Total Reflection of Neutrons on Cobalt

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April 13, 1949

ATTENTION has recently been called to the possibility of producing polarized neutron beams by reflection from magnetized iron mirrors.[1] The indices of refraction differ for the two neutron spin states, since their magnetic scattering amplitudes are opposite in sign. The resultant difference in critical angle of total reflection can be used to separate the spin components.

For Fe, the coherent nuclear amplitude exceeds the magnetic amplitude, so that the index of refraction is less than one for both spin states, and both are capable of total reflection. Since the critical angle is proportional to neutron wave-length, two wave-lengths (one for each spin state) will overlap. This circumstance prevents attainment of complete polarization, since intensity requirements dictate the use of a fairly broad band of neutron energies.

It is interesting to note that by reflecting neutrons from a cobalt mirror magnetized along the beam direction one can obtain an exact analog of the Nicol prism. The coherent scattering cross section of Co is ~ 1.8 barns[2] compared to 10.3 barns for Fe. At the same time, the magnetic amplitude for Co is $\sim 4.6 \times 10^{-13}$ cm, which is only slightly below the value 6.0×10^{-13} for Fe, so that for Co the magnetic amplitude *exceeds* the nuclear amplitude. Consequently, the refractive indices for the two spin states lie on opposite sides of unity for *all* wave-lengths, and only one of the spin components is capable of undergoing total reflection. With an arbitrarily broad spectrum of incident neutrons, the mirror will reflect a completely polarized beam.

D. J. Hughes and his associates are now conducting reflection experiments with Fe and Co.

[1] O. Halpern, Phys. Rev. **75**, 343 (1949).
[2] C. G. Shull and E. O. Wollan, unpublished.

On the "Magic Numbers" in Nuclear Structure

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A SIMPLE explanation of the "magic numbers" 14, 28, 50, 82, 126 follows at once from the oscillator model of the nucleus,[1] if one assumes that the spin-orbit coupling in the Yukawa field theory of nuclear forces leads to a strong splitting of a term with angular momentum l into two distinct terms $j=l\pm\frac{1}{2}$.

If, as a first approximation, one describes the field potential of the nucleons already present, acting on the last one added, as that due to an isotropic oscillator, then the energy levels are characterized by a single quantum number $r=r_1+r_2+r_3$, where r_1, r_2, r_3 are the quantum numbers of the oscillator in 3 orthogonal directions. Table I, column 2 shows the multiplicity of a term with a given value of r, column 3 the sum of all multiplicities up to and including r. Isotropic anharmonicity of the potential field leads to a splitting of each r-term according to the orbital angular momenta l (l even when r is odd, and vice versa), as in Table I, column 4. Finally, spin-orbit coupling leads to the l-term splitting into $j=l\pm\frac{1}{2}$, columns 5 and 6, whose multiplicities are listed in column 7.

The "magic numbers" (column 8) follow at once on the assumption of a particularly marked splitting of the term with the highest angular momentum, resulting in a "closed shell structure" for each completed r-group, together with the highest j-term of the next succeeding r-group. This classification of states is in good agreement with the spins and magnetic moments of the nuclei with odd mass number, so far as they are known at present. The anharmonic oscillator model seems to us preferable to the potential well model,[2] since the range of the nuclear forces is not notably smaller than the nuclear radius.

TABLE I. Classification of nuclear states.

1 Oscillator-quantum number r	2 Multiplicity	3 Sum of all multiplicities	4 Orbital momentum l	5 Total angular momentum j	6 l_j-symbol	7 Multiplicities	8 Magic numbers
1	2	2	0	1/2	$s_{1/2}$	2	
2			1	3/2	$p_{3/2}$	4	
	6	8		1/2	$p_{1/2}$	2	
3			2	5/2	$d_{5/2}$	6	14
				3/2	$d_{3/2}$	4	
	12	20	0	1/2	$s_{1/2}$	2	
4			3	7/2	$f_{7/2}$	8	28
				5/2	$f_{5/2}$	6	
			1	3/2	$p_{3/2}$	4	
	20	40		1/2	$p_{1/2}$	2	
5			4	9/2	$g_{9/2}$	10	50
				7/2	$g_{7/2}$	8	
			2	5/2	$d_{5/2}$	6	
				3/2	$d_{3/2}$	4	
	30	70	0	1/2	$s_{1/2}$	2	
6			5	11/2	$h_{11/2}$	12	82
				9/2	$h_{9/2}$	10	
			3	7/2	$f_{7/2}$	8	
				5/2	$f_{5/2}$	6	
			1	3/2	$p_{3/2}$	4	
	42	112		1/2	$p_{1/2}$	2	
7			6	13/2	$i_{13/2}$	14	126
				11/2	$i_{11/2}$	12	
			4	9/2	$g_{9/2}$	10	

A more detailed account will appear in three communications to Naturwissenschaften.[3]

[1] See, e.g., H. A. Bethe and R. Bacher, Rev. Mod. Phys. **8**, 82 (1937), pars. 32–34.
[2] Which anyhow does not lead to a very different term-sequence compared with that of an anharmonic oscillator, see reference 1.
[3] (a) Haxel, Jensen, and Suess, Naturwiss. (in press). (b) Suess, Haxel, and Jensen, Naturwiss. (in press). (c) Jensen, Suess, and Haxel, Naturwiss. (in press).

Concerning the Abundance of Atmospheric Carbon Monoxide

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April 19, 1949

IN October of 1941 the 4.7-micron region of the solar spectrum was examined by the author at the Lowell Observatory, Flagstaff, for evidence of the carbon monoxide fundamental. The observation was made with a 2400-lines/inch grating in an f/5-Pfund type spectrometer of focal length 30 inches. Galvanometer deflections were recorded photographically. The solar spectrum was compared with laboratory observations,[1] but no conclusive evidence could be deduced for the existence of spectroscopically detectable quantities of carbon monoxide in the atmosphere above the observatory. The adequacy of the solar spectrum can be judged from the fact that carbon dioxide fine structure (some of it since traced to ν_3 of $C^{13}O_2^{16}$), which is twice as difficult to resolve as carbon monoxide fine structure, was abundantly present and clearly resolved.

One notes with interest, therefore, Migeotte's recent observation of the carbon monoxide fundamental as a prominent feature in the solar spectrum at Columbus, Ohio.[2]

The purely local nature of atmospheric abundance of carbon monoxide is emphasized by its absence over Flagstaff, and further emphasized by the extremly large variations in the amount over Columbus as reported by Migeotte. There is a temptation to regard the presence of carbon monoxide in the air as a consequence of its nature as a product of incomplete combustion. It would be most instructive to follow the course of carbon monoxide above Columbus as a function of fuel consumption and local weather.

Amongst the minor constituents of the atmosphere, carbon monoxide must be regarded as a local manifestation, unlike nitrous oxide, for example, which was recently shown to be world wide in an essentially constant amount.[3]

[1] S. E. Whitcomb and R. T. Lagemann, Phys. Rev. **55**, 181 (1939).
[2] M. V. Migeotte, Phys. Rev. **75**, 1108 (1949).
[3] Arthur Adel, Astrophys. J. **90**, 627 (1939); **93**, 509 (1941); Shaw, Sutherland, and Wormell, Phys. Rev. **74**, 978 (1948).

On the Excited States of Li^7

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April 20, 1949

THE ground state of Li^7 has nuclear spin $I=3/2$, and the possibility[1] that the 480-kev excited state is the other state of the doublet, ${}^2P_{\frac{1}{2}}$, is favored[2] by the existence of a transition[3] to this state by K-capture in Be^7. However, when thermal neutrons impinge on B^{10}, which[4] has $I=3$, the transition by alpha-emission leads[5] almost entirely (93 percent) to the excited state of Li^7, and this strong preference seems to demand[6] a selection rule based on a large angular momentum associated with the excited level, considerably larger than the value $I=1/2$. In keeping with the assumption of spherically symmetric exchange interactions, which have been used[7] fairly successfully to correlate nuclear stability properties, it was suggested[6] that the excited level of Li^7 might be the two states of an unresolved ^{2}F, having $I=7/2$ and $5/2$. In the light of recent observations[8] which display the energy groups as sharp peaks and agree quite closely on the excitation energy (and barring a pervasive selection rule), this would require very small spin-orbit coupling (<10 kev).

The objection to this latter scheme[9] is that it leaves the total orbital angular momentum L too nearly a good quantum number, and the Be^7 K-capture to the excited level would be forbidden with $\Delta L=2$. Retention of L as a quantum number is perhaps an oversimplification. It is noteworthy that agreement could be obtained both with the Be^7 K-capture and the $B^{10}(n, \alpha)Li^7$ transitions to the excited state by the assumption that this excited state is single and simply has $I=5/2$. This assumption is also compatible with the observed lifetime[10] of the excited state, which may be attributed to a magnetic dipole transition moment of plausible magnitude,[10] but requires an electric quadrupole transition moment considerably larger than estimated from nuclear dimensions. It agrees as well with the observed angular distribution[11] of the reaction $Li^6(d, p)Li^7$ at low energies, wherein the spherical symmetry of the long-range protons and a term as high as $\cos^4\theta$ in the short-range protons may be explained, only if the excited state has $I\geq 5/2$, in which case the explanation involves compound states 0^+, 2^+ (both competing with alphas[12]) and 5^-. Since $I=5/2$ seems to be the only single value compatible with all the observations, the problem is to make this assumption theoretically plausible.

The shell structure apparent as "magic numbers" in nuclear stability has been correlated by Mayer[13] with the trend of nuclear spins and magnetic moments by postulating $j-j$ coupling for the individual nucleons, which requires strong spin-orbit coupling in most nuclei. There are sufficiently few exceptions to the general experimental agreement[13, 14] that the scheme has a strong empirical appeal in spite of its sharp divergence from previous concepts.[7] Perhaps the most serious discrepancy is found in Li^6, where two nucleons each with $j=3/2$ would be expected[14] to combine to make $I=3$, rather than 1 as observed, but there might be exceptionally small spin-orbit coupling in this nucleus because of the exceptionally weak binding of these nucleons. Then Li^7 might also be expected to have somewhat weaker spin-orbit coupling than normal for p orbits, and a 480-kev doublet splitting would seem ample to be compatible with a splitting of 2 Mev or more in heavier nuclei as might be required to have an influence on shell structure. This would leave the excited state a ${}^2P_{\frac{1}{2}}$. Because this is incompatible with the $B^{10}(n, \alpha)Li^7$ and the $Li^6(d, p)Li^7$ data, we wish also to consider the possibility of stronger spin-orbit coupling in Li^7.

In extreme $j-j$ coupling, one obtains the low states by coupling three vectors $j=3/2$. Because of the exclusion principle, the two neutron vectors make $J_\nu=0$ in the ground state, and $J_\nu=2$ combining with j_π to make $I=7/2$, $5/2$, $3/2$, $1/2$ in the simplest description of the next higher states, and we may expect that some acceptable choice of nucleon interactions would make their energies ascend in this order. Enumeration of the higher states with various nucleons having $j=1/2$ rather than $3/2$ shows that there are in all two states with $7/2$, five states with $5/2$, eight with $3/2$, and six with $1/2$. In a second-order calculation departing from extreme $j-j$ coupling, one might expect very roughly that the extent of depression of the lowest state with a given I, due to the familiar second-order "repulsion" in energy, would be greatest for the value of I which characterizes the greatest number of states. By this criterion, the first-order ground state would be depressed most and remain the ground state, and $I=5/2$ would be depressed more than $I=7/2$, which does indeed make it a fairly plausible first excited state. Unfortunately, in an approach to the intermediate coupling situation from the opposite extreme of $L-S$ coupling with the ${}^2P$ assumed lowest in first order as in earlier calculations,[7] the same plausibility argument favors $I=1/2$ as the first excited state. This suggests that the interactions which one assumes to provide the $j-j$ coupling scheme may have to deviate from previous concepts so severely as to alter the order of the multiplets calculated by neglecting spin-orbit coupling. Another difficulty is that recent observations[15] fail to detect further excited states in Li^7 up to about 1.6 Mev, and this qualitative discussion unfortunately does not suffice to explain this gap.

[1] D. R. Inglis, Phys. Rev. **50**, 783 (1936); G. Breit and J. R. Stehn, Phys. Rev. **53**, 459 (1938).
[2] G. Breit and J. K. Knipp, Phys. Rev. **54**, 652 (1938).
[3] N. P. Heydenburg and G. L. Locher, Phys. Rev. **53**, 1016 (1938); unpublished data of H. T. Richards and R. M. Williamson (12 percent to excited state).
[4] Gordy, Ring, and Burg, Phys. Rev. **74**, 1191 (1948).
[5] J. K. Bøggild, Kgl. Danske Vid. Sels. Math.-fys. Medd. **23**, 4, 26 (1945).
[6] D. R. Inglis, Phys. Rev. **74**, 1876 (1948).
[7] E. Feenberg and E. Wigner, Phys. Rev. **51**, 95 (1937); E. Feenberg and M. Phillips, Phys. Rev. **51**, 597 (1937), *et al.*
[8] Buechner, Strait, Stergiopoulos, and Sperduto, Phys. Rev. **74**, 1569 (1948); Rubin, Snyder, Lauritsen, and Fowler, Phys. Rev. **74**, 1564 (1948).
[9] As discussed by Wigner, Primakoff, and Feenberg, Feld, and others.
[10] L. C. Elliott and R. E. Bell, Phys. Rev. **74**, 1869 (1948); Rasmussen, Lauritsen, and Lauritsen, Phys. Rev. **75**, 199 (1948).
[11] Krone, Hanna, and Inglis, Phys. Rev. **75**, 335 (1949).
[12] R. Resnick and D. R. Inglis, Phys. Rev. **75**, 1291 (1949).
[13] M. G. Mayer, Phys. Rev. (to be published).
[14] E. Feenberg and K. C. Hammack, Phys. Rev. (to be published).
[15] Unpublished observations of Buechner and co-workers on $Be^9(d,\alpha)Li^7$ and $Li^6(d,p)Li^7$, of Heydenburg and Inglis on $Li^6(d,p)Li^7$, and of Resnick and Hanna and of Inglis on $Be^9(d,\alpha)Li^7$.

Meson Exchange and Spin-Orbit Coupling in Nuclei

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April 20, 1949

THE single-nucleon spin-orbit coupling required by recent conjectures[1-3] concerning the prevalence of a $j-j$ coupling scheme in most nuclei is too strong a coupling to be