one to expect that the inexact numerical factor of the term $(ka)^{4N}$ will be in close accord with the correct value.

[1] C. J. Bouwkamp, this issue.
[2] H. Levine and J. Schwinger, Phys. Rev. **74**, 958 (1948).

Two-Component Wave Equations

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RECENTLY some interest has come up in two-component wave equations. As is well known, those wave equations, in contrast to the four-component Dirac equation, can only be made covariant with respect to the Lorentz group, not including covariance with respect to reflections.

The following antilinear two-component equation presents some interest,

$$\gamma^\kappa(\partial/\partial x^\kappa - i\varphi_\kappa)\psi = \mu\psi^*, \tag{1}$$

(ψ^* = conjugate complex of ψ), where

$$\tfrac{1}{2}(\gamma^{\kappa *}\gamma^\lambda + \gamma^{\lambda *}\gamma^\kappa) = -g^{\kappa\lambda}1, \tag{2}$$

with $g^{00} = +1$, $g^{11} = g^{22} = g^{33} = -1$. In the case $\varphi_\kappa = 0$, we obtain by iteration the ordinary linear second-order wave equation. Equation (2) can be satisfied with

$$\gamma^0 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3}$$

With the Bargmann operator

$$\alpha = -\tilde{\alpha} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{4}$$

we can form a real current vector

$$s^\kappa = \psi^\dagger \alpha \gamma^\kappa \psi \tag{5}$$

($\psi^\dagger$ = Hermitian conjugate).

Let us denote a Lorentz transformation by

$$\partial/\partial x^{\lambda'} = a_\lambda{}^\kappa \partial/\partial x^\kappa, \quad s^\lambda = a_\kappa{}^\lambda s^{\kappa'}. \tag{6}$$

Lorentz covariance of Eqs. (1) and (5) can be formulated in the following way. Let us take α and γ^κ as a *fixed* set of matrices (3) and (4) and transform the other quantities so that (1) and (5) go over into

$$\gamma^\kappa(\partial/\partial x^{\kappa'} - i\varphi_\kappa')\psi' = \mu\psi^{*\prime}, \tag{7}$$

$$s^{\kappa'} = \psi^{\dagger\prime}\alpha\gamma^\kappa\psi'. \tag{8}$$

If ψ transforms under the Lorentz transformation (6) as

$$\psi' = S^{-1}\psi, \tag{9}$$

the covariance of Eqs. (1) and (7) demands

$$\gamma^\kappa a_\kappa{}^\nu = S^{*-1}\gamma^\nu S, \tag{10}$$

and the covariance of (5) and (8) requires

$$\alpha = S^\dagger \alpha S^*, \tag{11}$$

Eq. (11) implies S to be an unimodular, i.e., $|S| = 1$. Therefore, the matrix S contains only 3 complex parameters which is sufficient to satisfy Eq. (10) with 6 real parameters $a_\kappa{}^\nu$ of the Lorentz group.

A transformation $S^{*-1}\gamma^\nu S$ with any matrix S, applied to the γ^ν will preserve the relations (2), i.e., any set of matrices differing from Eq. (3) by an arbitrary S transformation will lead to nothing new.

The set of matrices

$$\gamma^0 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{12}$$

however, is another solution of (2), not differing from (2) by an S transformation. If Eqs. (1) and (3) represent the motion of a positive charge in the potential φ_κ, then Eqs. (1) and (12) represent the motion of a negative charge in the same potential φ_κ, which is readily seen by forming the conjugate complex of Eqs. (1) and (12) and comparing it with Eqs. (1) and (3).

The wave equation (1) is covariant with respect to gauge transformations $\varphi_\kappa'' = \varphi_\kappa + \partial\Lambda/\partial x^\kappa$,

$$\text{with} \quad \left.\begin{aligned} &\psi'' = S^{-1}\psi, \quad \gamma^{\kappa''} = S^{*-1}\gamma^\kappa S, \quad \alpha'' = S^\dagger\alpha S^* \\ &S = 1\exp(-i\Lambda), \end{aligned}\right\} \tag{13}$$

i.e., a not unimodular S. It is readily seen that $s^{\kappa''} = s^\kappa$.

The charge conservation law

$$\partial s^\kappa/\partial x^\kappa = 0 \tag{14}$$

follows from Eq. (5) by differentiation, observing that the left-hand side is, as s^κ, a real quantity and the right-hand side, by virtue of Eq. (1), purely imaginary.

We are investigating solutions of the *set* of Eqs. (1) with any matrices γ^ν satisfying Eq. (2). If ψ_1 and ψ_2 are two solutions of Eqs. (1) and (2), we can superimpose them with arbitrary real coefficients provided we have first adjusted their relative phases by a transformation (13) to the same gauge, i.e., to the same matrices γ^ν. In this theory, therefore, the phase relations between superimposable ψ-functions are fixed.

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Neutron Diffraction by Gases

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BY use of a modified crystal spectrometer at the heavy water pile to select neutrons of 0.07-ev energy, the intensity of neutrons scattered from gases was measured in the angular range 5° to 90°. The gases studied, oxygen and carbon dioxide, were contained in a steel vessel at room temperature and approximately 60 atmospheres pressure. A schematic diagram of the experimental arrangement is shown in Fig. 1.

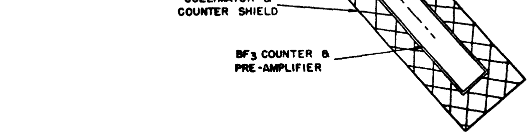

FIG. 1. Schematic diagram of neutron crystal spectrometer as modified for gas scattering experiments.

Since the scattering was not very intense, precautions were taken to reduce the background of fast neutrons penetrating the counter shield and slow neutrons scattered from the steel walls of the gas cell. The latter would be serious if the steel were not crystalline with a powder diffraction pattern. As there is no line of this pattern inside an angle of 30°, at small

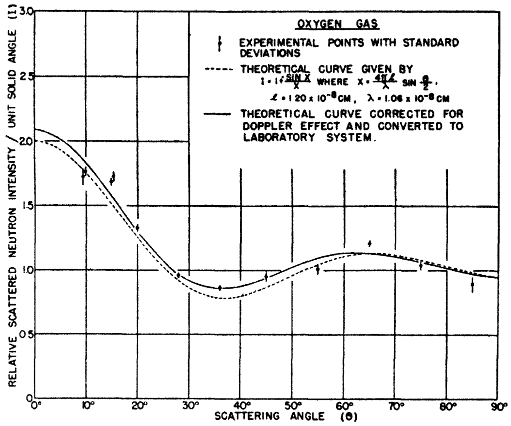

FIG. 2. Angular distribution of 0.07-ev neutrons scattered from oxygen gas.

angles the counter may be exposed to steel which is in the neutron beam without receiving too large a background. For angles above 30° baffles were arranged to prevent neutrons singly scattered from the steel from reaching the counter. Despite these precautions, at most angles background counts and true counts were approximately equal. The background from doubly scattered neutrons is appreciable ($\sim$10 percent), and an approximately calculated correction has been applied. No attempts were made to reduce it in the work here reported, but for work on other gases a system of baffles inside the gas cell has been planned.

In Figs. 2 and 3 are plotted the experimental results together with simple theoretical curves for both gases. (The vertical scale has been arbitrarily adjusted to fit the curves to the experimental points.) It will be seen that there is evidence of a pattern due to the form of the molecule.

The quantum-mechanically correct method of calculating the scattering requires a separate calculation for each possible rotational transition of the molecule (inelastic, elastic, and superelastic), and the summation of intensities after allowing for Doppler effect and the motion of the center of mass.[1] For a heavy molecule this requires a very large computational effort and has not been attempted. The curves drawn in Figs. 2 and 3 were obtained by a semiclassical calculation in which the neutron is represented by a wave, but the molecule is replaced by a rigid system of point scatterers. The method assumes elastic scattering only and is the same as the normal procedure for x-ray scattering,[2] with the atomic structure factors replaced by constants. No account has been taken of paramagnetism in the oxygen molecule,[3] but small corrections have been applied for Doppler effect and conversion to the laboratory system of coordinates. The good agreement with experiment in the case of oxygen leads one to expect that the exact formulas condense, to a close approximation, to the semiclassical result.[4] Of interest in this connection is a forthcoming letter by J. A. Spiers on the scattering of slow neutrons by deuterium gas.

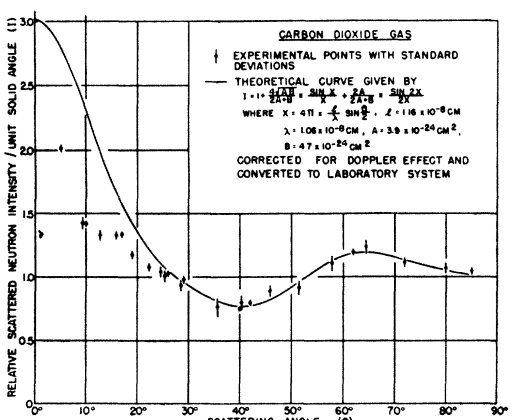

FIG. 3. Angular distribution of 0.07-ev neutrons scattered from carbon dioxide gas.

Carbon dioxide, unlike oxygen, shows a marked deficiency in intensity at small angles. A possible cause of this is intermolecular interference, which in a liquid causes the scattering to be small for angles between zero and the first diffraction ring. The carbon dioxide in our experiment is near the liquid phase, and, as shown by Eisenstein and Gingrich[5] for x-ray scattering from argon, the vapor pattern may exhibit features characteristic of the liquid. If our conditions are compared with theirs, by the Law of Corresponding States, it is found that the deficiency in scattering is in good agreement but that the carbon dioxide shows no excess scattering corresponding to the liquid peak found for argon.

The study of gas scattering was suggested by Dr. B. W. Sargent. The spectrometer was the work of Mr. A. J. Pressesky, Mr. P. R. Tunnicliffe, and one of us (DGH).

[1] J. Schwinger and E. Teller, Phys. Rev. **52**, 286 (1937); M. Hamermesh and J. Schwinger, Phys. Rev. **69**, 145 (1946).
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[3] O. Halpern, Phys. Rev. **72**, 746 (1947).
[4] See E. Fermi and L. Marshall, Phys. Rev. **71**, 666 (1947), Section 4.
[5] A. Eisenstein and N. S. Gingrich, Phys. Rev. **62**, 261 (1942).

Inelastic Scattering of 14.5-Mev Neutrons by Lead*

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ACCORDING to the statistical theory of nuclei[1] half the collisions of neutrons of wave-length short compared to the nuclear radius will result in inelastic scattering or absorption. The inelastically scattered neutrons are expected to have an approximately Maxwellian distribution in energy corresponding to the temperature of the residual nucleus.[2]

Previous investigations of the inelastic scattering by heavy nuclei of neutrons of energy above 3 Mev have been carried out by Grahame and Seaborg,[3] Soltan,[4] and Amaldi *et al.*[5] The results indicate reasonable agreement with theory for the magnitude of the cross section for inelastic scattering and absorption, but Grahame and Seaborg find for a Ra-Be source almost no inelastically scattered neutrons between the energies of 3 and 7 Mev, and Soltan finds no inelastically scattered neutrons between 3 and 11 Mev for Li+D neutrons.

The availability of 14.5-Mev neutrons with an anergy spread of only ± 0.5 Mev permits measurements of inelastic scattering in which the previous difficulties caused by the distribution in energy of the primary neutrons are not present. 14.5-Mev neutrons were obtained by bombarding a thick tritium gas target with deuterons from the Los Alamos electrostatic generator.

A lead sphere of radius about one-half mean free path for energy loss by the neutrons (4 cm) was constructed with a small central spherical cavity in which threshold detectors could be placed. In such a spherical geometry the effects of elastic scattering do not need to be considered since just as many neutrons are elastically scattered into the detector as are elastically scattered out, if the radius of the sphere is small compared with the distance from the source (40 cm). The threshold detectors were prepared in the form of long, thin foils which were rolled up tight to approximate a pseudosphere when placed in the cavity and which could be unrolled and wrapped helically around a Geiger tube.

A threshold detector should have, ideally, zero detection