of $\mathfrak{L}$ values. The known isomers fall under $\mathfrak{L}=3$, 4, and 5. Theoretical and experimental studies of internal conversion help to fix the multipole order and electric or magnetic character of the transition. A number of isomeric transitions have been studied sufficiently well to permit an unambiguous determination of both $\mathfrak{L}$ and ΔI.[19,20]

Table V lists all known odd isomers in the A and B classifications up to N or $Z=81$ and in the last column the $\mathfrak{L}$ values determined from Wiedenbeck's chart and the analysis of internal conversion.

Knowledge of the $\mathfrak{L}$ value in an isomeric transition reduces the ambiguity in the assignment of configurations to the ground and isomeric states. In particular at N or $Z=39$ the change in parity implied by $\mathfrak{L}=5$ removes Eq. 6(a) from the list of possibilities leaving only 6(b). Support for this conclusion is supplied by the spin and magnetic moment of ${}_{39}Y^{89}$. The states involved in the isomeric transitions are probably all included among the possibilities

$$\begin{aligned} &{}^2G_{9/2}\to{}^2P_{1/2}, \quad \Delta I=4,\\ &{}^2G_{11/2}\to{}^2P_{1/2}, \quad \Delta I=5,\\ &{}^2G_{11/2}\to{}^2P_{3/2}, \quad \Delta I=4. \end{aligned} \tag{13}$$

We conclude that the single particle levels occur in the order $2s$, $3p$, and $5g$ at N or $Z=39$, the spacing of the first pair being large and that of the second pair small.

At $Z=41$ there is again a change in parity and the low configurations are probably

$$(1s)^2(2p)^6(3d)^{10}(4f)^{14}(2s)^2\begin{bmatrix}(2p)^5(5g)^2\\(2p)^6(5g)^1\end{bmatrix}\begin{matrix}\text{odd parity}\\\text{even parity}\end{matrix} \tag{14}$$

giving rise to a ${}^2G$ ground state (compare with spin and moment of ${}_{41}Cb^{93}$]) and a ${}^2P$ excited state.

The crossovers occurring during the filling of the $5g$ shell introduce several possibilities for configurations involved in isomerism at N or $Z=43$, 45, 47, and 49. These configurations are

$$(1s)^2(2p)^6(3d)^{10}(4f)^{14}\begin{bmatrix}(3p)^5(5g)^{2n+2}\\(3p)^6(5g)^{2n+1}\end{bmatrix}\begin{matrix}\text{odd parity}\\\text{even parity,}\end{matrix} \tag{15a}$$

$$(1s)^2(2p)^6(3d)^{10}(4f)^{14}\begin{bmatrix}(5g)^{2n}(3p)^1\\(5g)^{2n+1}\end{bmatrix}\begin{matrix}\text{odd parity}\\\text{even parity,}\end{matrix} \tag{15b}$$

$$(1s)^2(2p)^6(3d)^{10}(4f)^{14}\begin{bmatrix}(2s)^1(5g)^{2n+2}\\(2s)^2(5g)^{2n+1}\end{bmatrix}\begin{matrix}\text{even parity}\\\text{even parity,}\end{matrix} \tag{15c}$$

$$(1s)^2(2p)^6(3d)^{10}(4f)^{14}\begin{bmatrix}(5g)^{2n}(2s)^1\\(5g)^{2n+1}\end{bmatrix}\begin{matrix}\text{even parity}\\\text{even parity.}\end{matrix} \tag{15d}$$

A modified version of (15a) and (15b) obtained by inserting a full $2s$ shell after $(4f)^{14}$ should also be included. Both $2s$ and $3p$ levels are required to cross $5g$ before the completion of the $5g$ shell at N or $Z=50$.

At N or $Z=49$ the configurations in (15b) probably produce the observed low states. These configurations imply a $2s$, $3p$ crossover before the completion of the $5g$ shell. A second crossover reversing the first may occur as the $4d$ shell fills up, since a retardation or reversal of trends should occur in that range. As noted earlier, the central particle density may increase or, at worst, decrease slowly during the filling of the $4d$ shell. The occurrence of $L=4$ in six isomeric transitions suggests a close competition between (15b) and (15d) and therefore a close spacing of $2s$ and $3p$ levels over most of the range involved ($41\leqq\cdot N$ or $Z\cdot\leqq 49$).

In the second island of isomerism ($63\leqq\cdot N$ or $Z\cdot\leqq 81$) $\mathfrak{L}=3$ occurs five times, $\mathfrak{L}=4$ two or three times and $\mathfrak{L}=5$ seven or eight times. The many isomeric transitions with change in parity ($\mathfrak{L}=3$ and 5) may be correlated with the configurations (8a), (8b), (10a), (10b), and (11). These all associate even parity with small spin ($1/2, 3/2, \cdots$) and odd parity with large spin ($\cdots 9/2, 11/2$). Two or three examples of isomerism with no parity change probably involve the configurations (9). Both isomeric and ground states have odd parity.

The isomerism of ${}_{56}Ba^{137}$ ($I=3/2$ in the ground state) provides experimental support for the configurations in Eq. (11). Here $\mathfrak{L}=5$[21] and $\Delta I=5$.[22]

[18] Marcellus L. Wiedenbeck, Phys. Rev. **69**, 567 (1946).

[19] A. C. Helmholz, Phys. Rev. **60**, 415 (1941).

[20] Bradt, Gugelot, Huber, Medicus, Preiswerk, Sherrer, and Steffen, Helv. Phys. Acta **XX**, 153 (1947).

[21] Townsend, Cleland, and Hughes, Phys. Rev. **74**, 499 (1948).

[22] A. C. G. Mitchell and C. L. Peacock, Phys. Rev. **75**, 197 (1949).

The ground state, generated by a $(4d)^{-1}$ configuration, has even parity. Thus the isomeric level has odd parity and $I=11/2$ or (more likely) 13/2. The larger value requires one or more units of orbital angular momentum supplied by the neutron group. Grave difficulties exist in the interpretation of the $_{55}Cs^{137}$ beta-decay. The absence of transitions to the ground state of the product nucleus is particularly puzzling.[22] A full discussion may be deferred until present uncertainty as to the spin and magnetic moment of $_{55}Cs^{137}$ has been removed (we are indebted to Professor G. Zacharias for the information that the unpublished assignment of $I=7/2$ to $_{55}Cs^{137}$ is provisional).

A second case in point is $_{56}Ba^{133}$ with $\mathfrak{L}=5$ for the isomeric transition. Assuming $I=3/2$ and even parity for the ground state (the configuration is $(4d)^5$; also $I=3/2$ for the isotopes of mass 135 and 137) the isomeric level has odd parity and $I=11/2$ or 13/2. A third example, $_{75}Re^{187}$, has $I=5/2$ in the ground state (the configuration is $(4d)^3$) and $\mathfrak{L}=3$ for the isomeric transition. Consequently the isomeric level has odd parity and spin 9/2 or 11/2.

Twenty examples of odd-odd isomers are known. Significant correlations with shell structure are difficult to establish primarily because there are few odd-odd nuclei for which both N and Z fall outside of the regions $29\pm ?$, 39–49, and 63–81.

Figure 1 exhibits the qualitative behavior of the single particle levels as deduced from the analysis of stability relations, spins, magnetic moments, and isomerism. The order and spacing of the single particle levels at N or $Z=83$ suggests an interpretation of the predominantly $^2H_{9/2}$ ground state of $_{83}Bi^{209}$. Among the low configurations are several produced by promoting one or more particles from the $6h$ or $4d$ shells into $2s$ and $3p$ orbits. The list of low configurations with odd parity capable of generating $^2H_{9/2}$ and $^2G_{9/2}$ states includes

$$(6h)^{-1}(2s)^2,\ (6h)^{-1}(2p)^2,\ (4d)^{-2}(2s)^2(3p)^1,$$
$$(4d)^{-2}(3p)^3,\ (6h)^{-1}(4d)^{-1}(3p)^2(2s)^1.$$

It is possible that the coupling within this large group of closely spaced configurations depresses a predominantly $^2H_{9/2}$ state below the $^2S_{1/2}$ and $^2P_{1/2}$ and $^2P_{3/2}$ states associated with the $(2s)^1$ and $(3p)^1$ configurations.

IV. BETA-DECAY

The empirical classification of beta-spectra with respect to the allowed or forbidden character of the transition provides information on the spins and parities of parent and daughter nuclei. Caution is required in interpreting much of the experimental data because of the complicated and incompletely understood structure of many transitions. Relevant experimental information on light nuclei beyond the $1s$ and $2p$ shells appears in Table VI. Values of

TABLE VI. Shell structure of light radioactive and product nuclei.

Nucleus	ft	Type of transition#	Configuration	Spin	Parity
$_7N^{16}(g)$	—	—	$(2p)^{-1}(2s)^1$	0, 1†, 2†	odd
$_8O^{16}(e)$	1.6×10^4	allowed (unfavored)	$(2p)^{-1}(2s)^1$	1	odd
$_8O^{16}(g)$	5.2×10^6	1st forbidden	closed	0*	even
$_9F^{18}(g)$	—	—	$(2s)^1(2s)^1$	1	even
$_8O^{18}(g)$	6×10^3	allowed (favored)	closed	0	even
$_8O^{19}(g)$	—	—	$(3d)^1$	3/2†, 5/2	even
$_9F^{19}(e)$	2.0×10^4	allowed (unfavored)	$(3d)^1$	3/2, 5/2	even
$_9F^{19}(g)$	3.4×10^5	allowed (unfavored)	$(2s)^1$	1/2*	even
$_9F^{20}(g)$	—	—	$(2s)^1(3d)^1$	1, 2†, 3†	even
$_{10}Ne^{20}(e)$	8×10^4	allowed (unfavored)	$(2s)^{-1}(3d)^1$	2	even
$_{10}Ne^{20}(g)$	$\gg 4\times 10^5$	$\geqq$2nd forbidden	closed	0	even
$_{11}Ne^{22}(g)$	—	—	$(3d)^1(3d)^1$	0, 1, 2, 3*, 4, 5	even
$_{10}Ne^{22}(e)$	10^8	2nd (?) forbidden	—	—	—
$_{10}Ne^{22}(g)$	$\gg 10^{10}$	highly forbidden	$(3d)^2$	0	even
$_{19}K^{38}(g)$	—	—	$(3d)^{-1}(3d)^{-1}$	0, 1, 2, 3†, 4, 5	even
$_{18}A^{38}(e)$	8×10^4	allowed (unfavored)	$(2s)^{-1}(3d)^{-1}$	2	even
$_{18}A^{38}(g)$	$\gg 10^6$	highly forbidden	$(3d)^{-2}$	0	even
$_{19}K^{40}(g)$	—	—	$(3d)^{-1}(4f)^1$	0, 1, 2, 3, 4*, 5, 6	odd
$_{20}Ca^{40}(g)$	$\sim 10^{15}$	$3d$ forbidden	closed	0	even
$_{18}A^{41}(g)$	—	—	$(3d)^{-2}(4f)^3$	1/2, 3/2, 5/2, 7/2†···	odd
$_{19}K^{41}(e)$	1.0×10^5	allowed (unfavored)	$(4f)^1(3d)^{-2}(4f)^2$	···5/2, 7/2···	odd
$_{19}K^{41}(g)$	4×10^8	2nd (?) forbidden	$(3d)^{-1}(4f)^2$	3/2*	even

Empirical classification.
* Experimental value.
† Preferred theoretical possibility.
e—Excited state; g—ground state.

TABLE VII. Shell structure of intermediate radioactive, isomeric and product nuclei.

Nucleus	ft	Type of transition	Configuration	Spin	Parity
$_{30}Zn^{69}(m)$	—	—	$(3p)^{-2}(5g)^1$	7/2, 9/2†	even
$_{30}Zn^{69}(g)$	—	IT; $\mathfrak{L}=5$	$(3p)^{-1}$	1/2†, 3/2	odd
$_{31}Ga^{69}(g)$	5×10^4	allowed (unfavored)	—	3/2*	odd
$_{32}Ge^{71}(m)$	—	—	$(3p)^{-2}(5g)^1$	7/2, 9/2†	even
$_{32}Ge^{71}(g)$	—	IT; $\mathfrak{L}\geqq5$	$(3p)^{-1}$	1/2†, 3/2	odd
$_{31}Ga^{71}(g)$	8×10^5	allowed (unfavored)	—	3/2*	odd
$_{34}Se^{83}(m)$	—	—	$(5g)^{-2}(3p)^1$	1/2†, 3/2	odd
$_{35}Br^{83}(g)$	1.5×10^5	allowed (unfavored)	$(3p)^1$	1/2, 3/2	odd
$_{36}Kr^{83}(m)$	1.4×10^5	allowed (unfavored)	$(5g)^{-4}(3p)^1$	1/2, 3/2	odd
$_{36}Kr^{83}(g)$	—	IT; $\mathfrak{L}=4$	$(5g)^{-5}(3p)^1(2s)^1$	7/2, 9/2*···	odd
$_{34}Se^{83}(g)$	—	IT(n.o.); $\mathfrak{L}\geqq5$	$(5g)^{-1}$	7/2, 9/2†	even
$_{35}Br^{83}(e)$	1.2×10^5	allowed (unfavored)	?	?	odd
$_{39}Y^{87}(g)$	—	—	$(3p)^{-1}$	1/2†, 3/2	odd
$_{38}Sr^{87}(m)$	?	allowed (unfavored)	$(5g)^{-2}(3p)^1$	1/2†, 3/2	odd
$_{38}Sr^{87}(g)$	—	IT; $\mathfrak{L}=5$	$(5g)^{-1}$	7/2, 9/2*	even
$_{37}Rb^{87}(g)$	—	K(n.o.); allowed	$(3p)^{-3}$	1/2, 3/2*···	odd
$_{38}Sr^{87}(g)$	5×10^{16}	3d forbidden	$(5g)^{-1}$	7/2, 9/2*	even
$_{38}Sr^{89}(g)$	—	—	$(4d)^1$	3/2, 5/2†	even
$_{39}Y^{89}(g)$	5×10^8	2nd (?) forbidden	$(3p)^{-1}$	1/2*(?), 3/2	odd
$_{40}Zr^{89}(m)$	—	—	$(5g)^{-1}$	7/2, 9/2†	even
$_{40}Zr^{89}(g)$	—	IT; $\mathfrak{L}=5$(?)	$(5g)^{-2}(3p)^1$	1/2†, 3/2	odd
$_{39}Y^{89}(g)$	6×10^5	allowed (unfavored)	$(3p)^{-1}$	1/2*(?), 3/2	odd
$_{38}Sr^{91}(g)$	—	—	$(4d)^3$	1/2, 3/2, 5/2†···	even
$_{39}Y^{91}(g)$	1.2×10^8	2nd (?) forbidden	$(3p)^{-1}$	1/2†, 3/2	odd
$_{40}Zr^{91}(g)$	5×10^8	2nd (?) forbidden	$(4d)^1$	3/2, 5/2†	even
$_{39}Y^{91}(e)$	5×10^6	1st forbidden	—	3/2, 5/2, 7/2	odd
$_{39}Y^{91}(m)$	—	γ; $\mathfrak{L}\leqq4$	$(3p)^{-2}(5g)^1$	7/2, 9/2†	even
$_{39}Y^{91}(g)$	—	IT; $\mathfrak{L}=5$	$(3p)^{-1}$	1/2†, 3/2	odd
$_{43}Tc^{99}(m)$	—	—	$(5g)^{2n}(2s)^1$	1/2	even
$_{43}Tc^{99}(m)$	—	—	$(5g)^{2n}(3p)^1$	1/2†, 3/2	odd
$_{43}Tc^{99}(g)$	—	γ; $\mathfrak{L}=4$ or 5	$(5g)^{2n+1}$	···9/2···	even
$_{44}Ru^{99}(g)$	10^{13}	highly forbidden	$(4d)^5$	1/2, 3/2†, 5/2···	even
$_{44}Ru^{103}(g)$	—	—	$(4d)^{-1}$	3/2, 5/2†	even
$_{45}Rh^{103}(e)$	1.3×10^6	1st forbidden (?)	—	—	odd(?)
$_{45}Rh^{103}(m)$	$\gg4\times10^7$	2nd forbidden	$(5g)^{2n}(2s)^1$	1/2	even
$_{45}Rh^{103}(g)$	$\gg4\times10^7$	2nd forbidden	$(5g)^{2n+1}$	···7/2, 9/2†	even
$_{48}Cd^{107}(g)$	—	—	$(4d)^{-1}$	3/2, 5/2†	even
$_{47}Ag^{107}(m)$	4.2×10^4	K capture, allowed (?)	$(5g)^{-5}(3p)^1(2s)^1$	7/2,† 9/2···	odd
$_{47}Ag^{107}(g)$	—	IT; $\mathfrak{L}=4$	$(5g)^{-4}(3p)^1$	1/2,* 3/2	odd
$_{46}Pd^{109}(g)$	—	—	—	—	—
$_{47}Ag^{109}(m)$	1.4×10^6	1st forbidden	$(5g)^{-5}(3p)^1(2s)^1$	···7/2†, 9/2	odd
$_{47}Ag^{109}(g)$	—	IT; $\mathfrak{L}=4$	$(5g)^{-4}(3p)^1$	1/2*, 3/2	odd
$_{48}Cd^{109}(g)$	—	—	$(6h)^1$	9/2†, 11/2	odd
$_{47}Ag^{109}(m)$	?	allowed (unfavored)	$(5g)^{-5}(3p)^1(2s)^1$	···7/2†, 9/2···	odd

† Preferred theoretical possibility.
* Experimental value.
e—Excited state; m—metastable state; g—ground state; n.o.—not observed.

f are taken from charts in the review article by Konopinski.[23] The empirical classification and the available theoretical calculations are followed in assessing the degree of forbiddenness of the transitions. The spin column contains the range of spin values permitted by the shell structure; experimental values are starred. Parity given in the last column is fixed uniquely by the shell structure. The discussion following the table is based on the Gamow-Teller selection rules as derived from tensor or axial-vector coupling:[23]

$\Delta I=0,\ \pm1$, no parity change, no $0\rightarrow0$ (allowed),
$\Delta I=0,\ \pm1,\ \pm2$, parity change (1st forbidden),
$\Delta I=0,\ \pm2,\ \pm3$, no parity change (2nd forbidden).

An additional selection rule due to Wigner[24] classifies the allowed transitions as favored (no change in supermultiplet, $ft\sim3000$) and unfavored

[23] Emil J. Konopinski, Rev. Mod. Phys. 15, 209 (1943).

[24] Eugene P. Wigner, Phys. Rev. 56, 519 (1939); L. W. Nordheim and F. L. Yost, Phys. Rev. 51, 942 (1937) present an informal, but, in essentials, equivalent statement of the distinction between favored and unfavored transitions.

(change in supermultiplet, but no change in parity, $ft \sim 10^5$). In simpler terms the transition is favored if the same space wave function occurs in both parent and daughter nuclei, while it may be allowed, but with a reduced value of the nuclear matrix element, if there is a substantial change in the space wave functions (change in symmetry or configuration). It is apparent from the empirical classification of transition types that the ft values of allowed (unfavored) and first forbidden transitions overlap, thus in extreme cases precluding positive identification of the transition types on the basis of these values alone. Very often the shell model provides the information needed to make the identification.

Discussion of Table VI

$_7N^{16}$: half-life 7.35 sec., β^- 3.8 Mev ($\sim$75 percent), 10.0 Mev ($\sim$25 percent), γ 6.2, 6.8 Mev. From the intensity ratio the "half-lives" of the separate components are 10 sec. (low energy transitions) and 30 sec. (high energy transitions). Both ^1P and ^3P levels occur among the low terms arising from the $(2p)^{-1}(2s)^1$ configuration in $_7N^{16}$. The same configuration and a lower supermultiplet yield a ^1P excited state for $_8O^{16}$.

$_9F^{18}$: half-life 6700 sec., β^+ 0.7 Mev. The transition is favored because both parent ($^3S_1$) and daughter ($^1S_0$) ground states belong to the same supermultiplet (and also the same configuration). A search for a gamma-ray (William J. Knox, Phys. Rev. **74**, 1192 (1948)) yielded a negative result.

$_8O^{19}$: half-life 27 sec., β^- 2.9 Mev ($\sim$70 percent), 4.5 Mev ($\sim$30 percent), γ 1.6 Mev; the half-lives of the separate components are 39 sec. (low energy transition) and 90 sec. (high energy transition). The high energy transition is particularly unfavored because the two ground states involved belong to different configurations ($(3d)^1$ and $(2s)^1$).

$_9F^{20}$: half-life 12 sec., β^- 5.0 Mev, γ 2.2 Mev. Both ^1D and ^3D levels occur among the low terms arising from the $(2s)^1(3d)^1$ configuration in $_9F^{20}$. The same configuration and a lower supermultiplet yield a $^1D_2$ excited state for $_{10}Ne^{20}$. It is also possible that the spectrum is complex; β^- 5.0 Mev to $_{10}Ne^{20}(g)$ and β^- 2.8 Mev to $_{10}Ne^{20}(e)$ followed by γ 2.2 Mev. In this case spin 1 would be preferred for $_9F^{20}(g)$ and the β^--transition to the ground state would be allowed (unfavored).

$_{11}Na^{22}$: half-life 10^8 sec., β^+ 0.56 Mev, γ 1.3 Mev. Again both singlets and triplets appear among the group of low levels in $_{11}Na^{22}$.

Empirically the transition to the ground state is highly forbidden whereas the quantum numbers of parent and daughter nuclei place it in the second forbidden category. However the matrix element involved does not appear in the more common 2nd forbidden transitions (presumably with $\Delta I = \pm 2$) and may be small enough to increase the empirical order of the transition.[23,25] This conjecture is confirmed by the $_4Be^{10} \rightarrow {}_5B^{10}$ transition (half-life 8×10^{13} sec., β^- 0.56 Mev, $ft \sim 5.6\times 10^{13}$). Here the spins are 0 and 3 and both parities are even. Moreover the magnetic moments of $_5B^{10}$ and $_{11}Na^{22}$ are almost identical (1.8 nuclear magnetons implying a $^3D_3$ ground state in both systems). Consequently the nuclear matrix elements of the beta-decay theory should have about the same value in both 10- and 22-particle systems for the transitions between ground states. With $f \sim 56$ at $Z = 10$ and β^+ 1.86 Mev the half-life of the transition to the ground state of $_{10}Ne^{22}$ is determined by the equation

$$56\, t_{22}\ (\text{ground state}) \sim 8\times 10^{13}\times 0.70 \text{ sec.},$$
$$t_{22}\ (\text{ground state}) \sim 10^{12} \text{ sec.} \qquad (16)$$

Thus only one transition out of ten thousand should go directly to the ground state, a small enough proportion to have escaped observation in spectrometer and early cloud-chamber studies.[26]

$_{19}K^{38}$: half-life 460 sec., β^+ 2.53 Mev, γ 2.15 Mev; almost certainly in cascade. For β^+ 2.53 Mev, $W_0 = 6.0$ mc^2 and $f \sim 165$ yielding $ft \sim 8\times 10^4$. If the β^+-spectrum were complex with the gamma-transition following β^+ 0.38 Mev, the number of gamma's per positron could not exceed $6\times 10^{-2}/165 \sim 4\times 10^{-4}$ (the ratio of the f values at $W_0 = 1.75$ and 6.0 mc^2). A change in configuration is required to make the transition to $_{18}A^{38}(e)$ unfavored.

$_{19}K^{40}$: half-life $\sim 3\times 10^{16}$ sec., β^- 1.35 Mev, γ 1.5 Mev (with K capture). The selection rules for the third forbidden transition include $\Delta I = \pm 4$ with change in parity. A summary of theoretical studies of this transition appears in reference 23.

$_{18}A^{41}$: half-life 6600 sec., β^- 1.18 Mev, 2.55 Mev (0.7 percent), γ 1.37 Mev; half-life of the transition to the ground state 10^6 sec. The transition to $_{19}K^{41}(e)$ requires a change in supermultiplet, but once more no change in configuration. The large ft value of the transition to the ground state implies a small nuclear matrix element for $\Delta I = \pm 2$ with change in parity.[25]

The special selection rule invoked in the discussion of the $_{18}A^{41}$ transition recurs in heavier systems. In several examples the empirical classification is 2nd forbidden while the shell model requires a change of parity and $\Delta I = \pm 2$ (1st forbidden by the G. T. selection rules derived from tensor coupling).

The extraordinary $_4Be^{10}$ decay provides an ex-

[25] N. Feather and H. O. W. Richardson, Proc. Phys. Soc. **61**, 452 (1948).

[26] K. H. Morganstern, at this laboratory, has observed a small number of long range positron tracks from the disintegration of a $_{11}Na^{22}$ source in a cloud chamber placed in a strong magnetic field.

ample of a transition [27–28] with no change in parity and $\Delta I=-3$. Here the selection rules classify the transition as second forbidden while the empirical classification is at least $3d$ forbidden. The same situation recurs[29] in the ${}_{11}Na^{22}(g)$ transition to ${}_{10}Ne^{22}(g)$. In view of these examples it is not astonishing to find a parallel situation for $\Delta I=\pm 2$ with change in parity.

Table VII exhibits the connection between the configurations suggested by the shell model, the selection rules for isomeric and radioactive transitions and the empirical evidence on such transitions. In several series of isomeric and radioactive transitions (notably at $A=71$, 87, 89, and 91) the information on the nature and order of the transitions when combined with the selection rules almost completely determines the unknown spins and parities. Such cases provide a searching test of the shell model. The series at $A=91$ is particularly interesting since no experimental spin values are available. Three series (at $A=69$, 71, and 91) exhibit the same pattern of spins and parities for the ground and isomeric levels of the nuclei with N or $Z=39$. A similar statement can be made for two series at $A=83$ and 87 involving $N=49$. The two indium isotopes ${}_{49}In^{113}$ and ${}_{49}In^{115}$, each with known spin 9/2 in the ground state and $\mathfrak{L}=5$ characterizing the isomeric transition, belong with ${}_{34}Sc^{83}$ and ${}_{38}Sr^{87}$. In a fifth series at $A=89$ the nucleus with $N=49$ exhibits an exact inversion of the ground state-isomeric state spin and parity relations observed in ${}_{34}Se^{83}$, ${}_{38}Sr^{87}$, ${}_{49}In^{113}$, and ${}_{49}In^{115}$.

Discussion of Table VII

${}_{30}Zn^{69}(m)$: half-life 5×10^4 sec., γ 0.44 Mev. Studies of internal conversion (reference 20) support the assignment $\mathfrak{L}=5$ taken from Wiedenbeck's chart.

${}_{30}Zn^{69}(g)$: half-life 3400 sec., β^- 1.0 Mev; an allowed (unfavored) transition to ${}_{31}Ga^{69}(g)$. The selection rules require spin 1/2 or 3/2 and odd parity for ${}_{30}Zn^{69}(g)$ in agreement with the shell model.

${}_{32}Ge^{71}(m)$: half-life 10^6 sec., γ 0.6 Mev; closer to $\mathfrak{L}=6$ than $\mathfrak{L}=5$ on Wiedenbeck's chart. Bearing in mind the scattering of experimental points on the chart the assignment to $\mathfrak{L}=5$ is not unreasonable.

${}_{32}Ge^{71}(g)$: half-life 1.4×10^5 sec., β^- 1.2 Mev; an allowed (unfavored) transition to ${}_{31}Ga^{71}(g)$. The odd parity assigned to ${}_{31}Ga^{69}(g)$ and ${}_{31}Ga^{71}(g)$ is derived from the odd parity of configurations containing an odd number of $4f$ wave functions. The possibility of using even parity seems ruled out by the small ft value in the Zn^{69}, Ga^{69} transition.

[27] M. Goldhaber, Phys. Rev. **74**, 1194 (1948).
[28] Gordy, Ring, and Burg, Phys. Rev. **74**, 1191 (1948).
[29] Luther Davis, Phys. Rev. **74**, 1193 (1948).

TABLE VIII. Beta-decay with change in parity.

Emitter	Radiation	Half-life (sec.)	W_0(mc^2)	f	ft
${}_{7}N^{16}$	β^-	7.35 (25%)	21.0	1.7×10^5	5.2×10^6
${}_{17}Cl^{38}$	β^-	2300 (53%)	11.3	9.2×10^3	4.0×10^7
${}_{18}A^{41}$	β^-	6600 (0.7%)	6.03	430	4.0×10^8
${}_{19}K^{42}$	β^-	4.5×10^4 (75%)	8.06	1.8×10^3	1.1×10^8
${}_{35}Br^{84}$	β^-	1800	11.4	1.8×10^4	3.2×10^7
${}_{36}Kr^{88}$	β^-	1.1×10^4	5.9	720	7.9×10^6
${}_{37}Rb^{88}$	β^-	1050	10.2	1.11×10^4	1.2×10^7
${}_{38}Sr^{89}$	β^-(no γ)	4.8×10^6	4.0	103	4.9×10^8
${}_{38}Sr^{90}$	β^-(no γ)	8×10^8	2.05	1.8	1.4×10^9
${}_{38}Sr^{91}$	β^-	3.6×10^4 (60%)	7.3	2.1×10^3	1.3×10^8
${}_{39}Y^{90}$	β^-(no γ)	2.2×10^5	5.4	500	1.1×10^8
${}_{39}Y^{91}$	β^-(no γ)	5.2×10^6	4.01	105	5.4×10^8
${}_{37}Rb^{87}$	β^-	2×10^{18}	1.26	0.025	5×10^{16}

${}_{34}Se^{83}(m)$: half-life 67 sec., β^- 3.4 Mev; an allowed (unfavored) transition to ${}_{35}Br^{83}(g)$. The odd parity is required by the failure to observe an isomeric transition to ${}_{34}Se^{83}(g)$ (with γ 0.26 Mev), implying $\mathfrak{L}=5$. Since ${}_{34}Se^{83}(g)$ is assigned even parity that of the isomeric level must be odd (assuming $\mathfrak{L}=5$ since $\mathfrak{L}=6$ seems unlikely).

${}_{34}Se^{83}(g)$: half-life 1500 sec., β^- 1.5 Mev, γ 0.17, 0.37, 1.1 Mev, an allowed (unfavored) or 1st forbidden transition to ${}_{35}Br^{83}(e)$ followed by a cascade of three gamma-rays.

${}_{35}Br^{83}(g)$: half-life 8600 sec., β^- 1.0 Mev; an allowed (unfavored) transition to ${}_{36}Kr^{83}(m)$.

${}_{36}Kr^{83}(m)$: half-life 6800 sec., γ 0.029, 0.046 Mev; an isomeric transition with $\mathfrak{L}=4$. The K/L conversion ratio of the 46-kev line requires $\Delta I=\pm 4$ (reference 20); in view of the known spin 9/2 of the ground state the isomeric level must have spin 1/2. The simplest possibility supplied by the shell model gives both levels even parity. However, the $(3g)^{-3}$ and $(3g)^{-4}(2s)^1$ configurations are rejected, and more complicated configurations involving the $3p$ orbit are accepted for two reasons: (a) the ft values of allowed and forbidden transitions should not overlap and (b) the properties of the odd silver isotopes associate the more complicated configurations with $Z=47$, hence, also with $N=47$.

${}_{39}Y^{87}(g)$: half-life 2.9×10^5 sec., K capture to ${}_{38}Sr^{87}(m)$. The selection rules and the quantum numbers required by the shell model make this an allowed (unfavored) transition. Considering Eq. (33b) of reference 23 for f_k, this interpretation is indeed reasonable.

${}_{38}Sr^{87}(m)$: half-life 9.7×10^3 sec., γ 0.38 Mev; near $\mathfrak{L}=5$ on Wiedenbeck's chart. Since ${}_{38}Sr^{87}(g)$ has spin 9/2 (experimental) and even parity (shell model) the isomeric state must have odd parity and spin 1/2. K capture to ${}_{37}Rb^{87}(g)$ is energetically possible and also allowed (unfavored). From Eq. 33b, reference 23, $f_k\sim 0.03$. Consequently, $t_k\sim 10^6$ sec. and ${}_{38}Sr^{87}(m)$ should exhibit a branching decay: 1 percent K capture and 99 percent IT. The K capture has not yet been observed.

$_{37}$Rb$^{87}(g)$: half-life 2×10^{18} sec., β^- 0.14 Mev, γ?; spins of both parent and daughter nuclei have been measured. The change in parity required by the shell model makes the transition 3d forbidden.

$_{38}$Sr$^{89}(g)$: half-life 4.7×10^{6} sec., β^- 1.5 Mev, no γ; in the empirical classification a 2nd forbidden transition to $_{39}$Y$^{89}(g)$. The theoretical classification is 1st forbidden with maximum possible spin change ($\Delta I=\pm 2$).

$_{40}$Zr$^{89}(m)$: half-life 270 sec., *IT* energy not known, but γ up to 1 Mev observed; $\mathfrak{L}=5$ indicated. The absence of transitions from $_{40}$Zr$^{89}(m)$ to $_{39}$Y$^{89}(g)$ associates large spin with the isomeric level and small spin with the ground level. An occasional reversal of the expected order is not unreasonable.

$_{40}$Zr$^{89}(g)$: half-life 2.9×10^{5} sec., β^+ 1.0 Mev; an allowed (unfavored) transition to $_{39}$Y$^{89}(g)$. With spin 1/2 and odd parity for $_{40}$Zr$^{89}(g)$ all requirements are met. In particular $\mathfrak{L}=5$ and $\Delta I=4$ in the isomeric transition from $_{40}$Zr$^{89}(m)$.

$_{38}$Sr$^{91}(g)$: half-life 3.6×10^{4} sec., β^- 3.2 Mev (60 percent), 1.3 Mev (40 percent), γ 1.3 Mev. Separate half-lives are 6×10^{4} sec. (ground state) and 9×10^{4} (excited state). In the empirical classification the transition types are 2nd forbidden to $_{39}$Y$^{91}(g)$: 1st forbidden to $_{39}$Y$^{91}(e)$ and 3d forbidden (at least) for the unobserved transition to $_{39}$Y$^{91}(m)$. The theoretical classification of the transition to $_{39}$Y$^{91}(g)$ is 1st forbidden with maximum possible spin change ($\Delta I=\pm 2$).

$_{39}$Y$^{91}(e)$: γ 1.3 Mev to $_{39}$Y$^{91}(m)$, no observed delay, no observed direct transitions to $_{39}$Y$^{91}(g)$.

$_{39}$Y$^{91}(m)$: half-life 3000 sec., γ 0.61 Mev; an isomeric transition on Wiedenbeck's $\mathfrak{L}=5$ curve. The unobserved transition to $_{40}$Zr$^{91}(g)$ is only 2nd forbidden. With β^- 2.14 Mev for this transition the

TABLE IX. Beta-decay with no change in parity.

Emitter	Radiation	Half-life (sec.)	W_0(mc^2)	f	ft
$_0$n^1	β^-	>900#	2.5	1.6	$>1.4\times 10^3$
$_1$H^3	β^-(no γ)	3.8×10^8	1.038*	2.9×10^{-6}†	1.1×10^3
$_4$Be^7	*K*-capture	3.7×10^6 (90%)	0.70	4.5×10^{-4}	1.8×10^3
$_6$C^{11}	β^+(no γ)	1200	2.95	4.1	4.9×10^3
$_7$N^{13}	β^+(no γ)	600	3.44	10.3	6.2×10^3
$_8$O^{15}	β^+	126	4.4	41	5.1×10^3
$_9$F^{17}	β^+	70	5.1	92	6.4×10^3
$_{10}$Ne^{19}	β^+	20.3	5.35	115	2.3×10^3
$_{12}$Mg^{23}	β^+	11.6	6.56	320	3.7×10^3
$_{14}$Si^{27}	β^+	4.9	8.2	890	4.4×10^3
$_{15}$P^{29}	β^+	4.6	8.2	860	4.0×10^3
$_{16}$S^{31}	β^+	2.6	8.6	1.1×10^3	2.9×10^3
$_{17}$Cl^{33}	β^+	2.4	9.14	1.4×10^3	3.4×10^3
$_{18}$A^{35}	β^+	1.88	9.7	1.9×10^3	3.6×10^3
$_{21}$Sc^{41}	β^+	0.87	10.74	3×10^3	2.6×10^3
$_2$He^6	β^-	0.89	7.9	1.0×10^3	0.9×10^3
$_{13}$Al^{26}	β^+	6	6.9(?)	400	2.4×10^3
$_4$Be^{10}	β^-	8×10^{13}	2.10	0.6	5×10^{13}
$_6$C^{14}	β^-	1.5×10^4	1.30	0.005	8×10^8
$_{15}$P^{30}	β^+	153	7.9(?)	710	1.1×10^5
$_8$O^{19}	β^-	29.5 (30%)	9.9	3.9×10^3	3.8×10^5
$_{10}$Ne^{23}	β^-	40	9.1	2.6×10^3	1.1×10^5
$_{13}$Al^{29}	β^-	400	5.9	340	1.4×10^5
$_{14}$Si^{31}	β^-(no γ)	1.02×10^4	4.6	98	10×10^5
$_{16}$S^{35}	β^-	7.5×10^6	1.33	0.023	1.8×10^5
$_{20}$Ca^{45}	β^-(no γ)	1.3×10^7	1.49	0.08	10×10^5
$_{22}$Ti^{45}	β^+	1.1×10^4	3.4	6.8	0.74×10^5
$_{25}$Mn^{51}	β^+	2.8×10^3	5.0	58	1.6×10^5
$_{27}$Co^{61}	β^-	6.3×10^3	3.6	42	2.6×10^5
$_{28}$Ni^{57}	β^+	1.3×10^5	2.33	0.46	0.60×10^5
$_{29}$Cu^{64}	β^-	4.6×10^4 (31%)	2.13	1.87	2.4×10^5
$_{29}$Cu^{64}	β^+	4.6×10^4 (15%)	2.30	0.39	1.2×10^5
$_{30}$Zn^{69}	β^-	3.4×10^3	3.0	15	0.51×10^5
$_{31}$Ga^{68}	β^+	4.1×10^3	4.8	42	1.7×10^5
$_{32}$Ge^{71}	β^+	1.4×10^5	3.4	5.6	7.8×10^5
$_{40}$Zr^{89}	β^+(no γ)	2.9×10^5	3.0	2.0	5.8×10^5
$_{46}$Pd^{112}	β^-(no γ)	7.6×10^4	1.4	0.14	0.11×10^5
$_{47}$Ag^{106}	β^+	1470	5.0	41	0.60×10^5
$_{49}$In^{114}	β^-	72	4.9	460	0.33×10^5
$_{50}$Sn^{121}	β^-(no γ)	9.7×10^4	1.8	1.3	1.3×10^5
$_{52}$Te^{127}	β^-(no γ)	3.3×10^4	2.5	13	4.3×10^5
$_{53}$I^{128}	β^-	1500 (93%)	5.0	600	9.6×10^5
$_{59}$Pr^{140}	β^+	210	5.8	83	0.17×10^5

Snell and Milker, Phys. Rev. **74**, 1217 (1948).
* Hanna, Pontecorvo, and Kirkwood, Phys. Rev. **75**, 983 (1949).
† E. J. Konopinski, private communication.

f value is 400. Assuming $ft \sim 4 \times 10^8$, $t \sim 10^6$ sec. Hence, the intensity of the β^--transition relative to the isomeric transition may be as large as 0.3 percent. A somewhat smaller intensity (by a factor of 10) would not be unreasonable.

${}_{39}Y^{91}(g)$: half-life 5.2×10^6 sec., β^- 1.53 Mev; in the empirical classification a 2nd forbidden transition to ${}_{40}Zr^{91}(g)$. The theoretical classification is 1st forbidden with maximum possible spin change ($\Delta I = \pm 2$).

Three examples of transitions in the empirical 2nd forbidden classification occur in Table VII. If ${}_{39}Y^{89}(g)$ and ${}_{39}Y^{91}(g)$ were assigned even parity the transitions could be interpreted theoretically as 2nd forbidden. However, even parity for the ground state at $Z = 39$ requires the substitution of a $2s$ orbit for $3p$ and yields a ${}^2S$ state for ${}_{39}Y^{89,91}(g)$. The magnetic moment of ${}_{39}Y^{89}(g)$ is in much better accord with a ${}^2P$ state. Also, the parity change in the isomeric transition ${}_{39}Y^{91}(m)$ to ${}_{39}Y^{91}(g)$ is hardly consistent with the available low configurations if the ground state has even parity; $(3p)^{-3}(2s)^1(5g)^1$ would do for the isomeric state, but does not look plausible.

An interesting series of radioactive and isomeric transitions occurs at $A = 95$ beginning with ${}_{40}Zr^{95}(g)$, continuing through ${}_{41}Cb^{95}(e)$, ${}_{41}Cb^{95}(m)$, ${}_{41}Cb^{95}(g)$, ${}_{42}Mo^{95}(e)$, and terminating at ${}_{42}Mo^{95}(g)$. Wiedenbeck's chart yields $\mathfrak{L} = 5$ for the isomeric transition in ${}_{41}Cb^{95}$. Excluding the possibility of a spin value exceeding 11/2 one of the states involved in the isomeric transition has spin 1/2 or 3/2. Inconclusive measurements yield 1/2 for the spin of ${}_{42}Mo^{95}(g)$. It is apparent that this value is incompatible with the absence of direct transitions from ${}_{41}Cb^{95}(m)$ or (g) to ${}_{42}Mo^{95}(g)$.

${}_{43}Tc^{99}(m)$: half-life 2×10^4 sec., γ 0.136 Mev; between $\mathfrak{L} = 4$ and $\mathfrak{L} = 5$ on Wiedenbeck's chart, $\mathfrak{L} = 5$ preferred.

${}_{43}Tc^{99}(g)$: half-life 10^{13} sec., β^- 0.32 Mev: not 4th forbidden (compare K^{40}) probably another example of the special selection rule $\Delta I = \pm 3$, no change in parity (compare 10 and 22 particle systems). The special selection rule and the low configurations for 43 and 45 particles fix the spin at 9/2 for ${}_{43}Tc^{99}(g)$ and 3/2 for ${}_{44}Ru^{99}(g)$. Both parities are even.

${}_{44}Ru^{99}$: $a \rightarrow \beta^-$ 0.46-Mev transition from ${}_{43}Tc^{99}(m)$ to ${}_{44}Ru^{99}(g)$ is either 1st forbidden or allowed (unfavored). With $f \sim 1.4$ the half-life for the direct transition is certainly less than 10^7 sec. The ratio of β^- to isomeric transitions is therefore certainly greater than 2×10^{-3}, but as yet, only the isomeric transition has been observed.

${}_{44}Ru^{103}$: half-life 3.5×10^6 sec., β^- 0.3 Mev, 0.8 Mev (weak), γ 0.56 Mev; 1st forbidden or allowed (unfavored) to ${}_{45}Rh^{103}(e)$ followed by gamma-transition to ${}_{45}Rh^{103}(m)$ and possibly also to

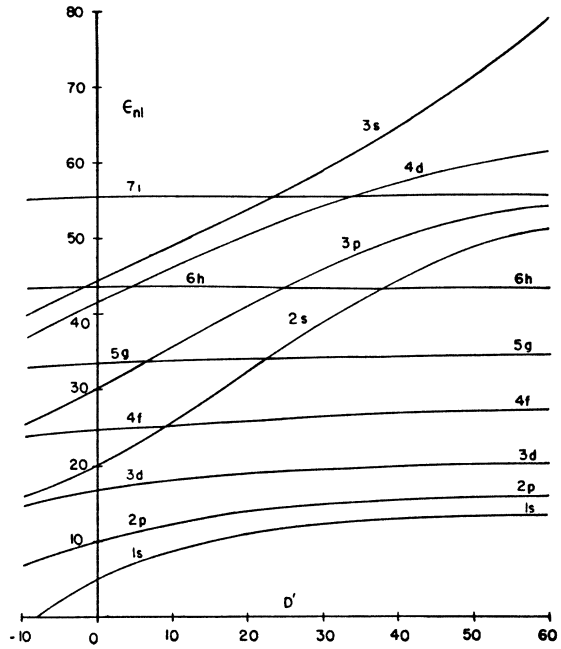

FIG. 2. Behavior of energy levels with increase of potential elevation D' in well of infinite depth and radius R. The elevation covers the range $0 \leqq r \leqq R/2$. ϵ_{nl} and D' in units $\hbar^2/MR^2$.

${}_{45}Rh^{103}(g)$; 2nd forbidden β^--transition to ${}_{45}Rh^{103}(m)$ and probably also to ${}_{45}Rh^{103}(g)$.

${}_{45}Rh^{103}(m)$: half-life 3×10^3 sec., γ 0.04 Mev; $\mathfrak{L} = 4$ from Wiedenbeck's chart.

${}_{46}Pd^{103}(g)$: half-life 1.5×10^6 sec., K capture (no γ) to ${}_{45}Rh^{103}(m)$; allowed (unfavored) excluding the unlikely possibility that the energy release is large ($W_0 \gg 1$).

${}_{48}Cd^{107}(g)$: half-life 2.4×10^4 sec., K capture, and β^+ 0.32 Mev (0.3 percent) to metastable state of ${}_{47}Ag^{107}$. With $W_0 \sim 1.63$ the f values are $f_k \sim 1.75$ and $f(\beta^+) \sim 5 \times 10^{-3}$. The ratio is 1/350 in agreement with the observed branching ratio. The small value of ft places the transition in the allowed (unfavored) classification.

${}_{48}Ag^{107}(m)$: half-life 40 sec., γ 0.094 Mev, an isomeric transition with $\mathfrak{L} = 4$ (Wiedenbeck's chart), and $\Delta I = 3$ (internal conversion and K/L ratios, reference 20). The spin (1/2) and magnetic moment (-0.10) of ${}_{47}Ag^{107}(g)$ show that the ground state is predominantly ${}^2P$. This result excludes the simpler possibilities of the shell model and requires the introduction of a ${}^2P$ orbit into both ground and metastable state configurations. The spin of the metastable state is therefore 7/2 (or possibly 9/2) and the parity is odd. A spin value 5/2, 7/2, 9/2, or 11/2 and odd parity are required by the selection rules for ${}_{48}Cd^{107}$. The only reasonable configuration for $N = 59$ is $(4d)^{-1}$ yielding a ${}^2D_{5/2}$ state of even parity. This discrepancy in parity is a serious difficulty for the shell model.

$_{46}Pd^{109}(g)$: half-life 4.7×10^4 sec., β^- 1.0 Mev to $_{47}Ag^{109}(m)$, probably 1st forbidden.

$_{47}Ag^{109}(m)$: half-life 40 sec., γ 0.087 Mev. The discussion parallels that for $_{47}Ag^{107}(m)$ down to the determination of the ground state configuration of the cadmium parent nucleus. In this case the only suitable (and reasonable) configuration for $N=61$ is $(6h)^1$ yielding the ground state $^2H_{9/2}$ with odd parity. The transition is then allowed and one may conclude that the K capture transition energy is quite small (<0.1 Mev).

The shell model is perhaps most reliable in determining the parity of ground and isomeric states. Such determinations are embodied in Tables VIII and IX for beta-transitions with and without changes in parity. Complex transitions are included only when the branching ratio has been measured, permitting the calculations of the "half-life" of the transition between ground states.

These tables exhibit remarkable features, in some respects at variance with generally accepted ideas. Excluding $_{37}Rb^{87}$, the *ft* values with change in parity spread out from 4.7×10^6 to 2.8×10^9 suggesting wide variations in the nuclear matrix elements involved in 1st forbidden transitions. The larger values of *ft* are still far too small to warrant a 3d forbidden assignment, hence must be explained by some special feature of the 1st forbidden transition. As already mentioned it is plausible that transitions with $\Delta I=\pm2$ and change in parity should appear empirically as 2nd forbidden.[25]

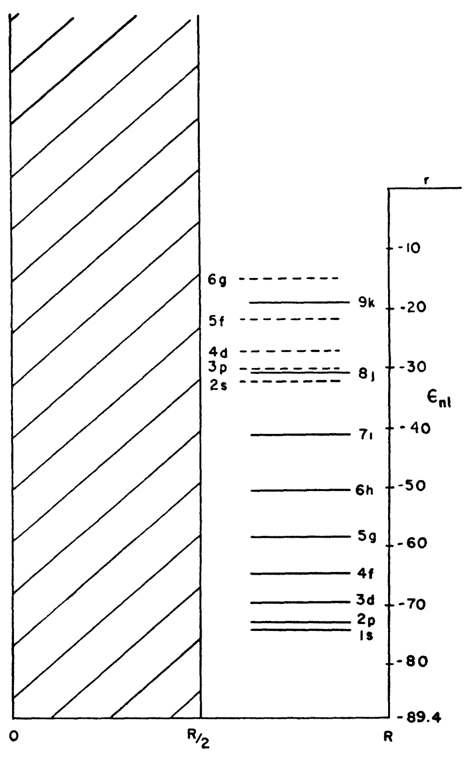

FIG. 3. Single particle energy levels in a well of finite depth, central elevation infinite. Unit of energy $\hbar^2/MR^2$.

The first group in Table IX extending from $_0n^1$ to $_{21}Sc^{41}$ is the mirror image transitions. *There is no evidence for any systematic trend with increasing atomic number.* Presumably the ground state wave functions of parent and daughter nuclei are practically identical from one end of the group to the other. The next group contains allowed singlet-triplet transitions. These two groups contain all transitions taking place within a supermultiplet (allowed and favored in Wigner's terminology). The third group is really a collection of diverse types. $_4Be^{10}$ has already been discussed in connection with the direct transition from $_{11}Na^{22}(g)$ to $_{10}Ne^{22}(g)$. The $_6C^{14}\rightarrow{}_7N^{14}$ transition is empirically 2nd forbidden although the spin changes by only one unit. Dr. H. Primakoff and one of the writers (E. F.) suggest the following explanation: the magnetic moment of $_7N^{14}$ requires a ground state containing 90 percent more or less of $^3D_1$ configurations. Since, however, an accurate calculation of the moment is not possible (because of relativistic effects) the ground state may be almost entirely $^3D_1$. The transition $^2S_0\rightarrow{}^3D_1$ is of course doubly forbidden even though the spin changes by only one unit. A small admixture of $^3S_1$ may dominate the beta-decay process and produce a close approximation to the allowed shape in the beta spectrum.[30] The remaining transition $_{15}P^{30}\rightarrow{}_{14}Si^{30}$ should be included in Wigner's allowed (unfavored) group. Here the transition is perhaps of the type $^3D_1+{}^3S_1\rightarrow{}^1S_0$ with $^3D_1$ predominant in the ground state of the parent nucleus, but not to the same extent as in $_7N^{14}(g)$.

The allowed (unfavored) group includes twenty-five examples ranging from $_8O^{19}$ to $_{59}Pr^{140}$. *Here also there is no evidence for a trend toward larger ft values with increasing atomic number.* All transitions in this group have a relatively small *ft* value ($\sim10^5$). In a few cases ($_{27}Co^{61}$, $_{40}Zr^{89}$, $_{46}Pd^{112}$, $_{47}Ag^{106}$, $_{49}In^{114}$, and $_{50}Sn^{121}$) the ground state configurations are uncertain because of crossovers and the classification is determined by the *ft* value and other relevant facts (isomerism, known spins and moments, etc.)

The absence of a trend in the allowed (unfavored) group was first noticed by Itoh.[31] A more extensive and precise analysis by Feather and Richardson[26] reveals a slight trend which might pass unnoticed in a table of *ft* values.

V. EIGENVALUE CALCULATIONS

Two calculations have been made to illustrate the behavior of single particle energy levels in a

[30] L. Feldman and C. S. Wu, Phys. Rev. **75**, 1286 (1949).

[31] Junkichi Itoh, Proc. Phys.-Math. Soc. Japan **22**, 531 (1940).

potential well containing a central elevation or depression. Figure 2 shows the levels in a well of infinite depth with the central half of the well occupied by an elevation ($D'>0$) or depression ($D'<0$). Another extreme situation appears in Fig. 3. Here the well depth is finite, but the central half is occupied by an elevation of infinite height. These extreme models reproduce the empirically established upward displacement of the $2s$, $3p$, and $4d$ group of levels, but fail to account for the cross-overs within the displaced group. Refinements in the shape of the potential are not likely to alter this situation.

The source of the discrepancy may be traced to the implicit incidence of configuration interaction on all the deductions and correlations of the shell model. Thus the levels traced in Fig. 1 are not true single particle levels such as might be generated by a potential function, but include the influence of all configurations which couple with the low configurations to produce the actual low states of nuclear systems. It is even plausible that the complete inversion of the $2s$, $3p$, and $4d$ levels occurring after the completion of the $5g$ shell simply expresses accurately the increasing importance of configuration interaction with increasing angular momentum. Studies of level density, based on the free particle model, show that the partial density of levels with a given angular momentum is a monatonic increasing function of angular momentum in intermediate and heavy nuclei.[32,33] Furthermore, in light nuclei, the maximum partial density occurs at $l=2$ (D states).[7] Configuration interaction should therefore, on the average, tend to depress states in the inverse order of their angular momentum (limited to $I \leqq 3$ in light nuclei).

Two relevant examples are provided by the ground states of ${}_5B^{10}$ and ${}_{11}Na^{22}$; in each Hartree type calculations place ${}^3S_1$ below ${}^3D_{1,2,3}$ and configuration interaction must be evoked to account for the observed ${}^3D_3$ ground states. Again configuration interaction is perhaps helpful in making the ground state of ${}_7N^{14}$ predominantly ${}^3D_1$.

Note added in proof.—The spin and parity assignments in the 89 and 91 series of isobars have been confirmed by recent measurements of beta-decay distribution functions (references in letter by Shull and Feenberg in this issue). These measurements support the completion of a p shell at $Z=40$ and the beginning of a d shell at $N=51$. The occurrence of long-lived isomeric states in six odd Tellurium isotopes (R. D. Hill, Washington meeting of the Physical Society, April 1949) fits neatly into the qualitative structure of Fig. 1.

[32] H. A. Bethe, Phys. Rev. **48**, 367 (1937).

[33] John Bardeen, Phys. Rev. **51**, 799 (1937).