### VII. CONCLUSIONS

The observation of distinct groups of recoil ions coincident with electron groups of much lower momentum seems explainable only on the hypothesis that a neutrino takes away the missing momentum in one package. Separate measurement of the missing energy shows it to have the correct magnitude required by the neutrino momentum. One cannot prove, without actually detecting the neutrino later, that it actually carries off the missing energy in the same direction as the missing momentum, but one is hard put to imagine a more likely place for the energy to be located.

Even without applying corrections for surface scattering of the recoils, a backward neutrino is convincingly rejected.

A summary of inferences about the form of the neutrino-electron interaction, based on the data of Fig. 10, is shown in Table I.

If the P 32 transition is second forbidden, the extreme forwardness of the neutrino with respect to the electron required by the angular momentum change, makes it unlikely that observations of the accuracy reported here could distinguish the original correlation due to the basic neutrino-electron interaction.

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# Polarization Effects of Vector-Mesons

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Adopting a vector-meson model for the $\pi$-meson we study (I) the polarization of $\pi$-mesons produced in nucleon-nucleon-collisions; (II) the angular distribution of $\mu$-mesons resulting from $\pi$-$\mu$-disintegrations, as depending on the polarization of the $\pi$-mesons; (III) the effect of electromagnetic fields on this polarization, in particular; (IV) the depolarization caused by the electric fields in matter. The theoretical results are encouraging for an experimental investigation.

### INTRODUCTION

In a previous paper[1] it was pointed out that if $\pi$-mesons have spin 1 it is very likely that polarized $\pi$-meson beams can be produced, and that various observable effects might result from this polarization. In order to substantiate these expectations, we want to present some theoretical estimates of certain polarization effects for vector-mesons. The choice of the vector-meson model may be justified by recalling that the $\pi$-mesons show a strong resemblance to the Yukawa mesons of the field theory of nuclear forces, indicating integer spin. In particular the fact that negative $\pi$-mesons captured by nuclei frequently produce large stars favors the assumption that the interaction of these mesons with nucleons is of the Yukawa type.[2] On the other hand, there is hardly any evidence so far to distinguish between spin 0 and spin 1. To decide this question, the most direct approach would be to prove or to disprove experimentally the existence of polarization effects.

[1] G. Wentzel, Helv. Phys. Acta 22 (1949), in press.
[2] J. A. Wheeler and J. Tiomno, Phys. Rev. 75, 1306 (1949).

### I. MESON PRODUCTION

First we want to study the polarization of vector-mesons produced by nucleon-nucleon-collisions. Let nucleons of momentum $\mathbf{p}_0$ impinge on a target; the nucleons embedded in the target shall be described in terms of a given (nuclear) potential field, which seems a reasonable approximation. The initial state of the system is represented by a superposition of plane waves (Dirac eigenfunctions of free spin $\frac{1}{2}$ particles, normalized):

$$\phi_{p_0\lambda_0}(\mathbf{r},\sigma)=u_{p_0\lambda_0}(\mathbf{r},\sigma)+\sum_{p'\lambda'}u_{p'\lambda'}(\mathbf{r},\sigma)(p'\lambda'|R|p_0\lambda_0).$$

$R$, or rather $S=1+R$, is the scattering matrix corresponding to the target potential assumed. The suffix $\lambda$ may be used to distinguish the charge states (proton, neutron) as well as the spin states of the nucleon. The transition to be studied, involving meson emission, is characterized by the matrix element

$$\phi_{p\lambda}^* H \phi_{p_0\lambda_0}, \quad H = g_l H_l + g_t H_t,$$

where $H_l$ and $H_t$ stand for the well-known interaction operators of the conventional vector-meson theories. The suffixes $l$ and $t$ refer to the fact that for nucleons at rest, i.e., in the "static" approximation, $H_l$ and $H_t$ describe the nuclear coupling of longitudinal and transverse mesons, respectively.[3]

We use the following notations: $\hbar = c = 1$; $M$ = nucleon mass; $\boldsymbol{\alpha}$ and $\beta$ = Dirac matrices of the nucleon, $\boldsymbol{\sigma}$ is its spin matrix ($\sigma_x = i\alpha_y\alpha_z$); $\tau$ = isotopic spin operator, describing transitions from the proton to the neutron state, or *vice versa*; $m$ = meson mass, $\mathbf{k}$ and $\omega_k = (m^2+k^2)^{\frac{1}{2}}$ its momentum and energy; $\mathbf{a}_k^*$ = meson creation operator (the $x, y, z$-components of this vector are the creation operators for mesons polarized parallel to the $x, y, z$ axis, respectively). Then

$$H_l = \exp(-i\mathbf{k}\cdot\mathbf{r})\mathbf{a}_k^* \cdot \{\boldsymbol{\alpha} - \mathbf{k}/m + (\omega_k/m - 1)((\boldsymbol{\alpha}\cdot\mathbf{k})\mathbf{k}/k^2)\}(m/\omega_k)^{\frac{1}{2}}\tau, \quad (1)$$

$$H_t = \exp(-i\mathbf{k}\cdot\mathbf{r})\mathbf{a}_k^* \cdot \{\boldsymbol{\alpha} + i[\mathbf{k}/\omega_k \times \boldsymbol{\sigma}] + (m/\omega_k - 1)((\boldsymbol{\alpha}\cdot\mathbf{k})\mathbf{k}/k^2)\}\beta i(\omega_k/m)^{\frac{1}{2}}\tau. \quad (2)$$

These expressions are to be inserted in

$$\phi_{p\lambda}^* H \phi_{p_0\lambda_0} = \sum_{p'\lambda'} (p\lambda|R^*|p'\lambda')(p'\lambda'|H|p_0\lambda_0) + \sum_{p''\lambda''} (p\lambda|H|p''\lambda'')(p''\lambda''|R|p_0\lambda_0) + \sum_{p'\lambda'p''\lambda''} (p\lambda|R^*|p'\lambda')(p'\lambda'|H|p''\lambda'') \times (p''\lambda''|R|p_0\lambda_0). \quad (3)$$

Here, the term $(p\lambda|H|p_0\lambda_0)$ has been omitted because the simple meson emission without scattering is forbidden by energy-momentum conservation. In the Born approximation, where the last term in (3) (as being quadratic in $R$) is neglected, the unitarity of the $S$-matrix requires $R^* = -R$.[4]

We want to show that in the particular case that the energy of the impinging nucleon, $E_0 = (M^2 + p_0^2)^{\frac{1}{2}}$, is just slightly above the threshold energy $M+m$, the polarization of the emitted mesons can be large, even almost complete, according to the Born approximation. Indeed, if $E_0$ approaches $M+m$, the velocities of the meson and of the outgoing nucleon ($k/\omega_k$ and $p/E$) become very small compared with $v_0 = p_0/E_0$. Therefore, in $(p'\lambda'|H|p_0\lambda_0)$, where $\mathbf{p}' = \mathbf{p}_0 - \mathbf{k}$, $\mathbf{k}/m$ can be neglected against $\boldsymbol{\alpha}$, so that the curly brackets in (1) and (2) reduce to $\boldsymbol{\alpha}$. Indeed, as is easily derived by a Lorentz-transformation:

$$(p'\lambda'|\exp(-i\mathbf{k}\cdot\mathbf{r})\boldsymbol{\alpha}\tau|p_0\lambda_0) \approx (p_0\lambda'|\boldsymbol{\alpha}\tau|p_0\lambda_0) = \mathbf{v}_0(\lambda'|\tau|\lambda_0),$$

$$(p'\lambda'|\exp(-i\mathbf{k}\cdot\mathbf{r})\boldsymbol{\alpha}\beta i\tau|p_0\lambda_0) \approx (p_0\lambda'|\boldsymbol{\alpha}\beta i\tau|p_0\lambda_0) = \mathbf{v}_0 \times (\lambda'|\boldsymbol{\sigma}\tau|\lambda_0),$$

where the matrices $(\lambda'|\cdots|\lambda_0)$ refer to a nucleon at rest. Hence, neglecting the other small terms:

$$(p'\lambda'|H_l|p_0\lambda_0) \approx (\mathbf{a}_k^*\cdot\mathbf{v}_0)(\lambda'|\tau|\lambda_0),$$

$$(p'\lambda'|H_t|p_0\lambda_0) \approx [\mathbf{a}_k^* \times \mathbf{v}_0]\cdot(\lambda'|\boldsymbol{\sigma}\tau|\lambda_0).$$

On the other hand, in the second term in (3), because $\mathbf{p}$ and $\mathbf{p}'' = \mathbf{p}+\mathbf{k}$ are small momenta,

$$|(p\lambda|H|p''\lambda'')| \ll |(p'\lambda'|H|p_0\lambda_0)|$$

while

$$|(p''\lambda''|R|p_0\lambda_0)| \approx |(p\lambda|R^*|p'\lambda')|$$

since $\mathbf{p}_0$ and $\mathbf{p}''$ are almost the same momenta as $\mathbf{p}'$ and $\mathbf{p}$; moreover, $\mathbf{p}'' - \mathbf{p}_0 = \mathbf{p} - \mathbf{p}' = \mathbf{p} + \mathbf{k} - \mathbf{p}_0$. Therefore, in the Born approximation, the first term in (3) is by far the largest, and the matrix element reduces to

$$g_l(\mathbf{a}_k^*\cdot\mathbf{v}_0)\textstyle\sum_{\lambda'} (p\lambda|R^*|p'\lambda')(\lambda'|\tau|\lambda_0) + g_t[\mathbf{a}_k^*\times\mathbf{v}_0]\cdot\textstyle\sum_{\lambda'} (p\lambda|R^*|p'\lambda')(\lambda'|\boldsymbol{\sigma}\tau|\lambda_0). \quad (4)$$

Now, consider the two simple cases of "pure" coupling:

(a) $g_l \neq 0$ and $g_t = 0$. The matrix element is proportional to $(\mathbf{a}_k^*\cdot\mathbf{v}_0)$, indicating that mesons with polarization perpendicular to $\mathbf{v}_0$ are not emitted.

(b) $g_l = 0$, $g_t \neq 0$. In this case the component of $\mathbf{a}_k^*$ parallel to $\mathbf{v}_0$ gives no contribution, i.e., polarization in this direction will not occur.

In these two cases we therefore expect complete polarization, parallel or perpendicular to the direction of the producing nucleon beam, respectively. In the case of a mixed interaction (both $g_l$ and $g_t \neq 0$) the degree of polarization will of course depend on the coupling ratio $|g_l/g_t|$. The polarization will also be somewhat lessened if the last term in (3) (quadratic in $R$) is appreciable. Finally it should be noted that we have disregarded intermediate states involving nucleon pair creation, but their contribution will be small since $Mv_0^2/2 \approx m \ll M$.

Although the assumption of a fixed nuclear potential in the target certainly is an oversimplification, one may nevertheless conclude that mesons of spin 1, produced in nucleon-nucleon-collisions, can be expected to be polarized to a considerable extent, as long as the velocities of the meson and

[3] See G. Wentzel, *Quantentheorie der Wellenfelder* (Vienna 1943); English translation (New York 1949), §14.

[4] Generally: $S^*S = (1+R^*)(1+R) = 1$; $R^* + R + R^*R = 0$.

the nucleon in the final state remain small as compared with the initial nucleon velocity $v_0$.

## II. $\pi$-$\mu$-DISINTEGRATION

Turning to the $\pi$-$\mu$-decay, we consider the disintegration of a vector-meson into two particles of spin $\frac{1}{2}$, with rest-masses $\mu$ and $\nu$, where $\nu$ may be zero. The theory of this process is well known from older $\beta$-theories since the $\mu$-meson, except for its larger mass, behaves like an electron. The matrix element of the transition may be written

$$u_{\mu p\lambda}{}^* K u_{\nu p_0\lambda_0}, \tag{5}$$

where $u_\nu$ refers to a negative energy anti-neutrino wave function. The operator $K$ may again be a linear combination of two terms similar to $H_l(1)$ and $H_t(2)$. In the rest system of the $\pi$-meson ($\mathbf{k}=0$) they reduce to

$$K_l=(\mathbf{a}\cdot\boldsymbol{\alpha}), \tag{6}$$

$$K_t=(\mathbf{a}\cdot\boldsymbol{\alpha})\beta i. \tag{7}$$

If the polarization of the initial $\pi$-meson is linear, **a** may be replaced by the unit vector in the direction of polarization; otherwise appropriate averages of the squared matrix elements must be taken.

Assuming a pure coupling of the type (6) or (7), one finds by standard methods for the probability that a $\mu$-meson is emitted into the solid angle $d\Omega$, in the rest system of the $\pi$-meson:

$$\text{const.}\,d\Omega\{1+(1-v_\mu{}^2)^{\frac{1}{2}}(1-v_\nu{}^2)^{\frac{1}{2}}\mp v_\mu v_\nu\cos 2\theta\}, \tag{8}$$

where the upper and lower signs refer to the coupling types (6) and (7), respectively; $v_\mu$ and $v_\nu$ are the velocities (magnitudes, in units $c$) of the $\mu$-meson and the "neutrino," as determined by energy and momentum conservation; $\theta$ denotes the angle which the direction of emission ($\mathbf{v}_\mu$ or $\mathbf{v}_\nu$) forms with the polarization vector **a**. If the rest mass of the neutrino vanishes ($v_\nu=1$), (8) becomes

$$\text{const.}\,d\Omega\{1\mp v_\mu\cos 2\theta\},\quad v_\mu=(m^2-\mu^2)/(m^2+\mu^2). \tag{9}$$

Adopting the experimental value $m/\mu=1.32$, we have $v_\mu=0.27$, and the anisotropy ratio would be $(1+v_\mu)/(1-v_\mu)=1.74$. Although this optimum ratio can only be reached in the case of a pure coupling ($K_l$ or $K_t$) and linear polarization, the effect seems large enough to be detectable even under less favorable conditions.

## III. EFFECTS OF ELECTROMAGNETIC FIELDS

In order to judge the observational possibilities, it is necessary to investigate how the polarization of the $\pi$-meson may be altered during its lifetime. Here, nuclear effects can be disregarded as unlikely, but electromagnetic forces, caused by external fields and by the matter traversed, may be important.

In this problem, since creation and annihilation processes are of no concern, it is convenient to describe the $\pi$-mesons by a non-quantitized wave field $\psi_\nu$, of four-vector character. In a given electromagnetic field

$$F_{\nu\mu}=\partial A_\mu/\partial x_\nu-\partial A_\nu/\partial x_\mu$$

($x_4=it$) the $\psi$-wave propagates according to the field equations[5]

$$\partial_\mu(\partial_\mu\psi_\nu-\partial_\nu\psi_\mu)-i\gamma eF_{\nu\mu}\psi_\mu=m^2\psi_\nu \quad [\partial_\mu=\partial/\partial x_\mu-ieA_\mu], \tag{10}$$

where $\gamma$ is an arbitrary numerical factor accounting for an anomalous magnetic moment of the $\pi$-meson. The four components $\psi_\nu$ are not independent, for it follows from (10) that

$$m^2\partial_\nu\psi_\nu=i(1-\gamma)eF_{\nu\mu}\partial_\nu\psi_\mu-i\gamma e(\partial F_{\nu\mu}/\partial x_\nu)\psi_\mu. \tag{11}$$

Here, the identity

$$\partial_\nu\partial_\mu-\partial_\mu\partial_\nu=-ieF_{\nu\mu}$$

has been used which may also serve to rewrite (10) in the form

$$\partial_\mu{}^2\psi_\nu-\partial_\nu(\partial_\mu\psi_\mu)-i(1+\gamma)eF_{\nu\mu}\psi_\mu=m^2\psi_\nu, \tag{12}$$

where $(\partial_\mu\psi_\mu)$ can now be eliminated by inserting (11).

In order to study the change of polarization caused by the field $F$, let us express $\psi_\nu$, according to

$$\psi_\nu=\varphi a_\nu,$$

by a scalar field $\varphi$ obeying the field equation

$$\partial_\mu{}^2\varphi=m^2\varphi.$$

$\varphi$ describes the motion of a spinless particle of mass $m$ under the same electromagnetic forces. Assuming that $a_\nu$ is a slowly varying field (i.e., neglecting second derivatives) we obtain from (12)

$$2(\partial_\mu\varphi)\partial a_\nu/\partial x_\mu=i(1+\gamma)eF_{\nu\mu}\psi_\mu+\partial_\nu(\partial_\mu\psi_\mu).$$

In the last term we want to neglect the derivatives of the $a_\mu$ and of the $F_{\nu\mu}$, supposing that both fields vary slowly in regions with dimensions $m^{-1}$; then

$$2(\partial_\mu\varphi)\partial a_\nu/\partial x_\mu =ie\{(1+\gamma)\varphi F_{\nu\mu}+(1-\gamma)m^{-2}(\partial_\nu\partial_\lambda\varphi)F_{\lambda\mu}\}a_\mu. \tag{13}$$

For the discussion of a small space region, we can choose a frame of reference in which the particle is almost at rest and, gauging away the potentials $A_\mu$ in this region, we will have

$$\varphi\approx e^{-imt},\quad \partial_\mu\varphi\approx -m\varphi\delta_{4\mu}. \tag{14}$$

[5] See H. C. Corben and J. Schwinger, Phys. Rev. 58, 953 (1940); G. Wentzel, reference 3, §13.

Then from (13), $\nu=4$, and from (11):

$$a_4 \approx -i\gamma e m^{-2} F_{4k} a_k + m^{-1} \partial a_k / \partial x_k.$$

If we exclude very strong electric fields, we may neglect in (13) $F_{k4}a_4$ against $F_{kl}a_l$ ($k, l=1, 2, 3$). In this approximation, (13) reduces to

$$\partial \mathbf{a}/\partial t = -(1+\gamma)(e/2m)[\mathbf{H}\times\mathbf{a}], \qquad (15)$$

indicating that the polarization vector $\mathbf{a}=(a_1, a_2, a_3)$ precesses around the direction of the magnetic field $\mathbf{H}$ with the angular velocity $(1+\gamma)e/2m$, corresponding to a Landé factor $g=1+\gamma$. In the special case of a circular polarization (e.g., $a_2=ia_1$, $a_3=0$), this provides for the well-known precession of the spin vector $i\mathbf{a}\times\mathbf{a}^*$; indeed, from (15), it follows that

$$\partial[\mathbf{a}\times\mathbf{a}^*]/\partial t = -(1+\gamma)(e/2m)[\mathbf{H}\times[\mathbf{a}\times\mathbf{a}^*]].$$

But (15) holds also if the polarization is, for instance, a linear one ($\mathbf{a}\times\mathbf{a}^*=0$).

Of course, the above equations can also be interpreted in terms of particles traveling on certain trajectories and carrying along a vector $a_\nu$ which may be considered as a function of the proper time $s$ of the particle. In (13), then, $(\partial_\mu\varphi)\partial a_\nu/\partial x_\mu$ goes over into $(im\varphi)da_\nu/ds$.

The first obvious application of Eq. (13) or (15) will be to $\pi$-mesons traveling in a magnetic field (as, for instance, mesons produced in a target located in the field of a cyclotron). The polarization vector which initially, according to (4), may be parallel or perpendicular to the primary nucleon beam, will change its direction along the trajectory of the meson, the total angle of precession being proportional to the time integral of the magnetic field and to the Landé factor $g=(1+\gamma)$. If $\pi$-$\mu$-disintegrations can be observed at various points of the trajectory, the anisotropic distribution of the $\mu$-meson velocities should reveal this precession, and the Landé factor should be measurable.

If the disintegration is observed after the $\pi$-meson has passed through dense matter, it will be important to know how the polarization is affected by the electric fields in the material. For fast particles ($v>1/137$) this question can be settled by applying the Born approximation.[6] However, we are interested in low velocities also because positive $\pi$-mesons may spend the major part of their lifetime ($\sim 10^{-8}$ sec.) at thermal velocities before they disintegrate. We content ourselves with an upper limit estimate sufficient to show that the depolarization of positive mesons is weak at non-relativistic velocities.

[6] O. Laporte, Phys. Rev. 54, 905 (1938); see also G. Wentzel, reference 3, §13.

## IV. DEPOLARIZATION

Consider a $\pi^+$-meson as a classical particle passing through the electric field $\mathbf{E}$ of an atom which may be assumed to be a central field. We choose a rectangular coordinate system with the $z$ axis perpendicular to the plane of motion and the $x$ axis parallel to $\mathbf{v}_2-\mathbf{v}_1$, $\mathbf{v}_1$ and $\mathbf{v}_2$ denoting the initial and final velocities of the particle. The polarization vector is, in the rest system, subject to an effective magnetic field $\mathbf{E}\times\mathbf{v}$ which has the $z$ direction; as to its magnitude, a simple geometrical argument shows that for a particle of positive charge $e$ (repulsive force $e\mathbf{E}$):

$$|\mathbf{E}\times\mathbf{v}| \leqq E_x v \leqq E_x v_1. \qquad (16)$$

The polarization vector $\mathbf{a}$ precesses around the $z$ axis, and its total angle of deflection is

$$\delta = (ge/2m)(\sin\alpha)\int dt |\mathbf{E}\times\mathbf{v}|$$

($\alpha$ = angle of $\mathbf{a}$ with the $z$ axis). Inserting (16), and noting that

$$e\int dt E_x = m(\mathbf{v}_2-\mathbf{v}_1)_x = m|\mathbf{v}_2-\mathbf{v}_1|,$$

we obtain

$$\delta \leqq (g/2)(\sin\alpha) v_1 |\mathbf{v}_2-\mathbf{v}_1|,$$

or introducing the scattering angle $\vartheta$ by

$$|\mathbf{v}_2-\mathbf{v}_1| = 2v_1 \sin\vartheta/2:$$

$$\delta \leqq g(\sin\alpha) v_1^2 \sin\vartheta/2. \qquad (17)$$

On account of the random directional distribution, the probable deflection $\Delta$ of the polarization vector after many atomic collisions is given by

$$\Delta^2 = \sum \delta^2.$$

In a time interval $dt$, $\Delta^2$ changes by

$$d(\Delta^2) = Nv dt \int_0^{2\pi} d\alpha \int_0^\infty d\rho \rho \delta^2, \qquad (18)$$

where $N$ denotes the number of atoms per unit volume. Inserting (17), the scattering angle $\vartheta$ is to be regarded as a function of the impact parameter $\rho$. If $v_1$ is still large enough that the particle can penetrate deeply into the atoms, we may use the Rutherford formula

$$\tan\vartheta/2 = \rho_1/\rho, \quad \rho_1 = Ze^2/mv_1^2,$$

where $Z$ = effective nuclear charge, and integrate in

(18) up to the atomic radius $R$:

$$d(\Delta^2) < Nvdt \cdot g^2(\pi/2)(Ze^2/m)^2 \log(1+R^2/\rho_1^2). \quad (19)^7$$

While traversing the matter, the meson is slowed down according to the well-known law

$$-dv = Nvdt \cdot (4\pi Ze^4/m_e mv^3) \log q, \quad (20)$$

where $m_e$ denotes the electronic mass ($m$ always means the mass of the $\pi$-meson); $\log q \sim 1$ for meson velocities substantially larger than the velocity of valence electrons. In this velocity range

$$d(\Delta^2) \lesssim |dv| v^3 Zm_e/m. \quad (21)$$

Integrated over a time interval during which $v$ decreases from an initial value $v_i$ to a lower value:

$$\Delta^2 \lesssim v_i^4 Zm_e/m. \quad (22)$$

This upper limit is certainly very small for non-relativistic initial velocities ($v_i \ll 1$).

For the lowest velocities where (20) no longer holds, it is sufficient to remember that according to (17)

$$\delta^2 \lesssim v^4$$

for positive mesons. Owing to this very small value, $\Delta^2$ can hardly grow to a value larger than (22), even during a lifetime as large as $10^{-8}$ sec., spent in dense matter.

This estimate seems to prove that for positive vector-mesons of at most some Mev energy, no appreciable depolarization is to be expected.

Since, according to Section I, slow $\pi$-mesons are most likely to be strongly polarized, one may hope that the photographic registration of $\pi$-$\mu$-disintegrations will reveal the polarization effects as discussed in Section II. In this connection it should be noted that if the meson at the end of its range in the photographic emulsion is still subject to a magnetic field, the polarization vector will go on precessing until the disintegration occurs, and since the individual lifetime may be shorter or longer, with an uncertainty of about $10^{-8}$ sec., the polarization effect will be blurred out in a too strong magnetic field. In order to keep the polarization well defined, the magnetic field strength should be kept below about 100 gauss, in the region of observation.

*Note added in proof.*—As Dr. I. I. Rabi pointed out, there is the possibility that the positive meson, before decaying, captures an electron from the surrounding matter such that the electron is bound in a hydrogen-like orbit; then the electron's magnetic field might cause a considerable depolarization of the meson, leading to a weaker anisotropy in the angular distribution of the $\mu$-mesons. For instance, if the electron is bound in an $s$-state (external magnetic fields being absent or weak) the distribution will be, instead of (9),

$$\text{const. } d\Omega\{1 \mp v_\mu[(6\cos^2\theta - 5)/9 + (1+\epsilon^2\tau^2)^{-1}4(3\cos^2\theta - 1)/9]\},$$

where $\epsilon$ is the hyperfine structure splitting ($\Delta E/\hbar$) of the $s$ level due to the magnetic moment of the $\pi$-meson, and $\tau$ the mean lifetime of the hydrogen-like state. For the ground state: $\epsilon \sim 10^{10}$ sec.$^{-1}$ (with $g \sim 1$), so that with a lifetime $\tau \sim 10^{-8}$ sec. the second term in the square bracket would be negligible, and the anisotropy would be reduced to about one-third, supposing that the capture process is a frequent occurrence. But actually its probability may be far below unity, and also the lifetime $\tau$ may be considerably shortened by chemical forces acting on the "pseudohydrogen atom" and tending to restore a diamagnetic state of the electrons. In either case, the larger anisotropy (9) is to be expected.

[7] As to the order of magnitude, (19) agrees with the cruder estimate (suggested to me by Dr. E. Teller):

$$\delta \sim (e/m)(Zev/\rho^2)(\rho/v),$$

where the last factor $(\rho/v)$ is the collision time; hence

$$\int d\rho\rho\delta^2 \sim (Ze^2/m)^2 \int d\rho/\rho \sim (Ze^2/m)^2.$$

This, however, is an overestimate in the case of slow positive particles ($mv^2 < Ze^2/R$, see below). On the other hand, for higher velocities ($v > Z/137$), the classical particle picture is no longer reliable, but the Born approximation confirms (19) as an upper limit estimate provided that $v \ll 1$. The factor $g^2$ in (19) appears to be replaced by $\gamma^2 = (g-1)^2$, indicating that for $\gamma = 0$, and $Z/137 < v \ll 1$, the depolarization is even weaker than one would expect from (19).