nuclear collisions, may be calculated by means of the stopping formula given by Bohr.[3] The calculation is done for two fragments; the (z, m)-values of these are chosen to be (39, 95) and (53, 139), corresponding to the most frequent values of the light and heavy group of fragments, respectively. As an example hydrogen may be considered. Curve a in Fig. 1 gives the calculated energy loss for the heavy fragment; it is obtained only as a function of differences of the abscissae and, hence, in the figure the end-point for curve a is chosen in accordance with the total range of the heavy group as given by Bøggild, Arrøe, and Sigurgeirsson. Combining curve a with the measured ionization curve, we obtain the fulldrawn curve representing the specific energy loss along the whole range. The curve for the light fragment is obtained in a similar manner.

The ionization values which were measured only on a relative scale have been normalized so as to give an initial energy of the light fragment of 86 Mev. The ratio of the energy of the light and heavy fragment is found to be 1.53, in agreement with the measured value.[4] The fact that the curves in H_2 and D_2 coincide over the first part of the range proves the insignificance of nuclear collisions for the corresponding rather high velocities of the fragments. For smaller velocities the different magnitude of the nuclear stopping in H_2 and D_2 is demonstrated by the splitting up of the curves. Of course, the assumption that the electronic stopping becomes zero just for $v<v_0$ and that nuclear stopping is negligible for $v>v_0$ is somewhat arbitrary; nevertheless, it leads to differences between the total ranges in D_2 and H_2 of 0.7 and 0.5 cm for the light and heavy fragment, respectively, which are in good agreement with the experiments.

When given in cm normal air, the extrapolated ranges in various gases are, in contrast to the total ranges, found to increase monotonically with the atomic number of the stopping gas. Corresponding to the smaller values of $R_{\text{ex}}^{\text{air}}$ the initial specific ionization by the fragments relative to that by α-particles is higher in the light than in the heavy gases. This result is in close agreement with the theoretical expectations when the initial charge of the fragments is assumed to be the same in all gases. Still, on careful consideration, the ionization in H_2 is found to be a little higher than should be expected on this assumption. The measurements thus seem to indicate a slightly higher charge in H_2 than in other gases, which may also be anticipated from the exceptionally low cross section for electron capture in H_2.

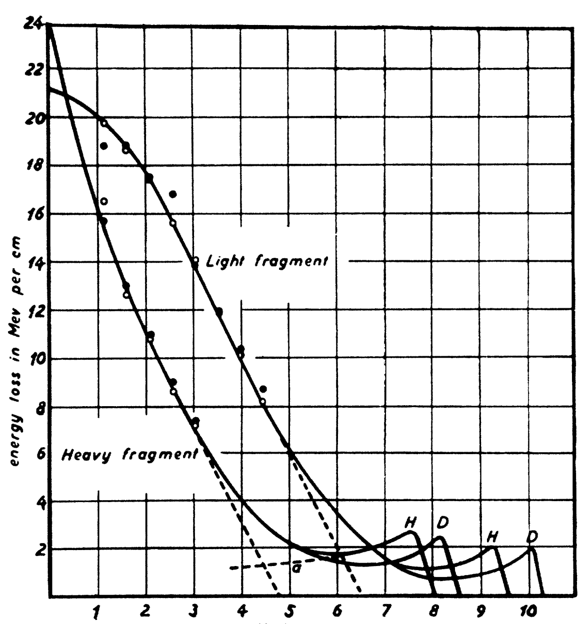

FIG. 1. The open circles give the result of measurements of the specific ionization in hydrogen. The full circles correspond to deuterium.

From the dE/dx-curves the energy and the velocity, as functions of the distance traversed, can be derived. Good agreement is obtained with the velocity-range relation found for argon by Bøggild, Brostrøm, and Lauritsen.[5] By means of the stopping formula the effective charge of the fragments can be calculated; the result agrees fairly well with the direct measurements of the total charge and its variation along range.[6]

A more detailed account of the present work will be published in the Communications of the Copenhagen Academy of Science.

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[3] N. Bohr, Kgl. Danske Vid. Sels. Math.-Fys. Medd. **18**, No. 8 (1948).
[4] Flammersfeld, Gentner, and Jensen, Zeits. f. Physik **120**, 450 (1943). W. Jentschke, Zeits. f. Physik **120**, 165 (1943).
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[6] N. O. Lassen, Kgl. Danske Vid. Sels. Math.-Fys. Medd. **23**, No. 2, (1945); Phys. Rev. **68**, 142 (1945); Phys. Rev. **69**, 137 (1946).

The Distribution of Arc-Chord Differences in Scattered Particle Tracks*

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April 11, 1949

IN a previous letter,[1] it was suggested that the scattering of particle tracks in emulsions might be readily measured by the difference between the actual length of track and the chord joining the two ends. In fact, an approximate expression for the average value of this difference was given, which was quite similar to the corresponding expression for the usual measure of scattering.

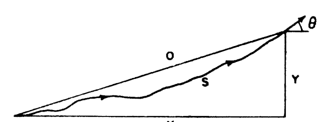

FIG. 1. A scattered track of arc length s, chord D, projection on the initial direction x, lateral displacement y, and angular deviation θ.

It is to be expected, however, that the statistical distributions in the two quantities should be qualitatively different. The arc-chord difference should be subject to relatively large fluctuations. A given, fairly large angle of scattering near the middle of the track will affect the entire measurement although a similar scattering near one end will have a much smaller effect. On the other hand, the sum of the squares of successive chord angles will not be greatly affected if one of the angles is large, and in fact the distribution of this quantity will certainly get narrower as the number of chords is increased. It is the object of this note to report on calculations of these two distributions that partially bear out these expectations.

The calculations are made for the case of constant energy which, of course, does not obtain in photographic emulsions, but which allows an explicit result that will serve to compare the two methods. We start with a diffusion equation for the combined distribution W of u, y and θ as functions of s. In these expressions, u stands for the difference between the arc length s and the projection x of the track along its initial direction; y is the lateral displacement from the x axis, and θ gives the direction of the track (Fig. 1). We have, assuming all angles to be small,

$$\frac{\partial W}{\partial s}+\frac{\theta^2}{2}\frac{\partial W}{\partial u}+\theta\frac{\partial W}{\partial y}=\frac{1}{\lambda}\frac{\partial^2 W}{\partial \theta^2}, \tag{1}$$

where λ represents a "scattering length"; $2/\lambda$ is the mean square projected scattering angle per unit path length.[2] This equation may be solved by assuming an exponential-type solution (LaPlace transform) in the variables u, y, and s, solving the resulting ordinary differential equation in θ, in terms of Hermite functions. The condition of convergence in θ yields an expression for the exponential coefficient of s and a $\delta(\theta)$ boundary condition yields a solution in the form of a double integral over the transformed variables for u and y with an infinite series of the Hermite functions for θ.

The series may readily be summed. The coefficient of the double integral is determined in the following manner: $W(u, y, \theta, s)$ is written in terms of the dimensionless variables $\xi = u\lambda/s^2$, $\nu = y(\lambda/s^3)^{\frac{1}{2}}$, and $\eta = \theta(\lambda/s)^{\frac{1}{2}}$. It may be shown from Eq. (1) that the resulting distribution is independent of s, which implies that the shape of the distribution is constant, the scale factors changing as s changes. Hence, a normalized distribution of this form yields for $s=0$, $W=\delta(u)\delta(y)\delta(\theta)$. There is a unique coefficient in the double integral which yields this behavior. Two further checks are that integration over y yields a solution to the diffusion equation for u, θ, and s as it should, and integration over u yields Fermi's distribution function in y and θ (Eq. (1) of reference 2).

To get the distribution in $v = s - D = s - (x^2+y^2)^{\frac{1}{2}} \cong u - y^2/2s$, we change variables from u to v, integrate the solution over ν and η, and using the dimensionless variable $\omega = v\lambda/s^2$ we arrive finally at the desired distribution

$$F(\omega)d\omega = \frac{d\omega}{2\pi i}\int_{a-i\infty}^{a+i\infty} z^{\frac{1}{2}} \sinh^{-\frac{1}{2}} z \exp(\tfrac{1}{2}z^2\omega)dz. \qquad (2)$$

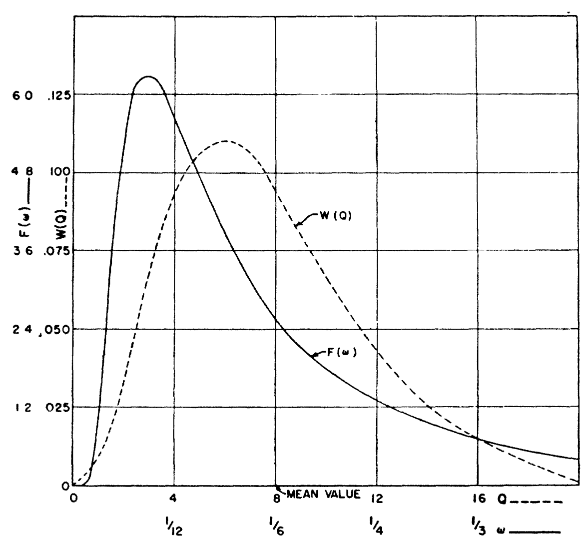

FIG. 2. $F(\omega)$, the distribution function for arc-chord differences, as a function of $\omega = \lambda(s-D)/s^2$ and $W(Q)$, the distribution function for the sum of squares of chord angles, as a function of $Q = (3\lambda/4s)\Sigma_{k1}\,\alpha_k^2$ (calculated for nine chords).

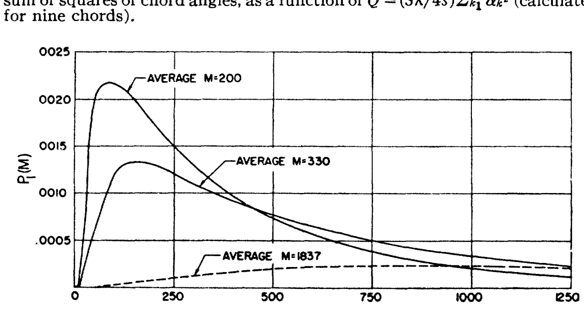

FIG. 3. $P_1(M)$, the arc-chord difference distribution, as a function of the apparent mass M.

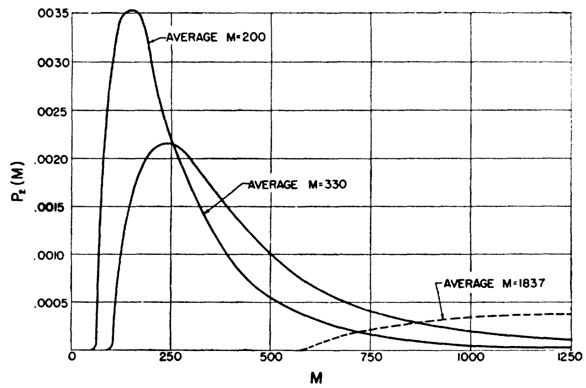

FIG. 4. $P_2(M)$ the sum-of-squares distribution for nine chords, as a function of the apparent mass M.

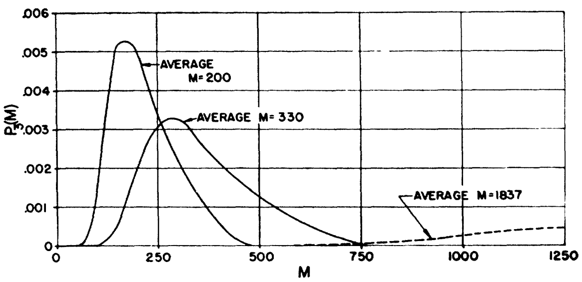

FIG. 5. $P_3(M)$ the sum-of-squares distribution (uncorrelated), for twenty-one chords, as a function of the apparent mass M.

The moments of the distribution are readily calculated: $\langle\omega\rangle_{Av} = \frac{1}{6}$; $\langle\omega^2\rangle_{Av} = 1/20$; $[\langle\omega^2\rangle_{Av} - \langle\omega\rangle_{Av}^2]^{\frac{1}{2}} = 0.89\langle\omega\rangle_{Av}$. The distribution itself may be calculated by expanding $\sinh^{-\frac{1}{2}}z$ in powers of e^{-z}; the kth term of the sum yields on integration an expression involving the parabolic cylinder function $D_{3/2}[(4k+1)/2\omega^{\frac{1}{2}}]$. These functions are expressible in terms of tabulated Toronto functions;[3] the final result of the computation is shown in Fig. 2. The skewness of this curve is quite apparent, the most probable value of ω being about $\frac{3}{8}$ the mean. The most important feature, however, is the long tail on the curve, indicating large fluctuations. (The skewness itself disappears if $\log\omega$ instead of ω is used as a measure of scattering, e.g., for reasons of constant relative error. But, the nearly symmetrical curve obtained is very broad.)

The distribution in $q = (E^2/e)\Sigma\alpha_k^{2\prime}$, the sum of the squares of angles between successive chords, may be found from the correlated probability for N such angles (reference 2, Eq. (31)). Standard probability calculus is used; we give here the result for nine track segments or eight chord angles (the improvement for more segments varies as $N^{\frac{1}{2}}$), in terms of the dimensionless variable $Q = 3\lambda qe/4sE^2$ where s is the length of one path segment:

$$W(Q)dQ = \frac{[\beta_1\beta_2\cdots\beta_8]^{\frac{1}{2}}}{2\pi i}\int_{-i\infty}^{i\infty} dt e^{Qt} \times[(t+\beta_1)(t+\beta_2)\cdots(t+\beta_8)]^{-\frac{1}{2}}, \qquad (3)$$

$$\beta_k = \left(2 - \cos\frac{kT\psi}{9}\right)^{-1}.$$

The integral is readily transformed into two real integrals suitable for numerical calculation, and Fig. 2 shows this curve also. Moments are readily found: $\langle Q\rangle_{Av} = 8$; $\langle Q^2\rangle_{Av} = 81.75$; $[\langle Q^2\rangle_{Av} - \langle Q\rangle_{Av}^2] = 0.53\langle Q\rangle_{Av}$. While the curve is not as broad or skew as that for $F(\omega)$, the fluctuations are still large.

A better representation of the efficacy of each method for estimating the masses of scattered particles is to plot each distribution as a function $P(M)$ of the "apparent mass" M which is the mass of a particle for which the given value of

ω or Q is the mean value. In this way, considerations of range and energy do not need to be explicitly included. Figure 3 shows $P_1(M)$ for the arc-chord differences, for three different average values of M; M is in units of the electron mass. Figure 4 shows $P_2(M)$ for the sums of squares of chord angles. Figure 5 shows $P_3(M)$ calculated for the *uncorrelated* probability for 20 chord angles with 21 segments. The result will be slightly broader if the correlation were taken into account.

The arc-chord difference is certainly a less precise measure of the scattering. However, the overlap of the curves in each graph shows the difficulty in using multiple scattering as an indication of the mass of the particle, and indicates the extent to which especially large or small observed values of ω or Q can eliminate, respectively, a large or small mass value from consideration. The separate chord-angle method can serve to distinguish between protons and mesons in most, but not all cases, which the arc-chord difference cannot do as well. Neither method has any reliability for distinguishing meson masses closer even than the values used in preparation of the figures.

* Research carried out at Brookhaven National Laboratory under the auspices of the AEC, while the author was on summer leave from Smith College.

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[2] W. T. Scott, Phys. Rev. (to be published).
[3] A. H. Heatley, Trans. Roy. Soc. Canada **37**, 13 (1943).

Scattering of Slow Neutrons by Deuterium Gas

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April 12, 1949

PHYSICAL intuition is often considered of little use as a guide to the outcome of computations based on quantum-mechanical formulas. Here is an instance of an elaborate computation which finally led to results very similar to those obtained from a simple picture, a fact probably not to be surmised from inspection of the original formulas.

It is well known that slow neutron scattering experiments in deuterium gas can give information on the neutron-deuteron interaction.[1]

Only the total cross section has been considered so far as the calculation of the differential cross section requires involved transformations from the center of mass systems to the laboratory system, owing to the thermal velocities of the molecules.

A relatively simple method of calculation in which this effect is accurately taken into account has been devised by the writer, and is being used to compute the angular distribution in the laboratory system of monokinetic neutrons of energy 0.07 ev scattered by deuterium gas at a temperature of 90°K.

Two points of general interest have arisen as a result of these computations:

(1) Although $kT\sim\frac{1}{10}$th of the incident neutron energy for the figures quoted, the assumption of molecules initially at rest would lead to relatively small errors in the computed differential cross section ($\sim$1 to 7 percent, although percentage corrections to individual transitions may be large), according to the results obtained so far, which are for angles of scattering in the laboratory of 30° to 120°.

(2) The computed curves closely resemble those obtained from a semi-classical picture in which the incident neutron is scattered by two deuterons at a fixed distance (0.74A) apart on an axis with fixed but random orientation,[2] provided relative changes of phase on scattering by the two scatterers of (i) 0 and (ii) π-radians are assumed. Thus, it was found that for each initial molecular rotational quantum number J (the computations covered $J=0$, 1, 2, 3), the contribution to the differential cross section due to all transitions to final states of the same parity, had the form of curve (i), while for parity change the contribution had the form of curve (ii).

The respective quantum-mechanical curves follow closely the semi-classical curves for angles of observation (lab. system) around 30°, but fall off from the latter with increasing angle of observation to approximately $\frac{2}{3}$ of the semi-classical value at 180°.

Analytically, the quantum-mechanical formulas become identical with the semi-classical ones described above if the energy lost or gained by the neutron in the various molecular transition is treated as negligible, which, in fact, of course, is far from being the case. The same result is not obtained by neglecting the mass of the neutron compared with that of the molecule, since if we do this there is no recoil of the deuterium molecule, and the results then differ widely from the correct quantum-mechanical results.

The two methods of calculation (quantum and semi-classical) would however be expected to yield identical results for those diatomic molecules whose masses are, in fact, large compared to that of the neutron,[3] but the relatively good agreement for deuterium is perhaps surprising.

These calculations, together with graphs exhibiting the dependence of the various partial cross sections on angle of scattering, will be published in due course by the National Research Council of Canada.

[1] M. Hamermesh and J. Schwinger, Phys. Rev. **69**, 145 (1946).
[2] See Section 4 of the paper by E. Fermi and L. Marshall, Phys. Rev. **71**, 666 (1947).
[3] N. Z. Alcock and D. G. Hurst, Phys. Rev. **75**, 1609 (1949).

Observations on the Blood of Cyclotron Workers

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April 20, 1949

ATTENTION is called to the finding of two types of unusual cells in the blood of personnel of the 130-in. cyclotron at Rochester, New York. These, are, respectively, very early mononuclear cells and cells which appear to be lymphocytes with bilobed or double nuclei. These cells, particularly the latter, occur in very small numbers, and it has been necessary to inspect several thousand leukocytes from each person in order to be assured of the finding. The "early mononuclear cells" are large (approximately 18–20μ in diameter), peroxidase negative, have deeply basophilic cytoplasm, and no nucleoli. The "lymphocytes with bilobed nuclei" tend to be somewhat larger than typical large lymphocytes. The nuclei stain slightly lighter than the nuclei of the other lymphocytes but are similar in consistency and have no nucleoli. The cytoplasm is clear, basophilic, peroxidase negative, and commonly contains a few typical azurophilic granules.

Both kinds of cells have been found in individuals not associated with the cyclotron who have abnormal blood smears due to infections, specifically in several cases of infectious mononucleosis and in one case of acute pharyngitis due to Hemolytic H. Influenza. There is, however, no indication that the findings in cyclotron personnel are due to infections.

Early mononuclear cells are occasionally found in routine films of "normal blood," however, lymphocytes with bilobed nuclei have not, to our knowledge, been described previously, and it is felt that they represent a true departure from the usual or normal blood picture. Although it is not possible to draw any conclusions relative to the significance of these cells as indicators of radiation damage, the findings may conceivably have such significance, and are presented at this time so that they may come to the attention of responsible individuals associated with cyclotrons in other institutions.

This work was supported by the joint program of the ONR and AEC.