thus

$$e_6 = s(\sin\theta)W/4 + \gamma(1-0.3S/W) + 1.70S^2I_s^2(\sin^2\theta)/4W(1+(\mu^*(1+\mu^*)/2))^{\frac{1}{2}}.$$

This leads to a minimum energy for

$$S_{\min} = 0.3\gamma/[1.70I_s^2/2(1+(\mu^*(1+\mu^*)/2))^{\frac{1}{2}}]\sin^2\theta = 0.3\gamma/\sqrt{2}A\sin^2\theta,$$

where $A = 3.60\times10^4$ as defined in Section 5. The energy is then

$$e_6 = s(\sin\theta)W/4 + \gamma(1-0.15S_{\min}/W)$$

(neglecting the small correction to the magnetostatic energy). Values of $S_{\min}$ so calculated are given in Table I and are about 3 times the measured values.

7. Comparison of Total Energies

In Fig. 34, the three energy expressions are plotted as functions of θ. The e_1 curve is valid over the range of validity of the μ^* method; since the latter is limited by the condition that θ is small, this curve is certainly accurate to $\theta > 2°$. The e_5 approximation, however, depends on having $b \cong a/2$; the range of $b=a$ to $b=a/3$ is shown as heavy and represents an extension of e_5 somewhat beyond its range of validity. Three approximations to the steep slope patterns are shown: γ corresponds to placing $S=0$ and neglecting magnetostriction; e_6 for $S=0$ adds the effect of magnetostriction; e_6 shows the best approximation with $S=S_{\min}$. Figure 34 is seen to explain the general trend of one type of pattern to another satisfactorily, the incidence of tree patterns at $\theta \cong 0.5°$ being given correctly. The weakest feature is the e_5 curve which has not been treated accurately enough to give properly the transition from tree to steep slope patterns.

PHYSICAL REVIEW VOLUME 75, NUMBER 1 JANUARY 1, 1949

A Simple Domain Structure in an Iron Crystal Showing a Direct Correlation with the Magnetization

H. J. Williams and W. Shockley
Bell Telephone Laboratories, Murray Hill, New Jersey

(Received August 30, 1948)

A hollow rectangle cut from a single crystal of 3.8 percent silicon iron has been studied with the aid of powder patterns and flux measurements. The edges and surfaces were all cut accurately parallel to ⟨100⟩, the directions of easy magnetization. The domain pattern consists of 8 domains, four forming an inner rectangle magnetized in one direction and the others forming an oppositely magnetized outer rectangle. Changes in magnetization occur by the growth of one set of domains at the expense of the other. In the saturated condition, each leg of the rectangle is one domain about $1.5\times0.1\times0.1$ cm in size. Implications of these results in connection with Barkhausen effect are discussed, and a method of measuring the energy of the Bloch wall is proposed.

THE theory of magnetic domains has been developed over a period of years to explain the gross magnetic properties of matter in terms of the behavior of smaller regions of substantially uniform magnetization. However, except for certain artificially simplified cases such as fine stressed wires or very small particles, it has not been possible to obtain a complete picture of the domain structure in any actual specimen and to show how it explains the state of magnetization and variations thereof. The experiments described below furnish an example of correlation between domain structure and magnetization for a specimen having a dimension of the order of one centimeter.

This specimen was in the form of a hollow rectangle (or "picture frame") of 3.8 weight percent silicon iron cut from a single crystal so as to have all edges and surfaces substantially parallel to [100] or equivalent directions. It had origi-

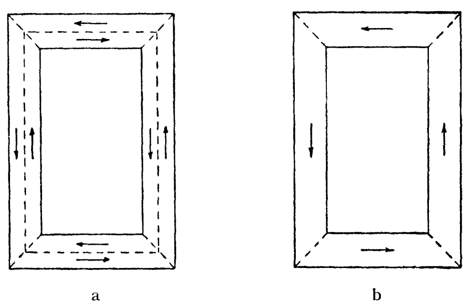


FIG. 1. Domain structure of the crystal with solid lines showing the outline of the crystal, dotted lines representing Bloch walls, and arrows indicating the directions of magnetization. (a) After applying a field; (b) after cooling from 1000°C.

nally been prepared and used for studies of magnetization along the [100] direction.[1] For these new experiments, the surfaces were carefully ground parallel to (001) planes so as to avoid the superficial "tree patterns"[2] and polished for use with the colloidal suspension used for observing domain patterns.[2,3]

The magnetization in the specimen has been determined by methods described below to consist essentially of eight domains arranged as shown in Fig. 1a. The domain walls shown by dashed lines are perpendicular to the paper and run through the specimen whose thickness is 0.074 cm. The width of the legs of the rectangle is 0.102 cm and the over-all dimensions are 1.9×1.3 cm. The pattern shown represents perfect flux closure in the sense that no magnetic poles are produced on any of the domain walls or external surfaces of the specimen. The magnetization changes by sidewise displacements of the walls so as to make the inner four domains, which represent flux in the clockwise sense, grow at the expense of the outer set, or vice versa.

Presumably the state of least energy can be obtained experimentally by heating the specimen above the Curie temperature and cooling it slowly. When this is done, it is found that the specimen is magnetized in four domains all running in one direction so that a state of saturation is produced (Fig. 1b). This domain structure would also seem to be theoretically the one of minimum energy. (It obviously has no magnetostatic or anisotropic energy and the wall energy is a minimum. There is some magnetostrictive energy; however, since the legs of the rectangle are long compared to their width, the stress will

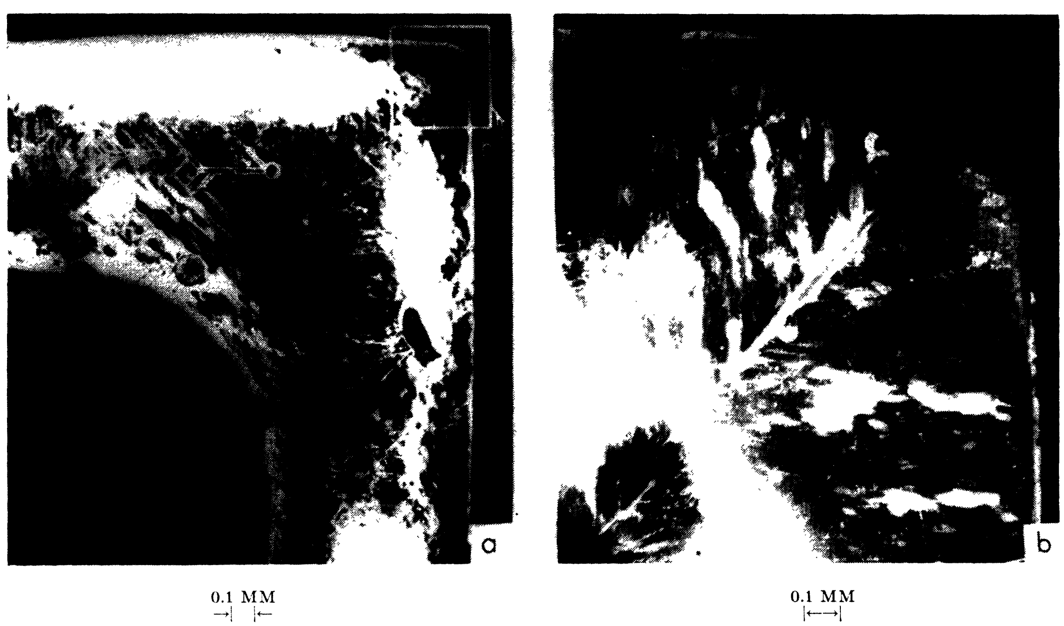


FIG. 2. (a) Pattern obtained on a corner of the crystal after cooling from 1000°C. (b) Small region shown in (a) obtained with greater magnification.

[1] H. J. Williams, Phys. Rev. **52**, 747–751 (1937).
[2] H. J. Williams, Phys. Rev. **71**, 646–647 (1947).
[3] H. J. Williams, R. M. Bozorth, and W. Shockley, Phys. Rev. **75**, 155 (1949).

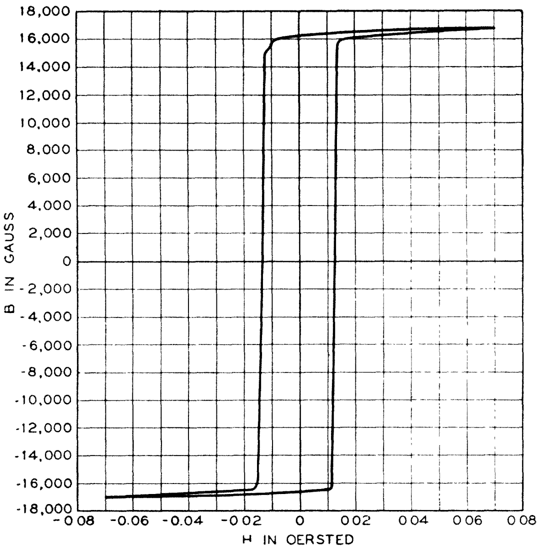


FIG. 3. Hysteresis loop traced on a Cioffi recording fluxmeter.

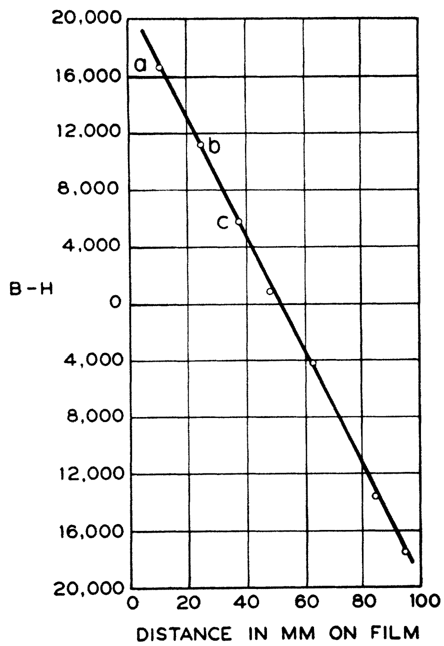
be much less than in a rectangle with a filled center. A rough estimate indicates that both the wall and magnetostrictive energy average about 1 erg/cm^3 so that any increased complexity of domain structure to reduce magnetostrictive energy would result in a net increase in total energy due to added wall energy.[4])

The magnetization pattern just discussed has been established by using the powder pattern technique[2,3] combined with measurements of flux with the Cioffi recording fluxmeter.[5] In Fig. 2, a pattern is shown as obtained after cooling from 1000°C. Similar patterns were also observed on the other corners. This pattern shows a Bloch wall bisecting the 90° angle at the corner of the specimen. Figure 2b shows a portion of the same pattern taken with a larger magnification. This pattern shows the Bloch wall and a number of elongated clusters of colloidal magnetite. In previous work[2] the direction of magnetization has been shown to be normal to these elongated clusters. The powder patterns, observed over the entire crystal, showed that the domain structure, except for a few superficial details due to crystal imperfections and irregularities of cross section, was as shown in Fig. 2. This domain arrangement has a net magnetization equal to the saturation. This feature was checked by putting primary and secondary windings around the specimen and then tracing a hysteresis loop (Fig. 3). Care was taken not to expose the specimen to magnetic fields previous to this. The results showed that the specimen had an initial magnetization, after cooling from above the Curie point of 17,000 gauss, in good agreement with the known saturation of the specimen, considering the uncertainty in exact cross section.

For intermediate states of magnetization the domain pattern of Fig. 4 was obtained. Figure 4 is a composite of three pictures of adjacent positions showing the wall at a corner and along part of a side of the crystal. The inner wall at the corner is due to excess material on the inside corner. The wall was easily traced along the specimen. Observation of the position of the wall on opposite sides of the crystal indicated that it extended straight through the specimen. According to this conclusion, the flux should vary linearly with position of the domain wall, varying from positive saturation to negative saturation, as the wall moves across the specimen. Within the

FIG. 4. Composite of patterns on three adjacent areas showing the domain structure represented in Fig. 1a.

[4] See C. Kittel, Phys. Rev. **70**, 965 (1946), or reference (3) for a more general discussion of this subject.
[5] P. P. Cioffi, Phys. Rev. **67**, 200 (1945).



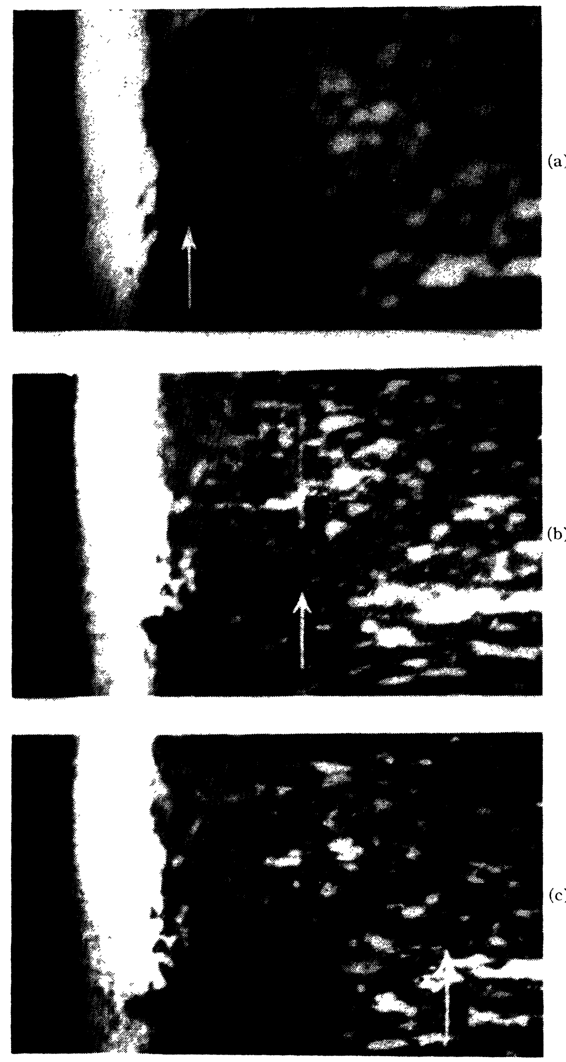


FIG. 5. Magnetization *vs.* displacement of the 180° Bloch wall, and patterns showing the Bloch wall in three different positions.

precision of measurement of the cross section, this was found to be the case. The wall was photographed at seven positions while tracing a hysteresis loop. Figure 5 shows the wall position for these photographs and the resultant plot of $B-H$ *vs.* wall position.

This experiment also throws light on the nature of the Barkhausen effect for specimens with large domains. According to early theories, the Barkhausen effect arises from the abrupt reversal of domains. In this specimen, on the contrary, a relatively steady progress of the domain boundary occurs. While this motion was being observed, however, the Barkhausen effect could be simultaneously heard over a loudspeaker and amplifier system. There are several possible mechanisms which may produce irregular or jerky motions of the Bloch wall. When the boundary moves past small holes or other slight imperfections appearing on the surface of the crystal, superficial domain structures around these regions unite with the wall, forming irregularities in it, a particular example of which is shown by the drawing in Fig. 6a. As the main wall continues to move, the walls extending from the holes to the main wall become more and more extended and finally break, suddenly forming a new structure

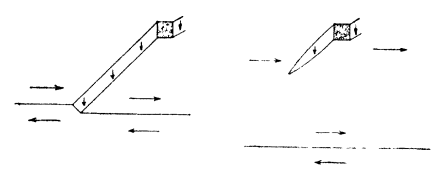


FIG. 6. (a) Drawing of the domain structure, around a square cavity, connected to the 180° Bloch wall. (b) Domain structure formed after breaking the connection between the cavity and the 180° Bloch wall.

around the hole, and the main wall straightens out (Fig. 6b). Figure 7 shows the actual patterns observed which correspond to the drawings of Fig. 6. Figure 7b is a superposition of the two patterns. The light lines show the original pattern (Fig. 7a) which was permitted to etch the surface, and the heavy lines show the final pattern corresponding to Fig. 6b. It is not possible at present to say whether the number and magnitude of Barkhausen impulses can be correlated with the behavior of these imperfections. However, it is evident that their behavior must account at least for some of the discontinuities observed in the magnetization process.

The same imperfections may possibly make important contributions to the coercive force of about 10^{-2} oersted observed for this specimen. From observations of patterns like those of Fig. 4 one estimates that there are about four such structures per cm showing on the surface. If there are other similar structures distributed uniformly in depth, there will, of course, be many more per unit area. Taking the diameter of the structure as 3×10^{-3} cm, we estimate that there will be about 10^3 per cm^2 of Bloch wall. Each of these exerts a force of about σC on the wall where $\sigma=2$ dynes/cm is the surface tension of the wall, and $C=\pi 3\times10^{-3}\doteq10^{-2}$ cm is the circumference of the contact with the wall. Thus the force on the wall due to these structures will be about $10^3\times2\times10^{-2}=20$ dynes/cm^2. When the wall is moved, there will be a tendency for all of the structures to pull back in one direction, giving a net force of this magnitude. The force per unit area on the wall due to a magnetizing field H is $2HI_s\doteq3000H$ for this material. Equating this force to the restoring force due to the structures leads to $H=0.006$ oersted, whereas the observed coercive force is about 0.012 oersted. At the present stage of these observations this agreement in order of magnitude may be fortuitous, and at most, all one can say is that the effect of surface tension on walls about imperfections may account for coercive force, and the abrupt changes when the walls break away may account for the Barkhausen effect.

Another explanation is that stresses produced during the formation of the crystal tend to hold certain small areas of the wall in position by interaction with the magnetostriction. This idea has previously been discussed by Kondorsky;[6] however, in that theory very considerable curvatures of the wall are assumed. In Fig. 4 it is seen that the wall is very nearly straight. This difference is due to the fact that the poles produced on a curved wall exert large forces on the wall and tend to make it plane. For this reason, even though the same forces may act as in the Kondorsky theory, the curvature of the wall will be small; however, some curvature is to be ex-

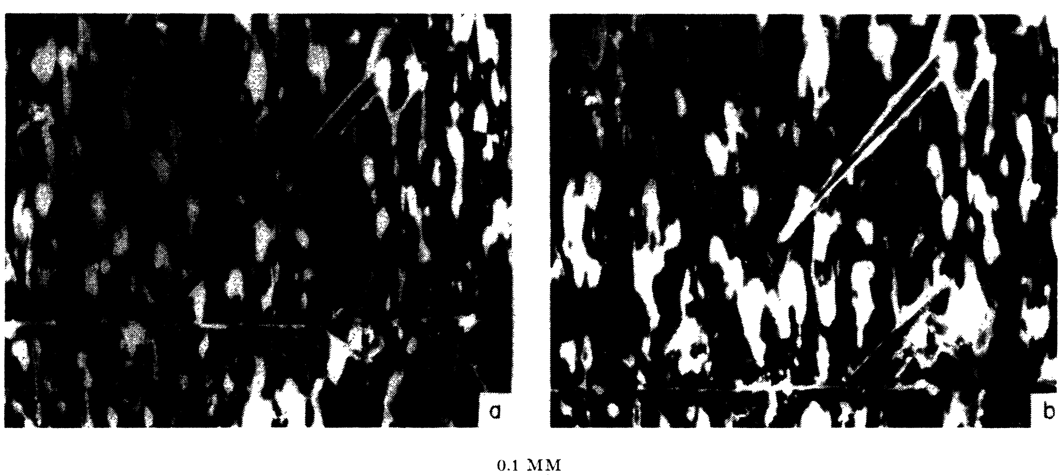


FIG. 7. Patterns corresponding to the domain structure shown in Fig. 6.

[6] E. Kondorsky, Phys. Zeit. Sowiet. 11, 597 (1937); R. Becker and W. Döring, *Ferromagnetismus* (J. Springer, Berlin, 1939), p. 205.

pected and may be detectable in more precise experiments.

The domain structure in this specimen permits an experiment which will measure directly the domain wall energy. This may be accomplished with the arrangement shown in Fig. 8. Here a current, J, is passed horizontally through the specimen from one point contact to another. This is equivalent to winding the upper half of the specimen with a primary in one direction and the lower half with a primary in the opposite direction. This will produce a magnetic field around the hollow rectangle in opposite directions in the top and bottom halves, as indicated in (b), and will distort the wall, as indicated in (c). This distortion can be detected by comparing the wall positions on the two sides, or by comparing the wall position on one side with the total flux measured on the fluxmeter. The offset W of the wall from its average position will be linear in J to a first approximation. Assuming that the wall is the same shape all around the rectangle (if it were not, large magnetostatic fields would be produced), we may equate the surface tension force per unit length of the wall in the y direction to the magnetic force tending to move it. This force is obtained from the integral of H around the rectangle which gives

$$\int Hds = (4\pi/10)Jy/T,$$

assuming the current is uniformly spread throughout the height of the specimen at the wall position. Hence the force per unit height on the wall is

$$\int 2I_sHds = (8\pi/10)I_sJy/T$$

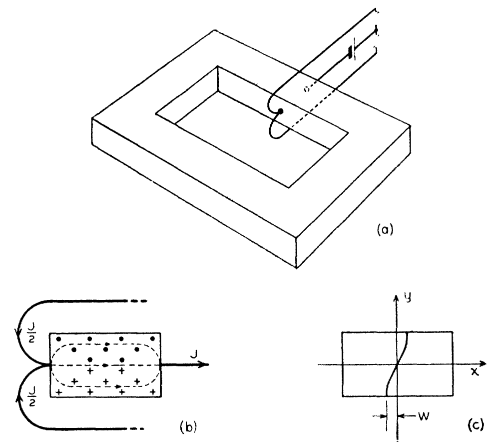


FIG. 8. Experimental arrangement proposed for a direct measurement of wall energy.

where I_s is the saturation magnetization. The force due to the surface energy of the wall is

$$L\sigma d^2x/dy^2$$

where L is the circumference around the rectangle. Equating these two forces and integrating subject to the boundary condition that $dx/dy=0$ at $y=\pm T/2$ gives

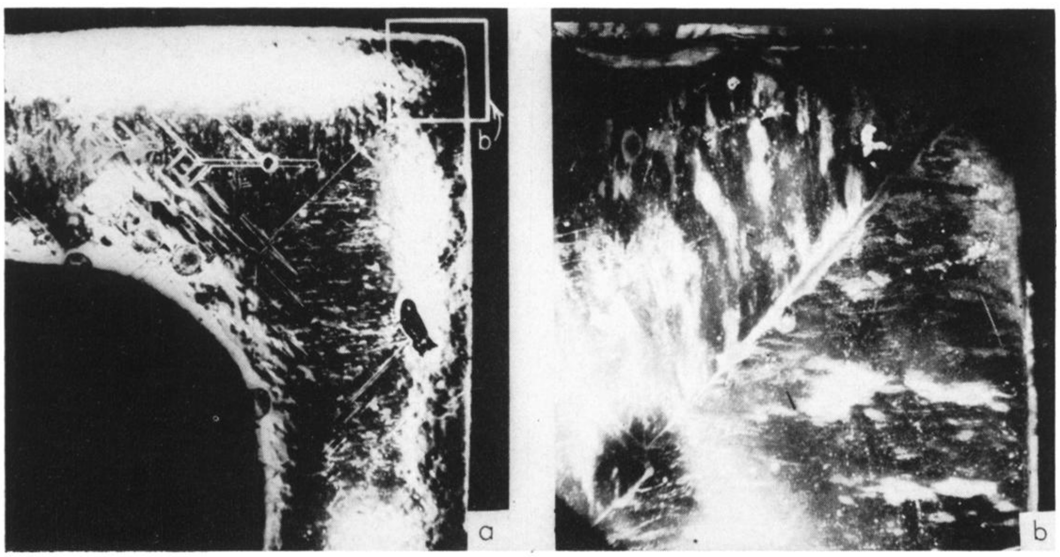
$$x = (2\pi/15)(I_sJ/L\sigma T)(y^3 - 3y(T/2)^2)$$

for the shape of the wall. The maximum value of x, shown as "W" in Fig. 8, is given by

$$W = (\pi/30)I_sJT^2/L\sigma.$$

Since I_s, J, T, and L are readily determined, this equation can be used to solve for σ as soon as W is measured. It is hoped that results of this experiment can be reported in the near future.

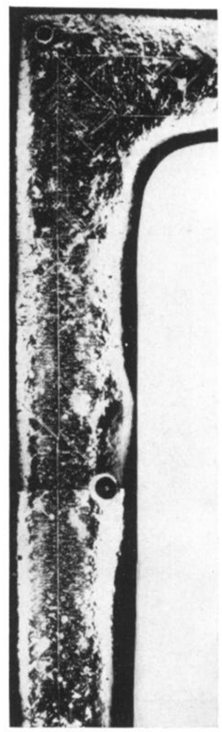
We are indebted to R. M. Bozorth and C. Kittel for suggestions and discussions, to W. L. Bond for assistance with an x-ray goniometer, and to J. G. Walker for assistance with the photography.



0.1 MM 0.1 MM

FIG. 2. (a) Pattern obtained on a corner of the crystal after cooling from 1000°C. (b) Small region shown in (a) obtained with greater magnification.

FIG. 4. Composite of patterns on three adjacent areas showing the domain structure represented in Fig. 1a.



|← 1 MM →|

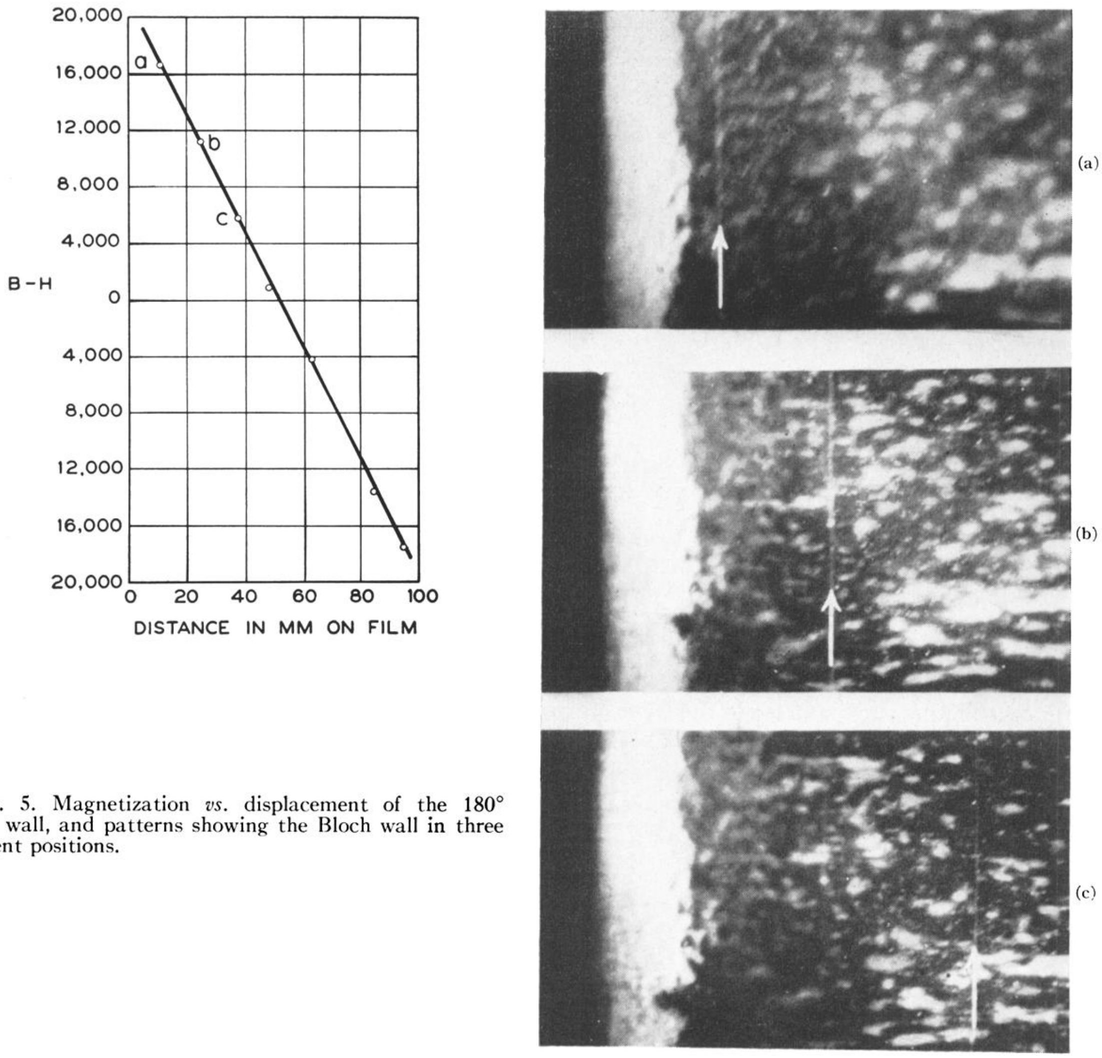


FIG. 5. Magnetization *vs.* displacement of the 180° Bloch wall, and patterns showing the Bloch wall in three different positions.

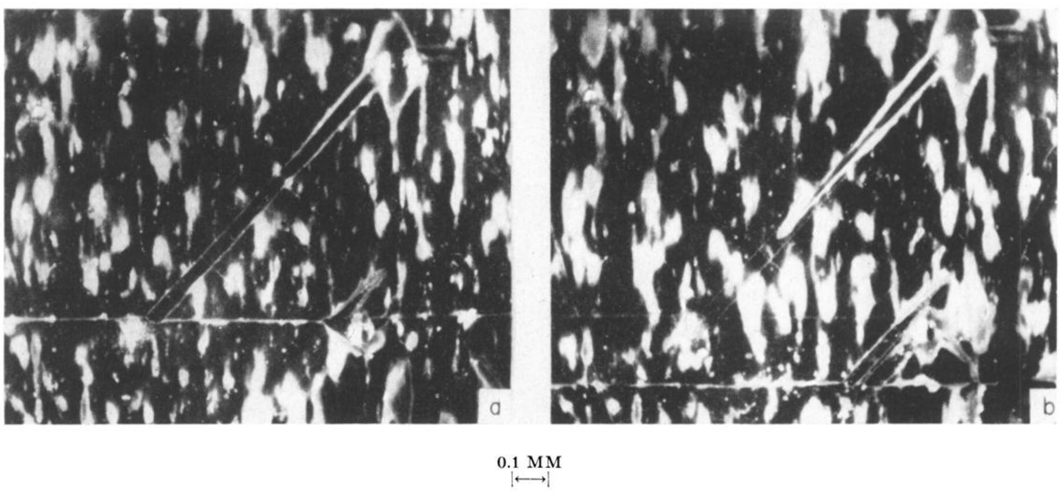


FIG. 7. Patterns corresponding to the domain structure shown in Fig. 6.