PHYSICAL REVIEW VOLUME 75, NUMBER 11 JUNE 1, 1949

Theory of Proposed Reactions Involving Polarized Protons

LINCOLN WOLFENSTEIN*
Department of Physics, University of Chicago, Chicago, Illinois
(Received January 17, 1949)

The use of fast polarized projectiles in nuclear reactions provides a possible tool for studying the spin-dependence of nuclear interactions. In the first section general properties of such reactions are discussed. For incident neutrons or protons and a value $L_{\max}$ for the maximum partial wave that need be considered in the incident beam, the results are summarized in theorems 3 and 4.

The following reactions involving polarized protons are shown to be possible: (a) production of polarized protons by the (n,p) reactions of N or He^3 using polarized thermal neutrons, (b) detection of polarization by means of the $Li^7(p,\alpha)\alpha$-reaction, and (c) production or detection by the resonance scattering of protons from helium. Considering these reactions alone and exploring the fundamental limitations on intensity, one finds that the intensity problem appears to be least critical if (c) is used for both production and detection. The polarization effect usually depends on at least one parameter which does not affect or affects much less critically the unpolarized result. The polarization effect caused by the spin-dependent Coulomb scattering is calculated and found to be less than 5 percent for atomic number Z less than 5, except for special cases. Protons retain their polarization while being slowed down.

I. INTRODUCTION

THE production of a beam of polarized elementary particles might provide a useful tool for the study of the spin-dependent interactions of these particles. During the past twenty years a number of methods have been suggested for accomplishing this. The most successful of these has been the polarization of thermal neutrons by scattering in iron in a magnetic field.[1] The use of these neutrons to study spin-dependent nuclear interactions is limited by their low velocity. Proposed methods for polarizing fast particles depend upon the presence of spin-orbit coupling in the scattering of the particles. Mott first proposed such a method for polarizing electrons in 1929,[2] but the experiments did not prove successful until recently.[3] Similar methods for polarizing fast neutrons have been discussed recently by Schwinger.[4,5] The experiments suggested by Schwinger have not as yet been attempted because of the difficulties associated with the double scattering of neutrons.

The present investigation concerns the possibilities of producing and utilizing polarized beams of protons. In the introduction the general properties of reactions involving polarized particles of spin $\frac{1}{2}$ are considered, while in succeeding sections specific reactions involving polarized protons are proposed and discussed. It will be assumed throughout that the target nuclei involved in the reactions are unpolarized.[6]

The spin function corresponding to a state of complete polarization is written

$$A_{\frac{1}{2}}s^{\frac{1}{2}}+A_{-\frac{1}{2}}s^{-\frac{1}{2}} \tag{1}$$

with $|A_{\frac{1}{2}}|^2+|A_{-\frac{1}{2}}|^2$ equal to unity. The expectation value of the components of the spin operator $\boldsymbol{\sigma}$ are:

$$\begin{aligned}\bar{\sigma}_x&=2Re(A_{\frac{1}{2}}{}^*A_{-\frac{1}{2}}),\\ \bar{\sigma}_y&=2Im(A_{\frac{1}{2}}{}^*A_{-\frac{1}{2}}),\\ \bar{\sigma}_z&=|A_{\frac{1}{2}}|^2-|A_{-\frac{1}{2}}|^2.\end{aligned} \tag{2}$$

A superposition of n states of the form (1) with arbitrary phase relations is expressed

$$\sum_{\lambda=1}^{n}\{A_{\frac{1}{2}\lambda}s^{\frac{1}{2}}+A_{-\frac{1}{2}\lambda}s^{-\frac{1}{2}}\}c_\lambda\epsilon_\lambda \Big/ \left(\sum_{\lambda=1}^{n}c_\lambda{}^2\right)^{\frac{1}{2}}, \tag{3}$$

where c_λ is the weight with which a particular state is introduced, and ϵ_λ is an arbitrary phase factor[7] which enters into any meaningful result (such as an expectation value) only in the form $\epsilon_\lambda{}^*\epsilon_{\lambda'}$, which is defined to be equal to $\delta_{\lambda\lambda'}$. In this notation an unpolarized spin state may be represented

$$\{s^{\frac{1}{2}}\epsilon_1+s^{-\frac{1}{2}}\epsilon_2\}/2^{\frac{1}{2}}. \tag{4}$$

For the states given by Eq. (3) the expectation value of the spin operator $\boldsymbol{\sigma}$ is

$$\bar{\boldsymbol{\sigma}}=\sum_{\lambda=1}^{n}c_\lambda{}^2\bar{\boldsymbol{\sigma}}_\lambda \Big/ \sum_{\lambda=1}^{n}c_\lambda{}^2, \tag{5}$$

where the components of $\bar{\boldsymbol{\sigma}}_\lambda$ are determined from

* Now at Carnegie Institute of Technology, Pittsburgh, Pennsylvania.

1 F. Bloch, Phys. Rev. **50**, 259 (1936); **51**, 994 (1937). A summary of the latest work is given by D. J. Hughes, J. R. Wallace, and R. H. Holtzmann, Phys. Rev. **73**, 1277 (1948).

2 N. F. Mott, Proc. Roy. Soc. **A124**, 425 (1929); **A135**, 429 (1932).

3 C. G. Shull, C. T. Chase, and F. E. Myers, Phys. Rev. **63**, 29 (1943).

4 Julian Schwinger, Phys. Rev. **69**, 681 (1946).

5 Julian Schwinger, Phys. Rev. **73**, 407 (1948).

6 Possibilities involving polarized target nuclei have been discussed by M. E. Rose, Phys. Rev. **75**, 213 (1949).

7 A similar factor is used by G. Breit and B. T. Darling, Phys. Rev. **71**, 402 (1947). The ϵ_λ's may be thought of as basis unit vectors, and the summation over λ indicated in Eq. (6a) as a vector summation.

Eq. (2). If one makes the substitution in Eq. (3)

$$\sum_{\lambda=1}^{n} A_{\pm\frac{1}{2}\lambda} c_\lambda \epsilon_\lambda \Big/ \left(\sum_{\lambda=1}^{n} c_\lambda^2\right)^{\frac{1}{2}} = A_{\pm\frac{1}{2}} \nu_{\pm\frac{1}{2}}, \qquad (6a)$$

where $\nu_{\frac{1}{2}}$ and $\nu_{-\frac{1}{2}}$ are unit vectors like ϵ_λ but are not mutually orthogonal, Eq. (5) yields a generalization of Eq. (2);

$$\begin{aligned} \bar{\sigma}_x &= 2Re(A_{\frac{1}{2}}{}^*\nu_{\frac{1}{2}}{}^* A_{-\frac{1}{2}}\nu_{-\frac{1}{2}}), \\ \bar{\sigma}_y &= 2Im(A_{\frac{1}{2}}{}^*\nu_{\frac{1}{2}}{}^* A_{-\frac{1}{2}}\nu_{-\frac{1}{2}}), \\ \bar{\sigma}_z &= |A_{\frac{1}{2}}|^2 - |A_{-\frac{1}{2}}|^2. \end{aligned} \qquad (6b)$$

The most complete knowledge that may be found experimentally concerning a spin state is the mean value of an arbitrary operator for that state. Since any operator may be written in terms of σ_x, σ_y, σ_z, and the unity operator, a spin state is completely designated by the values of $\bar{\sigma}_x$, $\bar{\sigma}_y$, $\bar{\sigma}_z$; that is, a vector $\bar{\boldsymbol{\sigma}}$. For each vector $\bar{\boldsymbol{\sigma}}$, and thus for each spin state, there exists a representation in which the spin functions are quantized in the direction defined by $\bar{\boldsymbol{\sigma}}$, and $\nu_{\frac{1}{2}}$ and $\nu_{-\frac{1}{2}}$ are orthogonal. In this coordinate system it is seen directly that the percentage of polarization P is given by

$$P^2 = \bar{\boldsymbol{\sigma}} \cdot \bar{\boldsymbol{\sigma}} = (\bar{\sigma}_x)^2 + (\bar{\sigma}_y)^2 + (\bar{\sigma}_z)^2. \qquad (7)$$

Since the right-hand side is a scalar, Eq. (7) holds in all coordinate systems.

Several general theorems concerning nuclear reactions involving polarized particles will now be demonstrated. (Theorems 1 and 1A not restricted to particles of spin $\frac{1}{2}$.)

Theorem 1.—If $L_{\max}$ is the maximum orbital angular momentum that need be considered in a nuclear reaction the maximum degree spherical harmonic that can enter into the angular distribution of the outgoing intensity is $2(L_{\max}+S)$, where S is the spin of the incident particle if it is polarized and S is zero if the particle is unpolarized. Furthermore, if $2(L_{\max}+S)$ is odd, the maximum degree is $2(L_{\max}+S-\frac{1}{2})$.

Theorem 1A.—The maximum degree is $2(L_{\max}+S+S')$ if the angular distribution of a specified state of polarization of an outgoing particle with spin S' is considered.

The following corollary follows immediately for particles of spin $\frac{1}{2}$:

Theorem 2.—The maximum degree spherical harmonic that can enter the angular distribution of the outgoing intensity is $2L_{\max}$ whether or not the incident particle is polarized. In particular, polarized particles can never be detected by a reaction in which only s-waves are involved.

These theorems may be proved most directly using a method given by Yang.[8] It will be convenient for later purposes, however, to follow the procedure of Eisner and Sachs.[9] (In both references Theorem 1 is proved for the case in which the incident particle is unpolarized.) A collision characterized by orbital angular momentum L of a projectile with spin S and a nucleus of spin I will be considered. The initial state may be written as a sum of products of functions of the spin and angle variables. If the nuclear spin is unpolarized the initial state in any coordinate system is written

$$\Psi_0 = \Sigma_{m_I m_S m_L} A_{m_S} B_{m_L} s^{m_S} b^{m_L} a^{m_I} \epsilon_{m_I}/(2I+1)^{\frac{1}{2}}, \qquad (8)$$

where s^{m_S} represents the spin state of the projectile with a spin component m_S along the z axis, and a^{m_I} and b^{m_L} similarly represent nuclear spin and orbital angular momentum states. A_{m_S} and B_{m_L} are the coefficients with which the states s^{m_S} and b^{m_L} are introduced.[10] The products $s^{m_S}b^{m_L}$ may be expressed in terms of the eigenfunctions Θ_j of the total projectile angular momentum j,

$$s^{m_S}b^{m_L} = \Sigma_j (SLjm_S+m_L|SLm_Sm_L)\Theta_j{}^{m_L+m_S},$$

using the usual transformation coefficients. The products $\Theta_j{}^{m_L+m_S}a^{m_I}$ may similarly be expressed in terms of the eigenfunctions ${}^i\chi_J{}^{m_L+m_S+m_I}$ of the total angular momentum J. (Since three angular momentum functions have been combined, the eigenfunctions for each J value are in general degenerate. Here a particular set of eigenfunctions has been selected according to the j values, and the members are distinguished by the left-hand superscript.) Equation (8) may now be written

$$\begin{aligned} \Psi_0 = \Sigma_{m_I m_S m_L j J} & A_{m_S} B_{m_L} (SLjm_S+m_L|SLm_Sm_L) \\ & \times (IjJm_I+m_S+m_L|Ijm_Im_S+m_L) \\ & \qquad \times {}^i\chi_J{}^{m_L+m_S+m_I}\epsilon_{m_I}/(2I+1)^{\frac{1}{2}}. \end{aligned}$$

The effect of a nuclear reaction is to transform each state ${}^i\chi_J{}^M$ into $\Sigma_K(2I+1)^{\frac{1}{2}}\,{}^K\rho_J{}^i\,{}^K\Phi_J{}^M$, where ${}^K\rho_J{}^i$ is independent of M, and ${}^K\Phi_J{}^M$ is one of the states of the outgoing particles having the same transformation properties as $\chi_J{}^M$. The outgoing wave function then is

$$\begin{aligned} \Psi_f = \Sigma_{m_I m_S m_L j J K}\, & {}^K\rho_J{}^i A_{m_S} B_{m_L} \\ & \times (SLjm_S+m_L|SLm_Sm_L) \\ & \times (IjJm_I+m_S+m_L|Ijm_Im_S+m_L) \\ & \qquad \times {}^K\Phi_J{}^{m_L+m_S+m_I}\epsilon_{m_I}. \end{aligned} \qquad (9)$$

The argument of Eisner and Sachs[9] can be followed from here on. If the coefficient of Φ in Eq. (9) is written as α, the absolute square of the outgoing wave function has the form[11]

$$\begin{aligned} \Psi_f{}^\dagger\Psi_f = \Sigma_{m_I}\Sigma_{m_S m_L J K}\, & \alpha^*{}_{m_S m_L J K}({}^K\Phi_J{}^{m_L+m_S+m_I})^\dagger \\ & \times \Sigma_{m_S' m_L' J' K'}\, \alpha_{m_S' m_L' J' K'} \\ & \qquad \times ({}^{K'}\Phi_{J'}{}^{m_L'+m_S'+m_I'}). \end{aligned} \qquad (10)$$

[8] C. N. Yang, Phys. Rev. **74**, 764 (1948).

[9] E. Eisner and R. G. Sachs, Phys. Rev. **72**, 680 (1947); L. Wolfenstein and R. G. Sachs, **73**, 528 (1948).

[10] A_{m_S} may in general contain phase factors like ϵ_λ. Thus for an initial unpolarized state of spin $\frac{1}{2}$, $A_{\frac{1}{2}} = \epsilon_1/2^{\frac{1}{2}}$, $A_{-\frac{1}{2}} = \epsilon_2/2^{\frac{1}{2}}$.

[11] $\Psi^\dagger$ is the conjugate transpose of Ψ. $\Psi^\dagger\Psi$ implies a summation over all the spin variables involved in Ψ.

If the product of the Φ's in Eq. (10) is reduced to a sum of terms $\Psi_J{}^\mu$ which transform under rotation like an eigenfunction of total angular momentum $\mathbf{J}$, the maximum value of μ that occurs is seen to be $2(L+S)$. Because this holds in all coordinate systems it follows that $\mathbf{J}$ must be less than $2(L+S)$, that is

$$\Psi_f{}^\dagger\Psi_f=\sum_{\mathbf{J}=0}^{\mathbf{J}=2(L+S)}\sum_{\mu=-\mathbf{J}}^{\mu=\mathbf{J}}C_{\mathbf{J}\mu}\Psi_{\mathbf{J}\mu}. \tag{11}$$

If $\Psi_f{}^\dagger\Psi_f$ is integrated over all the independent variables except the outgoing direction of one particle, it follows that the maximum degree spherical harmonic that can enter in the angular distribution of the intensity of this particle is $2(L+S)$. Furthermore, the parity of each term entering Eq. (9) is the same being determined by L, the intrinsic parties of the initial particles, and the conservation of parity. It follows that each product obtained in squaring Ψ_f has even parity, and consequently only even spherical harmonics enter the angular distribution. Theorem 1 is now proved for the case of a single L value. Theorem 1A may be proved in a similar manner.

The argument is easily extended to the case in which more than one orbital angular momentum is effective with L in the previous results interpreted as the maximum orbital angular momentum $L_{\max}$. Equation (10) takes the form

$$\Psi_f{}^\dagger\Psi_f=\sum_{m_I}(Q^\dagger L_{\max}+Q^\dagger L_{\max}-1+\cdots)\times(QL_{\max}+QL_{\max}-1+\cdots).$$

The terms $QL_{\max}{}^\dagger QL_{\max}$ have been considered. By the previous argument the terms $QL_{\max}{}^\dagger QL_{\max}-1$ when reduced to the form (11) can contain no $\mathbf{J}$ value greater than $(L_{\max}+L_{\max}-1+2S)$. Furthermore, these terms must have odd parity. It follows that the maximum degree spherical harmonic which can arise from these terms is one less than that which can arise from the terms already considered.

For particles of spin $\frac{1}{2}$ the outgoing intensity is related to the initial direction of polarization by the following theorem:

Theorem 3.—(a) Polarization parallel to the axis of incidence (z axis) has no effect on the intensity.

(b) Assuming initial polarization along the y axis, the effect of the polarization on the angular distribution of the outgoing intensity is to produce a left-right asymmetry given by

$$\sum_{n=0}^{2L_{\max}-1}a_n\cos^n\theta\sin\theta\cos\phi,$$

where θ is the angle of scattering and ϕ is the azimuth angle measured from the x axis. If only one parity need be considered in the incident wave (that is, only odd or only even L), n is restricted to odd values.

To prove Theorem 3(a) an outgoing beam is considered with intensity I_0 in a given direction $(x_0,\ y_0,\ z_0)$. Symmetry about the axis of incidence (z axis) must obtain because no other direction is defined; therefore, the intensity is I_0 at $(x_0,\ -y_0,\ z_0)$. Now consider the transformation

$$y'=-y,\quad x'=x,\quad z'=z.$$

The only effect of this on the incident wave is to reverse the direction of polarization (because the spin is an axial vector); for the outgoing wave it gives the intensity I_0 at $(x_0,\ -y_0,\ z_0)$. Thus the intensity is the same for the two directions of polarization.

The proof of 3(b) will be given in two steps, the first of which parallels the proof just completed. Let the intensity at $(x_0,\ y_0,\ z_0)$ be I_0. In this case the transformation $y'=-y$ does not alter the incident state but gives the intensity I_0 at $(x_0,\ -y_0,\ z_0)$:

$$\begin{aligned}I(-y_0)&=I(y_0),\\ I(-\phi)&=I(\phi).\end{aligned} \tag{12}$$

Setting m_L equal to zero in Eq. (10), $\Psi_f{}^\dagger\Psi_f$ contains products of the form $(\Phi_J{}^{m_I+m_S})^\dagger(\Phi_{J'}{}^{m_I+m_S'})$. Analyzing Φ_J into products of the outgoing spin and orbital functions, carrying out the sums over the spin variables, and designating the remaining orbital functions $Y_l{}^M$, one is left with products of the form $(Y_l{}^{m_I+m_S+m})^*(Y_{l'}{}^{m_I+m_S'+m})$. Since $(Y_l{}^M)^*$ equals $\pm Y_l{}^{-M}$, these are of the form

$$Y_l{}^{-(m_I+m_S+m)}\,Y_{l'}{}^{m_I+m_S'+m}.$$

Consequently, in Eq. (11) the maximum absolute value of μ is the maximum absolute value of $m_S'-m_S$, namely, 1. (For the unpolarized case, the maximum value of μ is zero.) For polarization parallel to the y axis, $A_{\frac{1}{2}}=1/2^{\frac{1}{2}}$, $A_{-\frac{1}{2}}=i/2^{\frac{1}{2}}$. For this case the sum of all the terms $(Y_l{}^{m_I+m_S+m})^\dagger Y_{l'}{}^{m_I+m_S'+m}$ for which $m_S=m_S'$ gives the unpolarized result, considering Eq. (4). The remaining terms for which $m_S\neq m_S'$ represent the difference between the polarized and unpolarized cases and reduce into those terms of Eq. (11) for which μ equals ±1. Thus the angular distribution of the intensity difference between the unpolarized and polarized cases may be expanded in spherical harmonics $\Psi_{\mathbf{J}}{}^1$ and $\Psi_{\mathbf{J}}{}^{-1}$. To satisfy Eq. (12) the combinations proportional to $\cos\phi$ must be chosen; these are

$$\begin{aligned}&\mathbf{J}=1\ \sin\theta\cos\phi,\\ &\mathbf{J}=2\ \sin\theta\cos\theta\cos\phi,\\ &\mathbf{J}=3\ (5\sin\theta\cos^2\theta-\sin\theta)\cos\phi,\quad \text{etc.}\end{aligned}$$

The maximum value of $\mathbf{J}$ is $2L_{\max}$ (Theorem 2), and odd values of $\mathbf{J}$ are forbidden if only incident waves of a single parity need be considered. Theorem 3(b) follows immediately.

The formalism employed here is clearly symmetrical in initial and final states. Theorems 1, 2, and 3 may therefore be easily transformed into theorems concerning the angular distribution of a specified state of polarization of an outgoing particle provided the incident particle is unpolarized. In particular, from Theorem 3 one obtains

Theorem 4.—For an incident unpolarized beam, the outgoing particle, if polarized, will be polarized perpendicular to the plane of the motion and the polarized intensity will have an angular distribution

$$\sum_{n=0}^{2L_{\max}-1} a_n \cos^n\theta \sin\theta.$$

The special case of elastic scattering will now be considered further. In the customary theory of scattering[12] extended to particles of spin $\frac{1}{2}$ the scattered wave may be written:

$$\Psi_f U(r) = f(\theta, \phi) u_0 U(r),$$

where $U(r)$ is essentially an outgoing spherical wave, u_0 is the initial spin state, and $f(\theta, \phi)$ is an operator operating on this state. An arbitrary operator f may be expanded

$$f(\theta, \phi) = g(\theta, \phi) + \boldsymbol{\sigma}\cdot\mathbf{h}(\theta, \phi). \tag{13}$$

The scattering cross section per unit solid angle is given by

$$\begin{aligned}\frac{dS}{d\omega} &= u_0^{\dagger} |f(\theta, \phi)|^2 u_0 \\ &= |g|^2 + |\mathbf{h}|^2 + 2\bar{\boldsymbol{\sigma}}_0\cdot Re(g^*\mathbf{h}),\end{aligned} \tag{14}$$

where $\bar{\boldsymbol{\sigma}}_0$ describes the initial spin state. The polarization of the scattered wave is determined from

$$\begin{aligned}\boldsymbol{\sigma}_f\frac{dS}{d\omega} &= u_0^{\dagger} f^{\dagger}(\theta, \phi)\boldsymbol{\sigma} f(\theta, \phi) u_0 \\ &= |g|^2\bar{\boldsymbol{\sigma}}_0 + 2Re(g^*\mathbf{h}) + 2\bar{\boldsymbol{\sigma}}_0 \times Im(g^*\mathbf{h}) \\ &\quad + 2\bar{\boldsymbol{\sigma}}_0\cdot\mathbf{h}^*\mathbf{h} - |\mathbf{h}|^2\bar{\boldsymbol{\sigma}}_0.\end{aligned} \tag{15}$$

For an initially unpolarized beam

$$\bar{\boldsymbol{\sigma}}_f = 2Re(g^*\mathbf{h})/\{|g|^2 + |\mathbf{h}|^2\}. \tag{16}$$

This expression, of course, is also equal to the percentage change in cross section introduced by polarizing the initial beam in the direction of $Re(g^*\mathbf{h})$, as obtained from Eq. (14). If the scattering is not dependent on the nuclear spin, it follows from Theorem 3 or 4 that $\mathbf{h}$ is normal to the plane of the motion and is proportional to $\sin\theta$. The last follows from the fact that $g(\theta, \phi)$ is as in usual scattering theory a polynomial in $\cos\theta$.

II. PRODUCTION OF POLARIZED PROTONS USING POLARIZED NEUTRONS

If polarized thermal neutrons[1] are used as projectiles in the (n, p) reactions of N^{14} or He^3 it is possible that the resultant protons may also be polarized. These possibilities are investigated following the formalism of Section I.[13]

For the (n, p) reaction with nitrogen, the spin I equals 1 so that J values of $\frac{3}{2}$ and $\frac{1}{2}$ are possible. Since both N^{14} and C^{14} have even parity, the final orbital angular momentum values l are limited to even numbers. Since C^{14} has zero nuclear spin, the values $J=\frac{3}{2}$ and $J=\frac{1}{2}$ must correspond to l equals 2 and 0, respectively.

Using Eqs. (9) and (6), the expectation value $\bar{\boldsymbol{\sigma}}$ of the proton spin may be calculated. The component of $\bar{\boldsymbol{\sigma}}$ normal to the plane defined by the outgoing particle direction and the original direction of polarization is zero, while the other components resolved perpendicularly and parallel to the outgoing direction are

$$\begin{aligned}&\bar{\sigma}_{\perp} \equiv \bar{\sigma}_x \cos\theta - \bar{\sigma}_z \sin\theta = \tfrac{1}{3}(1+W)\sin\theta, \\ &\bar{\sigma}_{\parallel} \equiv \bar{\sigma}_x \sin\theta + \bar{\sigma}_z \cos\theta = -\tfrac{1}{3}(1-2W)\cos\theta, \\ &W = \{|\rho_{3/2}|^2 + 2Re(\rho_{3/2}\rho_{\frac{1}{2}}^*)\}/\{|\rho_{3/2}|^2 + \tfrac{1}{2}|\rho_{\frac{1}{2}}|^2\}.\end{aligned} \tag{17}$$

Here the z axis is the direction of the original neutron polarization, and the outgoing particle direction is given by the usual spherical coordinates θ and ϕ with ϕ equal to zero. The fact that no higher power of $\sin\theta$ or $\cos\theta$ than the first can enter Eq. (17) can be demonstrated on general grounds for this case.

The penetrability of the Gamow barrier for the outgoing protons is about 0.001 for outgoing d-waves ($J=\frac{3}{2}$) and 0.14 for outgoing s-waves ($J=\frac{1}{2}$). As a consequence it may be shown that the probability that the experimental cross section is primarily due to the $J=\frac{3}{2}$ resonance is only about one-sixth the probability that it is primarily due to the $J=\frac{1}{2}$ resonance. If the $J=\frac{3}{2}$ resonance has only a small effect, the outgoing proton will have a polarization of about 33 percent for θ equal to 90°.

For the (n, p) reaction with He^3, the spin I equals $\frac{1}{2}$ so that J values of 1 and 0 are possible. Since the product H^3 nucleus has spin $\frac{1}{2}$ the value $J=1$ may correspond to a final orbital angular momentum l of either 0 or 2, while the value $J=0$ corresponds to l equals 0. For the s-disintegration the proton energy is above the Coulomb barrier, while for the d-disintegration the barrier penetrability is only

[12] N. F. Mott and H. S. W. Massey, *Theory of Atomic Collisions* (Oxford University Press, New York, 1933).

[13] I wish to thank Dr. Frank C. Hoyt for showing me the work he had initiated on this problem.

about 0.02. It seems reasonable therefore to ignore the d-disintegration completely; one then obtains in the same manner as before

$$\begin{aligned}\bar{\sigma}_{\perp}&=-\tfrac{2}{3}\sin\theta(1+R\cos\gamma)/(1+\tfrac{1}{3}R^2),\\ \bar{\sigma}_{\parallel}&=\tfrac{2}{3}\cos\theta(1+R\cos\gamma)/(1+\tfrac{1}{3}R^2),\\ Re^{i\gamma}&=\rho_0/\rho_1.\end{aligned}\tag{18}$$

If R is close to zero the proton polarization is about 67 percent, while if R is equal to infinity there can naturally be no polarization because only angular momentum $J=0$ is involved. The polarization is found to be at least 20 percent for a wide range of values of R and γ.

III. DETECTION OF POLARIZATION BY THE $Li^7(p, \alpha)\alpha$-REACTION

A possible reaction for the detection of polarized protons is[14]

$$Li^7+H^1\rightarrow 2He^4.$$

For unpolarized protons the angular distribution of the reaction may be represented by[15]

$$(dS/d\omega)_0\sim 1+A(E)\cos^2\theta+B(E)\cos^4\theta,\tag{19a}$$

where θ is the angle between the line of centers of the leaving particles and that of the incident particles, and E is the initial kinetic energy. This angular distribution has been successfully analyzed[16,17] on the assumption that the ground state of Li^7 has odd parity so that only odd orbital angular momenta are effective in the reaction. In particular, s-waves are not effective in the reaction, which makes it especially favorable for the detection of polarization. If p-waves alone are considered it follows from the general analysis that the only possible effect of polarizing the incident protons along the z axis is to add to the unpolarized angular distribution a term

$$(dS/d\omega)_p\sim C(E)\sin\theta\cos\theta\cos\varphi.\tag{19b}$$

If f-waves also are considered a still further term may be added;

$$D(E)\sin\theta\cos^3\theta\cos\phi.$$

An explicit expression for $C(E)$ will be worked out for the case of p-waves alone. Since $I=\frac{3}{2}$ (for Li^7), $L=1$, and $S=\frac{1}{2}$, and since the final state has even parity and no spin, there are three factors $\rho_J{}^i$: $\rho_0{}^{\frac{3}{2}}$, $\rho_2{}^{\frac{1}{2}}$, and $\rho_2{}^{\frac{3}{2}}$. The final intensity may be obtained by squaring Eq. (9) and interpreting $\Phi_J{}^M$ as the spherical harmonics $Y_J{}^M$. For initial protons unpolarized this yields

$$\begin{aligned}(dS/d\omega)_0={}&\tfrac{1}{6}|\rho_0{}^{\frac{3}{2}}|^2+\tfrac{5}{6}|\rho_2{}^{\frac{1}{2}}|^2+\tfrac{5}{6}|\rho_2{}^{\frac{3}{2}}|^2\\ &-\{\tfrac{5}{6}Re(\rho_2{}^{\frac{1}{2}}\rho_2{}^{\frac{3}{2}*})-(5^{\frac{1}{2}}/6)Re(\rho_0{}^{\frac{3}{2}}\rho_2{}^{\frac{3}{2}*})\\ &+(5^{\frac{1}{2}}/6)Re(\rho_0{}^{\frac{3}{2}}\rho_2{}^{\frac{1}{2}*})\}(3\cos^2\theta-1).\end{aligned}\tag{20}$$

For polarization along the y axis the difference $(dS/d\omega)_p$ between the polarized and unpolarized intensities is

$$\begin{aligned}(dS/d\omega)_p={}&\{(5^{\frac{1}{2}}/2)Im(\rho_0{}^{\frac{3}{2}}\rho_2{}^{\frac{1}{2}*})\\ &-(5/2)Im(\rho_2{}^{\frac{1}{2}}\rho_2{}^{\frac{3}{2}*})\}\sin\theta\cos\theta\cos\phi.\end{aligned}\tag{21}$$

From Eqs. (19a), (19b), (20), and (21),

$$\frac{C(E)}{A(E)}=\frac{5^{\frac{1}{2}}Im(\rho_2{}^{\frac{1}{2}}\rho_2{}^{\frac{3}{2}*})-Im(\rho_0{}^{\frac{3}{2}}\rho_2{}^{\frac{1}{2}*})}{5^{\frac{1}{2}}Re(\rho_2{}^{\frac{1}{2}}\rho_2{}^{\frac{3}{2}*})+Re(\rho_0{}^{\frac{3}{2}}\rho_2{}^{\frac{1}{2}*})-Re(\rho_0{}^{\frac{3}{2}}\rho_2{}^{\frac{3}{2}*})}.\tag{22}$$

The fractional change in the scattered intensity caused by polarization has a maximum for $\theta=45°$ and $\phi=0°$, given by

$$r=\frac{(dS/d\omega)_p}{(dS/d\omega)_0}=\frac{C/A}{1+2/A}.\tag{23}$$

Since A ranges between 1 and 2, for energies from

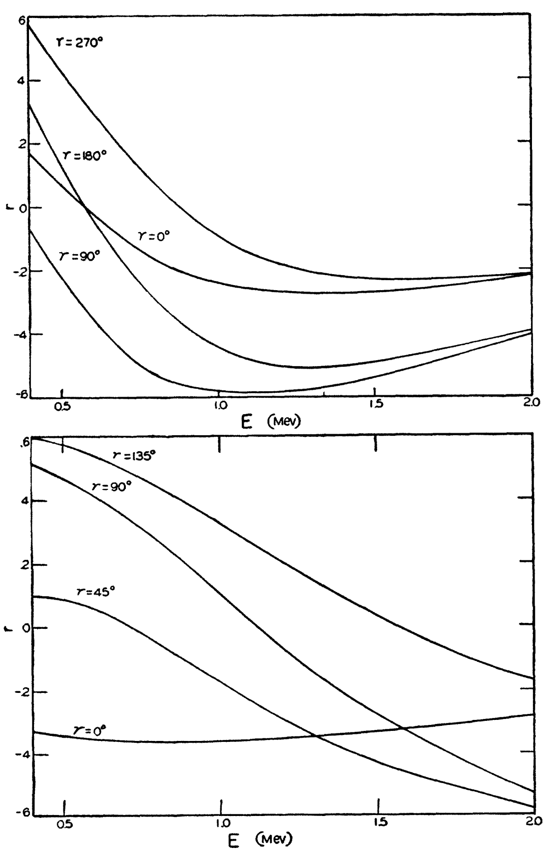

FIG. 1. Fractional change r in the scattered intensity as a result of the polarization of protons in the $Li^7(p,\alpha)\alpha$ reaction for $\theta=45°$ and $\phi=0°$ as a function of proton energy E and phase factor γ. Upper curve: one $J=0$ and one $J=2$ resonance assumed. Lower curve: two $J=2$ resonances assumed. (A change of 180° in γ changes the sign of r.)

[14] I am indebted to Dr. Robert G. Sachs for this suggestion and for discussions concerning it.

[15] Heydenburg, Hudson, Inglis, and Whitehead, Phys. Rev. **73**, 241 (1948).

[16] C. Critchfield and E. Teller, Phys. Rev. **60**, 10 (1941).

[17] David R. Inglis, Phys. Rev. **74**, 21 (1948).

0.5 to 2.0 Mev, an appreciable effect is expected unless (C/A) is in the neighborhood of zero. Equation (22) shows that (C/A) may vanish only if—

(1) $\rho_2^{\frac{1}{2}}=0$,
(2) $Im(\rho_0^{\frac{1}{2}}\rho_2^{\frac{1}{2}*})=5^{\frac{1}{2}}Im(\rho_2^{\frac{1}{2}}\rho_2^{\frac{3}{2}*})$.

Both of these conditions are very special and there is no reason to believe that they hold. Furthermore, even if the second condition were valid at one energy it would not be expected to hold over a sizeable energy interval. From the previous equations upper limits $r_{\max}$ may be set on the possible absolute values of r. If only $J=2$ is involved

$$r_{\max}=(3^{\frac{1}{2}}/2)(3+2A-A^2)^{\frac{1}{2}}/(A+2).$$

Using the experimental value of A,[15] this gives a maximum r of 0.36 at 1-Mev energy and of 0.6 at energies of 0.5 and 2.0 Mev. If both $J=0$ and $J=2$ are involved, for the values of A of interest,

$$r_{\max}=\tfrac{1}{4}(63+66A-A^2)^{\frac{1}{2}}/(A+2).$$

At 1 Mev this limit is 0.86, while at 0.5 and 2.0 Mev it is 0.95.

The use of the resonance formula to determine the value of r (Eq. 23) is not generally possible for two reasons: (1) it is not possible from the experimental data to determine all the constants in the resonance formula for the unpolarized angular distribution; (2) even if these constants are determined there remains completely undetermined a phase factor[18] which critically affects $(dS/d\omega)_p$ although not affecting $(dS/d\omega)_0$. However, it was thought of interest to calculate the polarization effect using the resonance formula for two cases:

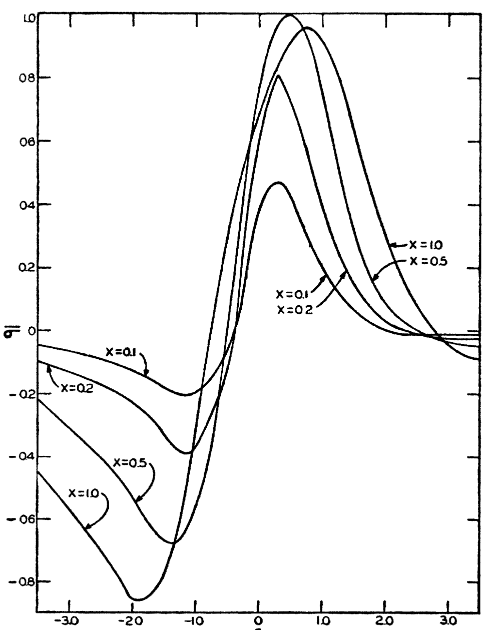

FIG. 2. Polarization $\bar{\sigma}$ of protons scattered from a doublet P resonance as a function of energy ϵ and splitting x. $\delta_0=45°$, $\eta=0$, $\theta=90°$.

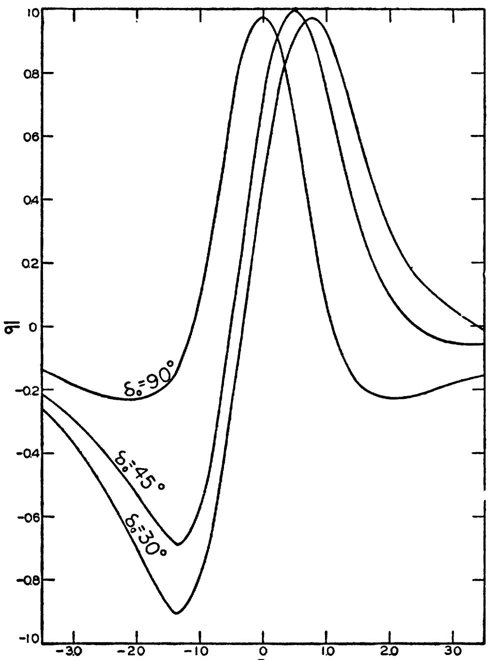

FIG. 3. Polarization $\bar{\sigma}$ of protons scattered from a doublet P resonance as a function of energy ϵ and s-phase shift δ_0. $x=0.5$, $\eta=0$, $\theta=90°$.

(a) A very broad resonance with $J=0$ plus a narrower resonance with $J=2$ was assumed, using a set of constants suggested by Inglis[17] to fit the experimental data.

(b) Two resonances with $J=2$ were assumed, each having a width of 1 Mev and located at 0 and 2 Mev,[19] using one set of constants that fit the data roughly.

The results are given in Fig. 1 as a function of proton energy E and the undetermined phase factor γ.

IV. POLARIZATION EFFECTS IN RESONANCE SCATTERING

The possibility of polarizing neutrons and also of detecting the polarization by means of the scattering from a resonance level that is split because of spin-orbit coupling was suggested by Schwinger.[4] In particular, he considered the scattering of neutrons with energy about 1 Mev from helium. Ex-

[18] This is the phase of the scattering from total spin 1 relative to that from total spin 2 (see the bottom of page 26 of reference 17). For purposes of calculation this phase factor was introduced by replacing $\sin\xi$ on page 14, of reference 16 by $\sin\xi e^{i\gamma}$. If two $J=2$ resonances are considered one obtains in a similar manner factors γ for the higher energy resonance and γ' for the lower energy. $(dS/d\omega)_0$ depends on $(\gamma-\gamma')$ but otherwise is independent of γ (or γ').

[19] This possibility has been suggested by R. Christy and S. Rubin, Phys. Rev. 71, 275A (1947).

periments[20] indicate that a p-wave resonance exists around this energy and that there is a considerable splitting between the $^2P_{3/2}$ and $^2P_{\frac{1}{2}}$ levels. A similar split resonance level would be expected around 2 Mev in the case of the scattering of protons from helium.

The scattering of a proton from a nucleus without spin is first considered in general with only one assumption: that incident orbital angular momenta L greater than 1 need not be considered. (For convenience, the polarization of an initially unpolarized beam is discussed, although, of course, one can directly convert this into the effect of initial polarization on the outgoing intensity.) In terms of the s-wave phase shift δ_0 and the two p-wave phase shifts $\gamma_{3/2}$ and $\gamma_{\frac{1}{2}}$, which correspond to total angular momenta J of $\frac{3}{2}$ and $\frac{1}{2}$, respectively, the scattering cross section for an unpolarized beam is*

$$\begin{aligned} dS/d\omega = \lambda\!\!\!^{-2}\{ | -(\eta/2s^2)\exp[-i\eta\ln s^2]+\sin\delta_0 e^{i\delta_0} \\ +\cos\theta[2\sin\gamma_{3/2}\exp[i(\gamma_{3/2}+\sigma_1)] \\ +\sin\gamma_{\frac{1}{2}}\exp[i(\gamma_{\frac{1}{2}}+\sigma_1)]]|^2 \\ +\sin^2\theta\sin^2(\gamma_{3/2}-\gamma_{\frac{1}{2}})\}, \end{aligned} \tag{24}$$

and the expectation value $\bar{\boldsymbol{\sigma}}$ of the spin after scattering is given by

$$\begin{aligned} \bar{\boldsymbol{\sigma}} dS/d\omega = -2\lambda\!\!\!^{-2}\sin\theta\sin(\gamma_{3/2}-\gamma_{\frac{1}{2}}) \\ \times\{\sin\delta_0\sin(\gamma_{3/2}+\gamma_{\frac{1}{2}}+\sigma_1-\delta_0) \\ -(\eta/2s^2)\sin(\gamma_{3/2}+\gamma_{\frac{1}{2}}+\sigma_1+\eta\ln s^2) \\ +3\cos\theta\sin\gamma_{3/2}\sin\gamma_{\frac{1}{2}}\}\mathbf{n}. \end{aligned} \tag{25}$$

Here

$$\begin{aligned} \lambda\!\!\!^{-} = \hbar/Mv, \quad \eta = Ze^2/\hbar v, \quad \sigma_1 = 2\tan^{-1}\eta, \\ s=\sin(\theta/2), \quad \mathbf{n} = (\mathbf{k}\times\mathbf{k}')/|\mathbf{k}\times\mathbf{k}'|, \end{aligned} \tag{26}$$

where v is the incident velocity, M is the reduced mass of the proton, and $\mathbf{k}$ and $\mathbf{k}'$ are incident and outgoing wave vectors, respectively. From Eq. (7) the percentage polarization is equal to the magnitude of $\bar{\boldsymbol{\sigma}}\cdot\mathbf{n}$, which will be written $\bar{\sigma}$.

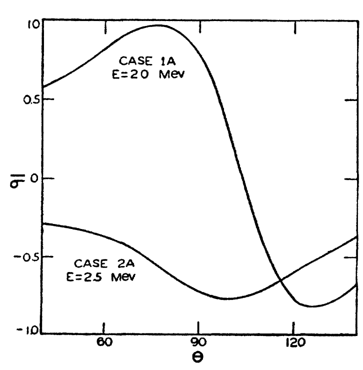

FIG. 4. Polarization $\bar{\sigma}$ of protons scattered from helium as a function of scattering angle θ under two assumptions.

For a small value of the splitting as measured by $(\gamma_{3/2}-\gamma_{\frac{1}{2}})$ the polarization effect is directly proportional to the splitting. It is of interest to compare this with the effect of the splitting on the cross section. Introducing a mean phase shift $\bar{\gamma}$ into Eq. (24) by the substitution

$$\begin{aligned} 2\sin\gamma_{3/2}\exp[i(\gamma_{3/2}+\sigma_1)]+\sin\gamma_{\frac{1}{2}}\exp[i(\gamma_{\frac{1}{2}}+\sigma_1)] \\ = -3ie^{i\sigma_1}[(1-T)/2]+3T\sin\bar{\gamma}\exp[i(\bar{\gamma}+\sigma_1)], \\ T=[1-8/9\sin^2(\gamma_{3/2}-\gamma_{\frac{1}{2}})]^{\frac{1}{2}}, \\ \bar{\gamma} = \tfrac{1}{2}\tan^{-1}\left\{\frac{2\sin 2\gamma_{3/2}+\sin 2\gamma_{\frac{1}{2}}}{2\cos 2\gamma_{3/2}+\cos 2\gamma_{\frac{1}{2}}}\right\}, \end{aligned} \tag{27}$$

one finds that the effect of the splitting on the cross section is proportional to $\sin^2(\gamma_{3/2}-\gamma_{\frac{1}{2}})$ and thus to $(\gamma_{3/2}-\gamma_{\frac{1}{2}})^2$ for small values of the splitting. Consequently sizeable polarization effects may exist in some cases even though the splitting cannot be detected from the cross section alone.

If the p-wave scattering is assumed to be due to a split resonance with energies $E_{3/2}$ and $E_{\frac{1}{2}}$ and a single width Γ, the phase shifts at an energy E are expressed

$$\begin{aligned} \gamma_{3/2} &= \cot^{-1}(\epsilon+x), \\ \gamma_{\frac{1}{2}} &= \cot^{-1}(\epsilon-x), \\ \epsilon &= [(E_{3/2}+E_{\frac{1}{2}})/2-E](2/\Gamma), \\ x &= (E_{3/2}-E_{\frac{1}{2}})/\Gamma, \end{aligned} \tag{28}$$

where ϵ and x are dimensionless forms of the energy and the splitting.[20] Substituting Eq. (28) into Eqs. (24) and (25), the polarization $\bar{\sigma}$ is found as a function of ϵ and x, the s-phase shift δ_0, and the Coulomb factor η. The results for θ equal to 90° and η equal to zero are shown in Figs. 2 and 3. (The general features of the curves are the same for values of η up to 0.3.) It should be noted that a change in sign in x merely changes the sign of $\bar{\sigma}$ and a change in sign of δ_0 merely changes the sign of ϵ. One sees that for x greater than 0.2 a polarization of more than 80 percent is generally obtainable at one energy. For smaller values of x, the polarization is approximately proportional to x. A secondary polarization maximum with the polarization reversed in sign also exists, which becomes more important as x increases and as δ_0 decreases. The two maxima are separated by an energy of the order of the resonance width, assuming this is not less than the separation of the two resonances. In general, there exists an angular interval about 90° with a width of at least 30° over which the average polarization effect is similar in magnitude to that at 90°.

The particular case of the scattering of protons by helium will now be considered. The experimental angular distributions seem to show the presence of a p-wave resonance between 2.0 and 3.0 Mev.[21] An analysis made on the basis of the

[20] T. A. Hall and P. G. Koontz, Phys. Rev. **72**, 196 (1947); H. Staub and H. Tatel, Phys. Rev. **58**, 820 (1940).

* It is to be noted throughout that θ is in the center-of-mass system.

[21] N. P. Heydenburg and N. F. Ramsey, Phys. Rev. **60**, 42 (1941); Freier, Lampi, Sleator, and Williams, Phys. Rev. **75**, 342 (1949).

earlier data and under the assumption that d-wave scattering is unimportant indicates that this p-wave resonance must be split. From this analysis the following conclusions about the polarization effect are drawn:

(a) The polarization for θ equal to 90° should reach a maximum value of about 80 percent at an energy between 2.0 and 2.5 Mev. (The earlier data goes only to 3.0 Mev and consequently does not allow the specification of a second polarization maximum.)

(b) The width of the polarization maximum may be as large as 1 Mev.

(c) The polarization $\bar{\sigma}$ as a function of scattering angle θ at an energy corresponding to a maximum of the polarization effect is given by one of the curves in Fig. 4. Cases 1A and 2A are distinguished by the assumed sign of δ_0: negative for 1A and positive for 2A.

These conclusions may be modified by consideration of the recent data.

V. POLARIZATION EFFECTS IN COULOMB SCATTERING

The earliest method suggested for the polarization of an elementary particle was to make use of the spin-orbit coupling in the Coulomb scattering of electrons.[2] Mott found that a polarization of the order of 30 percent should be possible at relativistic velocities and large atomic numbers. Recently Schwinger[5] has proposed a method for polarizing neutrons employing the interference between the spin-dependent Coulomb scattering and the nuclear scattering at small scattering angles where the two are of the same magnitude. For the case of protons, which will now be considered, however, the spin-independent Coulomb scattering greatly decreases this effect at small angles.

A classical treatment of this interaction between the proton spin and the Coulomb field is not significant at any energy as may be shown from an uncertainty principle argument.[22] In the quantum-mechanical approach, the proton is treated by the Dirac equation with an added Pauli term to describe the anomalous magnetic moment:

$$(E-eV+\beta mc^2)\Psi=-c\boldsymbol{\alpha}\cdot\mathbf{p}\Psi -(e\hbar/2Mc)(\mu_a/i)\beta\boldsymbol{\alpha}\cdot\mathbf{E}\Psi, \quad (29)$$

where V is the Coulomb potential, $\mathbf{E}=-\nabla V$, and μ_a is the anomalous moment in units of $e\hbar/2Mc$. The discussion of this equation will be restricted to the condition $v^2/c^2\ll 1$. An approximate solution for the scattered wave valid under the conditions, $\alpha^2\equiv(Z/137)^2\ll 1$ and $\eta<1$, is the following:

$$f(\theta)=-\lambda\!\!\!^{-}(\eta/2)\{\csc^2(\theta/2)-(v^2/c^2)(\mu_a+\tfrac{1}{2})i\boldsymbol{\sigma}\cdot\mathbf{n}\cot(\theta/2)\}\exp[-i\eta\ln\sin^2(\theta/2)], \quad (30)$$

where $\mathbf{n}$ is the unit normal to the plane of motion. This result is identical with the Born approximation except for the addition of the characteristic Coulomb phase factor, which multiplies not only the ordinary Coulomb scattering but the spin-dependent scattering as well.[23]

From Eqs. (13), (16), and (30), it follows that to this approximation the Coulomb scattering gives no polarization effect. The highest order polarization effect is obtained by adding to $f(\theta)$ of Eq. (30) the nuclear scattering

$$f_n(\theta)=\lambda\!\!\!^{-}[\sin\delta_0 e^{i\delta_0}+3\cos\theta\sin\delta_1 e^{i(\delta_1+\sigma_1)}+\cdots]. \quad (31)$$

From Eq. (16) one then obtains for the expectation value of the spin for scattering at an angle θ

$$\bar{\boldsymbol{\sigma}}=\mathbf{n}(\mu_a+\tfrac{1}{2})\lambda\!\!\!^{-}\eta(v^2/c^2)\cot(\theta/2)\left\{\frac{\cos(\eta\ln s^2)Im(f_n)+\sin(\eta\ln s^2)Re(f_n)}{|f_n-(\lambda\!\!\!^{-}\eta/2s^2)e^{-i\eta\ln s^2}|^2+\frac{1}{4}(\mu_a+\frac{1}{2})^2(v^4/c^4)\lambda\!\!\!^{-2}\eta^2\cot^2(\theta/2)}\right\}, \quad (32)$$

where $\mathbf{n}$, η, s, and σ_1 are defined in Eq. (26). The region of validity of the equation is energy $E>0.025Z^2$ Mev, $E\ll 465$ Mev, and $(Z/137)^2\ll 1$; these conditions follow from $\eta<1$, $(v^2/c^2)\ll 1$ and $\alpha^2\ll 1$, respectively.

The polarization effect is seen to vanish for both $\theta=180°$ and $\theta=0°$. The order of magnitude at moderate angles may be obtained by setting the product of $\lambda\!\!\!^{-}$, the fractional expression in Eq. (32), and $\cot\theta/2$ equal to unity, giving

$$\bar{\sigma}\simeq(u_a+\tfrac{1}{2})\eta(v^2/c^2)=1.23\cdot 10^{-3}Z[E(\text{Mev})/2.5]^{\frac{1}{2}}.$$

Calculations of the maximum polarization effect considering only s-wave nuclear scattering are given in Table I. These results were calculated from Eq.

TABLE I. Maximum polarization effect caused by Coulomb scattering and nuclear s-wave scattering as a function of s-wave phase shift δ_0, atomic charge Z, and proton energy E. $\epsilon=E$ (Mev)/2.5 Mev. $\bar{\sigma}$ = maximum polarization. θ = angle of scattering for maximum effect.

	$Z=1\epsilon^{\frac{1}{2}}$		$Z=5\epsilon^{\frac{1}{2}}$		$Z=10\epsilon^{\frac{1}{2}}$	
δ_0	$\bar{\sigma}/\epsilon$	θ	$\bar{\sigma}/\epsilon$	θ	$\bar{\sigma}/\epsilon$	θ
90°	0.0032	31°	0.0068	66°	0.0076	90°
60°	0.0072	30°	0.026	64°	0.026	96°
30°	0.026	39°	0.090	90°	0.020	130°
15°	0.100	52°	0.032	138°	—	—
−30°	0.0016	54°	0.0032	68°	0.0038	80°
−60°	0.0024	39°	0.0062	62°	0.0084	76°

[22] This result is to be expected from the general argument given in reference 12, p. 45.

[23] For $\mu_a=0$ Eq. (30) is obtained by Mott (see reference 2) as the first terms in the expansion of the exact solution of Eq. (29) in powers of α^2. The extension of the spin-dependent part of the solution to include a non-zero anomalous moment follows from considering Mott's expansion as a perturbation-theory solution of the second-order equation corresponding to Eq. (29).

(32) neglecting the term in (v^4/c^4) in the denominator and thus are not accurate for large values of $\bar{\sigma}$ (greater than 0.2). In general the polarization effect varies only slowly with θ; the width of the maxima are of the order of 30°. The very large polarization effects occurring for certain positive phase shifts result from a large amount of cancellation in the denominator of Eq. (32) and therefore are associated with minima in the scattered intensity. For energies much greater than $0.025Z^2$ Mev ($\eta\ll 1$) it may be shown that for a given value of δ_0, the maximum polarization $\bar{\sigma}$ is proportional to $Z^{\frac{1}{2}}E^{\frac{1}{2}}$. This may be used to extrapolate some of the data of Table I.

In the special case of proton-proton scattering this effect is of a smaller order of magnitude because polarization effects can occur only for the triplet state, for which there is no s-wave scattering. A considerably larger polarization effect for this case is likely to be that due to the tensor force in the nuclear interaction. This too, however, is extremely small (less than 0.1 percent at 7 Mev) under any reasonable form of interaction. A discussion of polarization effects in $n-p$ and $p-p$ scattering will be given in a later paper.

VI. DEPOLARIZATION OF POLARIZED PROTONS

For experimental purposes it is important to know the extent to which polarized protons are depolarized by various types of scattering as they are slowed down. A beam of completely polarized protons is considered; the percentage depolarization $(1-P_f)$ after passing through a thickness dt containing N scattering centers per unit volume will be expressed

$$1-P_f=S_{\text{dep}}Ndt. \tag{33}$$

P_f is the average polarization of all particles scattered through an angle less than θ_0, plus all particles not scattered; that is, of all particles to be included experimentally in the direct beam. It follows from Eqs. (5) and (7) that

$$P_f^2=\frac{\left|\left(1-Ndt\int(dS/d\omega)d\omega\right)\bar{\boldsymbol{\sigma}}_0+Ndt\int_{<\theta_0}\bar{\boldsymbol{\sigma}}_f(dS/d\omega)d\omega\right|^2}{\left[1-Ndt\int_{>\theta_0}(dS/d\omega)d\omega\right]^2}.$$

Neglecting terms in $(dt)^2$ this gives

$$P_f=1-Ndt\int_{<\theta_0}(1-\bar{\boldsymbol{\sigma}}_0\cdot\bar{\boldsymbol{\sigma}}_f)(dS/d\omega)d\omega,$$
$$S_{\text{dep}}=\int_{<\theta_0}(1-\bar{\boldsymbol{\sigma}}_0\cdot\bar{\boldsymbol{\sigma}}_f)(dS/d\omega)d\omega. \tag{34}$$

Substituting from Eqs. (14) and (15)

$$S_{\text{dep}}=2\int_{<\theta_0}\{|\mathbf{h}|^2-|\bar{\boldsymbol{\sigma}}_0\cdot\mathbf{h}|^2\}d\omega. \tag{35}$$

If $\mathbf{h}$ is a well defined vector, that is, independent of the spin of the scatterer, the integration over ϕ may be carried out using the fact that $\mathbf{h}$ must be perpendicular to the plane of the motion and independent of ϕ in magnitude. This gives

$$S_{\text{dep}}=(1+\cos^2\xi)2\pi\int_0^{\theta_0}|\mathbf{h}(\theta)|^2\sin\theta d\theta, \tag{36}$$

where ξ is the angle between the initial polarization and the axis of incidence. It is to be noted that the depolarization is proportional to the square of the spin-dependent interaction and is not affected by interference between the spin-dependent and spin-independent scattering. Since the depolarization is found to be small the total depolarization for a thickness t may be given by $S_{\text{dep}}Nt$. In this case some particles will be included in the calculation that are scattered through a total angle greater than θ_0 due to plural small angle scattering, thus somewhat overstimating the depolarization. It will prove satisfactory for present purposes, however, to obtain an upper limit on the depolarization.

Five causes of depolarization are considered.

(1) Elastic Coulomb scattering from the nucleus. Using Eqs. (30) and (36), replacing the lower limit of the integral in the latter by an angle $\theta_{\min}$, and assuming $\theta_{\min}\ll\theta_0\ll 1$ one finds

$$(S_{\text{dep}})_1=2\pi(1+\cos^2\xi)(\mu_a+\tfrac{1}{2})^2\times(Ze^2/Mc^2)^2\ln(\theta_0/\theta_{\min}). \tag{37}$$

The angle $\theta_{\min}$ is determined by the screening of the Coulomb field by the atomic electrons; for a screening radius r_0

$$\theta_{\min}=\lambda\!\!\!^{-}/r_0. \tag{38}$$

A classical calculation based on the precession of the spin in the magnetic field also gives Eq. (37) but $\lambda\!\!\!^{-}$ in Eq. (38) must be replaced by the classical minimum distance of approach. Letting θ_0 equal 5.7°, $\cos\xi=0$, and $r_0=5.3\cdot 10^{-2}/Z^{\frac{1}{3}}$ one obtains at reasonable energies the order-of-magnitude result

$$(S_{\text{dep}})_1=6\cdot 10^{-30}Z^2. \tag{39}$$

(2) Inelastic Coulomb scattering from atomic electrons. As might be expected, this gives a

smaller depolarization than the previous case for $Z=1$ and depends linearly rather than quadratically on Z. Therefore

$$(S_{\text{dep}})_2 < 6\cdot 10^{-30} Z. \quad (40)$$

(3) Interaction of the proton magnetic moment with the magnetic moment of the atomic electrons. It is interesting to note that this interaction is of the same order of magnitude as the Coulomb interaction. The Coulomb interaction is proportional to

$$\mu_0(e\hbar/Mc)(e/r^2)(v/c),$$

while this interaction is proportional to

$$\mu_0(e\hbar/Mc)(e\hbar/mc)(1/r^3),$$

where m is the electron mass and μ_0 is the effective proton moment. The minimum value of r for a collision with an electron is $(\hbar/mv)$; substituting this in the above expressions shows them to be equal. A Born approximation calculation substituted into Eq. (35) gives the result that Eq. (40) also holds for $(S_{\text{dep}})_3$. Adding up all the electromagnetic effects

$$(S_{\text{dep}})_{\text{electromagn.}} < 6\cdot 10^{-30}(Z^2+2Z). \quad (41)$$

(4) Nuclear scattering involving spin-orbit coupling but no spin-spin coupling. An example is the scattering from helium considered in Section IV. For such a case

$$|\mathbf{h}| = K\lambda\!\!\!^{-} \sin\theta,$$

where K is of the order of unity and the proportionality to $\sin\theta$ follows from the statement at the end of Section I. Substituting this into Eq. (36)

$$(S_{\text{dep}})_4 = (1+\cos^2\xi)(\pi/2)\lambda\!\!\!^{-2}K^2\theta_0^4$$

which is at most of the order of $10^{-4}\lambda\!\!\!^{-2}$.

(5) Nuclear scattering involving only spin-spin coupling. Let the nuclear spin be I and

$$f_+(\theta) = f(\theta) \text{ for total spin } S = I+\tfrac{1}{2},$$
$$f_-(\theta) = f(\theta) \text{ for total spin } S = I-\tfrac{1}{2}.$$

If $\mathbf{I}$ is a vector in the direction of the nuclear spin with magnitude $I(I+1)$, $f(\theta)$ for both total spins may be written in the form of Eq. (13) with

$$\begin{aligned} g(\theta) &= [(I+1)f_+(\theta)+If_-(\theta)]/(2I+1), \\ \mathbf{h}(\theta) &= \mathbf{I}[f_+(\theta)-f_-(\theta)]/(2I+1). \end{aligned} \quad (42)$$

Substituting from Eq. (42) in Eq. (35), averaging over-all directions of $\mathbf{I}$, and assuming $f_+(\theta)$ and $f_-(\theta)$ to be fairly constant about $\theta = 0°$,

$$(S_{\text{dep}})_5 = \frac{4\pi}{3}\frac{I(I+1)}{(2I+1)^2}|f_+(0)-f_-(0)|^2\theta_0^2. \quad (43)$$

Writing the scattered waves in terms of phase shifts as in Eq. (31) one obtains

$$(S_{\text{dep}})_5 = \frac{4\pi}{3}\frac{I(I+1)}{(2I+1)^2}\theta_0^2\lambda\!\!\!^{-2}\Big|\sum\nolimits_L (2L+1) \times \sin(\delta_L^+ - \delta_L^-)e^{i(\delta_L^+ + \delta_L^-)}e^{i\sigma_L}\Big|^2. \quad (44)$$

If the phase-shift differences involved are large this will be the largest of all the depolarization effects and $(S_{\text{dep}})_5$ will be of the order of magnitude of $0.01\lambda\!\!\!^{-2}$.

As a typical example the depolarization of 7-Mev protons stopped in hydrogen is considered. For this case

$$\delta_L^{\pm} = \tfrac{1}{2}(1\pm(-1)^{L+1})\delta_L.$$

The depolarization $(1-P_f)$ of the completely stopped protons due to all causes is only about 10^{-5}.

VII. CONCLUSION

It has been attempted in this paper to emphasize the general features of nuclear reactions involving polarized protons. In conclusion, it is worth while to indicate the practicability of experiments using solely the reactions considered here. Because any experiment on polarization effects involves of necessity a double scattering, the problem of obtaining sufficient intensity is a critical one. The intensity will be maximized if the experiment is designed merely to detect the polarization effect, so that good resolution of angles and energies is not required. Such experiments are simplified by the fact that polarization may be detected by the qualitative feature of a left-right asymmetry in the second scattering.

If the initial intensity is I_0 the intensity after a scattering or a nuclear reaction is

$$s_1I_0 = (Nt)_1\int (dS/d\omega)_1 d\omega_1 I_0,$$

where N is the number of scattering centers per unit volume, t is the thickness, and the limits of the integral are determined by the experimental geometry. The polarized intensity equals

$$r_1s_1I_0 = (Nt)_1\left|\int (\bar{\sigma}_f dS/d\omega)_1 d\omega_1\right| I_0.$$

Similarly after the second scattering (or nuclear reaction) the symmetrical intensity may be written $s_2s_1I_0$, while the additional scattering to the right or left resulting from the polarization effect may be written $r_2s_2r_1s_1I_0$. After time T the total number of events observed to one side is

$$s_2s_1I_0T(1+r_1r_2) \pm [s_2s_1I_0T(1+r_1r_2)]^{\frac{1}{2}},$$

and to the other side

$$s_2s_1I_0T(1-r_1r_2) \pm [s_2s_1I_0T(1-r_1r_2)]^{\frac{1}{2}}.$$

The difference is approximately

$$s_2 s_1 I_0 T(2r_1r_2) \pm 2[s_2 s_1 I_0 T]^{\frac{1}{2}}.$$

Setting the arbitrary criterion that the statistical error should be less than one-tenth of the effect to be measured, the total number of incident particles required is

$$I_0 T > 100/(s_2 s_1 r_2^2 r_1^2). \tag{45}$$

The required values of I_0T have been estimated from Eq. (45) for several experiments on the assumption that the scattering thickness and the solid angle of the scattering are determined from the following considerations: (1) the incident energy in each scattering act must be well-defined relative to the width of the polarization maximum, (2) the final proton must be able to reach the counting apparatus, (3) within these limitations product sr^2 is to be maximized for each reaction. Order-of-magnitude results follow:

A. Polarized protons produced by $He^3(n,p)$ reaction and detected by $Li^7(p,\alpha)$ reaction. Assuming a 100 percent concentration of He^3,

$$I_0 T > 1.5 \cdot 10^{14}/C^2P^2,$$

where P is the polarization of the protons after the first reaction (Eq. (18)), and C is defined by Eq. (19b). If C^2 has a value of 0.3 in the energy interval around 0.5 Mev and P is equal to 0.5, a flux of about $5 \cdot 10^{11}$ polarized thermal neutrons per second would be required for one hour.

B. Polarized protons produced by scattering from helium and detected by $Li^7(p,\alpha)$ reaction. Assuming that the solid angle for the helium scattering is chosen symmetrically about 90°,

$$I_0 T > 2 \cdot 10^{16}/(r_{90^\circ})^2 C^2,$$

where (r_{90°) is the polarization for 90° scattering from helium. If (r_{90°) has a value of 0.7, and C has a value of 1 in the energy interval about 1.5 Mev, 2.0 microampere-hour of protons around 3.0 Mev is required.

C. Double scattering from helium. This is particularly hard to estimate because it has not been possible to determine the expected energy dependence of the polarization effect. If a resonance actually exists, it appears to be as broad as the proton energy loss caused by 90° scattering. It therefore seems possible that the initial scattering could be done at the high energy side of a polarization maximum and the second scattering at the low energy side of the same maximum. Another possibility is that suggested by Schwinger for the case of neutrons; that is, using one polarization maximum for the first scattering and a second maximum (see Figs. 2 and 3) for the second scattering. In either case one has very roughly

$$I_0 T > 2 \cdot 10^{13}/(r_{90^\circ})_1^2 (r_{90^\circ})_2^2.$$

This gives about 0.005 microampere-hour of 3.0- to 4.0-Mev protons required.

Thus neither of the last two possibilities can be ruled out solely on the basis of the fundamental limitations considered here. On this basis the double scattering experiment from helium appears the most practical. Both of the last two are made more difficult by the fact that the energy at which the polarization effect occurs for each of the two reactions must be determined experimentally. It must be emphasized that important practical limitations have not been considered here; in particular, limitations on the usable solid angle in each reaction imposed by the experimental geometry and the background count. Consequently it cannot be concluded whether any of the possibilities considered here is practical at this time.

I wish to express my thanks to Professor Edward Teller for the suggestion of this problem and for many helpful discussions concerning it. I also wish to express my appreciation of a DuPont fellowship in physics, which I held while carrying out this research.