Relativistic Terms in the Interaction between Nucleons in Pseudoscalar and Vector Meson Theory

L. VAN HOVE
*Institute for Theoretical Physics, Copenhagen, Denmark**
(Received January 3, 1949)

The second-order interaction between nucleons in the pseudoscalar and vector meson theory is calculated, all terms being taken into account. It is shown that in both cases the exact expression for the interaction caused by virtual exchange of mesons involves contact interaction terms which cancel the well-known direct coupling terms of the Hamiltonian. The remaining interaction is essentially different from the expressions given by usual derivations. Its singularity for small separations r of the nucleons, usually obtained to be in r^{-3} for pseudoscalar and vector fields, is reduced for the former and not for the latter, so that no singularity can be removed by a mixture of the mentioned fields. For pseudoscalar mesons, all interaction terms are proportional to some power of the nucleon velocity, whereas for vector mesons there is a non-vanishing static approximation in r^{-1}. It seems impossible from the present treatment to draw precise and reliable conclusions about the deuteron.

I. INTRODUCTION

THE forces between nucleons resulting from meson fields are usually calculated either by neglecting all velocity-dependent variables of the nuclear particles (static interaction),[1] or by neglecting only recoil energies of the nucleons in emission and absorption of virtual mesons.[2] In the latter case the perturbation method is used and the first non-vanishing part of the interaction is of second order in the coupling constants. Our purpose is the investigation of the second-order interaction, all terms being taken into account. We treat here the cases of pseudoscalar and vector meson fields; for both types of mesons the consideration of all terms makes it possible to separate from the interaction caused by virtual exchange of mesons expressions proportional to $\delta(\mathbf{r})$, where $\mathbf{r}$ is the relative position vector of two nucleons. Such expressions are usually called contact interaction terms. When the Lagrange function describing interacting nucleon and meson fields contains only coupling terms between nucleons and mesons and no term of direct coupling between nucleons, it is a well-known fact that the derived Hamiltonian contains direct coupling terms. These give rise to contact interactions between nucleons, and, as we shall see, cancel exactly the contact interaction terms mentioned above. The remaining second-order interaction turns out to be essentially different from the usual expressions. Its singularity for small values of the nucleon separation r, which was found to be in r^{-3} for both pseudoscalar and vector fields, is reduced for the former but not for the latter. Hence no singularity is removed in the relativistic region by mixing both fields, as was done for the static approximation by Møller and Rosenfeld.[1]

Our treatment of the second-order interaction is completely independent of the neutral or charged character of the mesons and of the corresponding charge dependence of nuclear forces. For the sake of simplicity we shall consider neutral mesons, and consequently the meson field will be described by real wave functions. The isotopic spin of the nucleons will play no part, and we may consider nucleons of definite charge which, we assume, obey the Dirac equation.

II. PSEUDOSCALAR MESON FIELD. EMISSION AND ABSORPTION MATRIX ELEMENTS

The Lagrange function of interacting nucleon and pseudoscalar meson fields is the sum of two terms for the free particles and a coupling term in which we take both types of coupling, pseudoscalar and pseudovector:

$$L = L_{\text{free nucl.}} + L_{\text{free mes.}} + L_1, \tag{1}$$

$$L_{\text{free nucl.}} = -(1/2i)\int[\psi^*(\boldsymbol{\alpha}\cdot\text{grad}\psi) - (\text{grad}\psi^*\cdot\boldsymbol{\alpha})\psi + \psi^*(\partial\psi/\partial t) - (\partial\psi^*/\partial t)\psi + 2iM\psi^*\beta\psi]d_3\mathbf{x}, \tag{2}$$

$$L_{\text{free mes.}} = -\tfrac{1}{2}\int[(\text{grad}\,U\cdot\text{grad}\,U) - (\partial U/\partial t)^2 + \kappa^2U^2]d_3\mathbf{x}, \tag{3}$$

$$L_1 = -\int\psi^*\{f_1\rho_2U + [f_2/\kappa]\times[(\boldsymbol{\sigma}\cdot\text{grad}\,U) + \rho_1(\partial U/\partial t)]\}\psi d_3\mathbf{x}. \tag{4}$$

The pseudoscalar U and the spinor ψ are the wave functions for mesons and nucleons; κ and M denote

* This work was performed during a stay in Copenhagen while the author was on leave from the University of Brussels, Belgium.
[1] C. Møller and L. Rosenfeld, Kgl. Danske Vid. Sels. Math.-Fys. Medd. 17, 8 (1940).
[2] N. Kemmer, Proc. Roy. Soc. A166, 127 (1938).

their respective masses in natural units ($\hbar = c = 1$); α, $\beta = \rho_3$, ρ_1, ρ_2, $\sigma = \rho_1\alpha = \alpha\rho_1$ are the usual Dirac matrices; the coupling constants f_1 and f_2 have no dimension in our units; the integrals are taken over three-dimensional space.

With the variable $\Pi = \delta L/\delta(\partial U/\partial t)$ canonically conjugage to U, the Hamiltonian is

$$H = H_{\text{free nucl.}} + H_{\text{free mes.}} + H_1 + H_{\text{dir.}}, \quad (5)$$

$$H_{\text{free nucl.}} = (1/2i)\int [\psi^*(\alpha \cdot \text{grad}\psi) - (\text{grad}\psi^* \cdot \alpha)\psi + 2iM\psi^*\beta\psi] d_3\mathbf{x}, \quad (6)$$

$$H_{\text{free mes.}} = \tfrac{1}{2}\int [(\text{grad}\, U \cdot \text{grad}\, U) + \Pi^2 + \kappa^2 U^2] d_3\mathbf{x}, \quad (7)$$

$$H_1 = \int \psi^* \{f_1\rho_2 U + [f_2/\kappa] \times [(\sigma \cdot \text{grad}\, U) + \rho_1\Pi]\}\psi d_3\mathbf{x}, \quad (8)$$

$$H_{\text{dir.}} = \tfrac{1}{2}(f_2/\kappa)^2 \int (\psi^*\rho_1\psi)^2 d_3\mathbf{x}. \quad (9)$$

Apart from the meson-nucleon interaction expressed by H_1, it contains a direct coupling $H_{\text{dir.}}$ between nucleons.

We use momentum representation and take all wave functions periodical with unit period in spatial coordinates. The plane wave states of the mesons will be denoted by their momentum $\mathbf{k}$, the corresponding energy being $\epsilon_k = (\kappa^2 + k^2)^{\frac{1}{2}}$. The nucleon plane wave states are written

$$\psi_m = \chi_m \exp(i\mathbf{p}_m \cdot \mathbf{x}). \quad (10)$$

$\mathbf{p}_m$ is the momentum of state m, and χ_m a matrix of four rows and one column satisfying Dirac equation

$$(\alpha \cdot \mathbf{p}_m + \beta M)\chi_m = E_m \chi_m, \quad (11)$$

where E_m is the energy of the state.

The operator H_1 accounts for emission and absorption of a meson by a nucleon. The matrix elements for these processes are obtained in the usual way by expansion of U and ψ in plane waves and use of the production and annihilation operators for mesons and nucleons. This gives the known expressions

$$(H_1)_{n+k\leftarrow m} = (H_1)_{m\leftarrow n+k}^* = [1/(2\epsilon_k)^{\frac{1}{2}}]\chi_n^* \times [f_1\rho_2 - i(f_2/\kappa)(\sigma \cdot \mathbf{k} - \rho_1\epsilon_k)]\chi_m,$$

where the meson momentum $\mathbf{k}$ is equal to the momentum change $\mathbf{p}_m - \mathbf{p}_n$ of the emitting or absorbing nucleon. By means of $\mathbf{k} = \mathbf{p}_m - \mathbf{p}_n$, $\sigma = \alpha\rho_1 = \rho_1\alpha$, Eq. (11), and $\rho_1\beta = -\beta\rho_1 = -i\rho_2$, we have successively

$$\begin{aligned} i\chi_n^*(\sigma \cdot \mathbf{k} - \rho_1\epsilon_k)\chi_m &= i\chi_n^*\rho_1(\alpha \cdot \mathbf{p}_m)\chi_m - i\chi_n^*(\alpha \cdot \mathbf{p}_n)\rho_1\chi_m - i\epsilon_k\chi_n^*\rho_1\chi_m \\ &= i\chi_n^*\rho_1(E_m - M\beta)\chi_m - i\chi_n^*(E_n - M\beta)\rho_1\chi_m - i\epsilon_k\chi_n^*\rho_1\chi_m \\ &= i(E_m - E_n - \epsilon_k)\chi_n^*\rho_1\chi_m - 2M\chi_n^*\rho_2\chi_m, \end{aligned}$$

and the matrix elements become

$$(H_1)_{n+k\leftarrow m} = (H_1)_{m\leftarrow n+k}^* = [1/(2\epsilon_k)^{\frac{1}{2}}]\chi_n^* \times [f_3\rho_2 - i(f_2/\kappa)(E_m - E_n - \epsilon_k)\rho_1]\chi_m, \quad (12)$$

with $f_3 = f_1 + (2M/\kappa)f_2$.

III. SECOND-ORDER INTERACTION IN PSEUDOSCALAR THEORY

In the Hamiltonian (5), the term H_1 of first order in the coupling constants is carried over to higher order by the canonical transformation $\exp(iS)$, defined in momentum representation by

$$S_{r'\leftarrow r} = i(H_1)_{r'\leftarrow r}/(E_r - E_{r'}). \quad (13)$$

E_r, $E_{r'}$ are the eigenvalues of the (diagonal) matrix $H_{\text{free nucl.}} + H_{\text{free mes.}}$. Apart from terms of order three and more in f_1 and f_2, the new Hamiltonian is

$$\exp(iS)H\exp(-iS) = H_{\text{free nucl.}} + H_{\text{free mes.}} + H_2 + H_{\text{dir.}}, \quad (14)$$

where H_2 is found to be

$$(H_2)_{r'\leftarrow r} = \tfrac{1}{2}\sum_{r''}[[1/(E_r - E_{r''})] + [1/(E_{r'} - E_{r''})]] \times (H_1)_{r'\leftarrow r''}(H_1)_{r''\leftarrow r}. \quad (15)$$

The matrix elements $(H_1)_{r''\leftarrow r}$ and $(H_1)_{r'\leftarrow r''}$ are given by (12), and the energy difference multiplying ρ_1 in this expression becomes equal (apart possibly from the sign) to the denominators $E_r - E_{r''}$ and $E_{r'} - E_{r''}$ of (15), respectively. The formal simplification thus introduced results in the appearance in H_2 of a contact interaction term. To see it, let us consider the simple case of a two-nucleon system with center of gravity at rest, and write down the matrix element of H_2 for an arbitrary transition $\Phi \rightarrow \Psi$ of the system. We expand Φ and Ψ in plane wave states (10):

$$\Phi = (1/\sqrt{2})\sum_{m_1, m_2} \Phi_{m_1 m_2}\psi_{m_1}^{(1)}\psi_{m_2}^{(2)},$$
$$\Psi = (1/\sqrt{2})\sum_{n_1, n_2} \Psi_{n_1 n_2}\psi_{n_1}^{(1)}\psi_{n_2}^{(2)}, \quad (16)$$

where the upper suffixes refer to the two particles.

The non-vanishing terms of the sums (16) satisfy

$$\Phi_{m_1m_2} = -\Phi_{m_2m_1}, \quad \Psi_{n_1n_2} = -\Psi_{n_2n_1},$$
$$\mathbf{p}_{m_1}+\mathbf{p}_{m_2}=\mathbf{p}_{n_1}+\mathbf{p}_{n_2}=E_{m_1}-E_{m_2}=E_{n_1}-E_{n_2}=0. \quad (17)$$

Introducing (12) into (15) and using (17) one gets, after some calculation,

$$(H_2)_{\Psi\leftarrow\Phi} = -(f_2/\kappa)^2 \sum \Psi_{n_1n_2}{}^*\chi_{n_1}{}^{(1)*}\chi_{n_2}{}^{(2)*} \times \rho_1{}^{(1)}\rho_1{}^{(2)}\chi_{m_1}{}^{(1)}\chi_{m_2}{}^{(2)}\Phi_{m_1m_2} + (W_1+W_2)_{\Psi\leftarrow\Phi}, \quad (18)$$

$$(W_1)_{\Psi\leftarrow\Phi} = -f_3{}^2 \sum \Psi_{n_1n_2}{}^*\chi_{n_1}{}^{(1)*}\chi_{n_2}{}^{(2)*} \times [\rho_2{}^{(1)}\rho_2{}^{(2)}/[\epsilon_k{}^2-(E_{m_1}-E_{n_1})^2]] \times \chi_{m_1}{}^{(1)}\chi_{m_2}{}^{(2)}\Phi_{m_1m_2}, \quad (19)$$

$$(W_2)_{\Psi\leftarrow\Phi} = i(f_2f_3/\kappa) \sum \Psi_{n_1n_2}{}^*\chi_{n_1}{}^{(1)*}\chi_{n_2}{}^{(2)*} \times [(E_{m_1}-E_{n_1})/[\epsilon_k{}^2-(E_{m_1}-E_{n_1})^2]] \times (\rho_1{}^{(1)}\rho_2{}^{(2)}+\rho_2{}^{(1)}\rho_1{}^{(2)})\chi_{m_1}{}^{(1)}\chi_{m_2}{}^{(2)}\Phi_{m_1m_2} \quad (20)$$

with $\mathbf{k}=\mathbf{p}_{m_1}-\mathbf{p}_{n_1}$. The first term of $(H_2)_{\Psi\leftarrow\Phi}$ in (18) does not depend explicitly on $\mathbf{p}_{m_1}$ and $\mathbf{p}_{n_1}$. Therefore, it gives an interaction energy proportional to $\delta(\mathbf{r})$, $\mathbf{r}$ being the relative position vector of the two nucleons. As can be seen immediately, that term is equal to $(-H_{\text{dir.}})_{\Psi\leftarrow\Phi}$ and the complete second-order interaction is

$$W = H_2 + H_{\text{dir.}} = W_1 + W_2. \quad (21)$$

For large relative momentum p or small relative distance r, the expectation value $\langle W_1\rangle$ becomes proportional to p or r^{-1} and $\langle W_2\rangle$ becomes proportional to p^2 or r^{-2}. For any real transition of the two-nucleon system, occurring with conservation of energy, the matrix element of W_2 vanishes. In particular, it has no diagonal elements in momentum representation and hence can be carried over to fourth order by canonical transformation. One will also observe that, according to (19) and (20), the second-order interaction vanishes for $f_3=f_1+(2M/\kappa)f_2=0$. A slightly longer calculation shows that all these features are still valid in an arbitrary system of reference, where the center of gravity of the particles is not at rest.

IV. AN ALTERNATIVE METHOD

Dyson[3] has shown that the pseudovector coupling term in (8) can be transformed into a pseudoscalar coupling term by a canonical transformation. This transformation $\exp(iS')$ is defined, in momentum representation, by a formula similar to (13):

$$S_{r'\leftarrow r}' = i(H_1')_{r'\leftarrow r}/(E_r - E_{r'}), \quad (22)$$

where H_1' differs from H_1 by the constant of pseudoscalar coupling:

$$H_1' = \int \psi^*\{-(2M/\kappa)f_2\rho_2 U + (f_2/\kappa)[(\boldsymbol{\sigma}\cdot\text{grad}\,U)+\rho_1\Pi]\}\psi d_3\mathbf{x}.$$

As a consequence of (12), the matrix elements (22) reduce to the simple form

$$(S')_{n+k\leftarrow m} = (S')_{m\leftarrow n+k}{}^* = [1/(2\epsilon_k)^{\frac{1}{2}}][f_2/\kappa]\chi_n{}^*\rho_1\chi_m.^4$$

The transformed Hamiltonian up to second-order terms is

$$\exp(iS')H\exp(-iS') = H_{\text{free nucl.}} + H_{\text{free mes.}} + H_1'' + H_2' + H_{\text{dir.}}, \quad (23)$$

where H_1'' involves pseudoscalar coupling only:

$$H_1'' = f_3\int(\psi^*\rho_2\psi)Ud_3\mathbf{x}, \quad f_3 = f_1 + (2M/\kappa)f_2,$$

and $H_2' + H_{\text{dir.}}$ gives a vanishing contribution to the second-order interaction between nucleons. Starting now with (23), the procedure of Section III gives the second-order interaction by means of canonical transformation $\exp(iS'')$ with $S''=S-S'$, S and S' being defined in (13) and (22), respectively. Instead of (21), one gets

$$W' = W_1,$$

given by (19). The discrepancy between W and W' is, of course, formally due to the difference between the transformation $\exp(iS)$ used in the preceding section and the product $\exp(iS'')\exp(iS')$ used here. In fact, the transformation V defined by

$$\exp(iS'')\exp(iS') = V\exp(iS)$$

removes the W_2 term from W and carries it over to fourth-order interaction.

One should expect the two methods to have the same physical consequences in second-order approximation; this is in agreement with the mentioned vanishing of all diagonal elements of W_2 and of its elements corresponding to real transitions. The deuteron problem will be discussed in the last section.[5]

[3] F. J. Dyson, Phys. Rev. **73**, 929 (1948). The author is greatly indebted to Dr. Luttinger who called his attention to Dyson's method and pointed out to him that it gives W_1 instead of W_1+W_2 as second-order interaction. Dyson investigates the case $f_1=0$. The generalization to $f_1\neq 0$ which we give here is quite immediate.

[4] This explains why S can be written in closed form in coordinate space as is done by Dyson.

[5] The conclusion of Dyson's treatment that pseudovector coupling has the same second-order effects as pseudoscalar coupling was previously reached by E. C. Nelson, Phys. Rev. **60**, 830 (1941), who transforms the Lagrange function (1) by means of the equations of motion deduced from it. This procedure seems not convincing to the present author, because the equations of motion deduced from the new Lagrange function are not equivalent to the previous ones.

V. VECTOR MESON FIELD

The Lagrange function of interacting nucleon and vector meson fields is again of form (1), but $L_{\text{free mes.}}$ and L_1 have to be replaced by

$$L_{\text{free mes.}}{}^v=-\tfrac{1}{2}\int\Big[(\text{curl}\mathbf{U}\cdot\text{curl}\mathbf{U})-(\{(\partial\mathbf{U}/\partial t)-\text{grad}\,U_0\}\{(\partial\mathbf{U}/\partial t)-\text{grad}\,U_0\})+\kappa^2(\mathbf{U}\cdot\mathbf{U}-U_0{}^2)\Big]d_3\mathbf{x},$$

$$L_1{}^v=-\int\psi^*\{g_1(\boldsymbol{\alpha}\cdot\mathbf{U}+U_0)+[g_2/\kappa][(\beta\boldsymbol{\sigma}\cdot\text{curl}\mathbf{U})-(\boldsymbol{\gamma}\cdot\{(\partial\mathbf{U}/\partial t)-\text{grad}\,U_0\})]\}\psi d_3\mathbf{x}.$$

The four wave functions $\mathbf{U}$, U_0 describe the mesons, g_1 and g_2 are dimensionless coupling constants, and $\boldsymbol{\gamma}$ is defined as usually: $\boldsymbol{\gamma}=-i\beta\boldsymbol{\alpha}=i\boldsymbol{\alpha}\beta$. Introduction of

$$\boldsymbol{\Pi}=\delta L/\delta(\partial\mathbf{U}/\partial t)$$

and elimination of U_0 by means of the field equations give the Hamiltonian of form (5), the three last terms being now

$$H_{\text{free mes.}}{}^v=\tfrac{1}{2}\int\Big[(\text{curl}\mathbf{U}\cdot\text{curl}\mathbf{U})+\kappa^2(\mathbf{U}\cdot\mathbf{U})+(1/\kappa^2)(\text{div}\boldsymbol{\Pi})^2+(\boldsymbol{\Pi}\cdot\boldsymbol{\Pi})\Big]d_3\mathbf{x},$$

$$H_1{}^v=\int\psi^*\{g_1(\boldsymbol{\alpha}\cdot\mathbf{U}-(1/\kappa^2)\,\text{div}\boldsymbol{\Pi})+(g_2/\kappa)[(\beta\boldsymbol{\sigma}\cdot\text{curl}\mathbf{U})-(\boldsymbol{\gamma}\cdot\mathbf{U})]\}\psi d_3\mathbf{x},$$

$$H_{\text{dir.}}{}^v=\tfrac{1}{2}(g_1/\kappa)^2\int(\psi^*\psi)^2d_3\mathbf{x}+\tfrac{1}{2}(g_2/\kappa)^2\int((\psi^*\boldsymbol{\gamma}\psi)\cdot(\psi^*\boldsymbol{\gamma}\psi))d_3\mathbf{x}.$$

Instead of transforming the matrix elements of $H_1{}^v$ by means of (11), as was done in Section II for H_1, it seems more convenient to deduce first the second-order interaction between two nucleons with center of gravity at rest. A canonical transformation defined by the formula analogous to (13) gives to the Hamiltonian the form (14), $H_{\text{free mes.}}$, $H_{\text{dir.}}$, and H_2 being replaced, respectively, by $H_{\text{free mes.}}{}^v$, $H_{\text{dir.}}{}^v$, and a matrix $H_2{}^v$ which is easily deduced.[6] For an arbitrary transition $\Phi\rightarrow\Psi$ of the two-nucleon system, using (16) and (17), one gets

$$\begin{aligned}(H_2{}^v)_{\Psi\leftarrow\Phi}=-\sum\Psi_{n_1n_2}{}^*\chi_{n_1}{}^{(1)*}\chi_{n_2}{}^{(2)*}[1/[\epsilon_k{}^2-(E_{m_1}-E_{n_1})^2]]\{g_1{}^2[(\boldsymbol{\alpha}^{(1)}\cdot\boldsymbol{\alpha}^{(2)})+(1/\kappa^2)\{k^2+(\boldsymbol{\alpha}^{(1)}\cdot\mathbf{k})(\boldsymbol{\alpha}^{(2)}\cdot\mathbf{k})\}]\\+(g_2/\kappa)^2[\epsilon_k{}^2(\boldsymbol{\gamma}^{(1)}\cdot\boldsymbol{\gamma}^{(2)})-(\boldsymbol{\gamma}^{(1)}\cdot\mathbf{k})(\boldsymbol{\gamma}^{(2)}\cdot\mathbf{k})+(\boldsymbol{\gamma}^{(1)}\cdot\boldsymbol{\gamma}^{(2)})(\boldsymbol{\alpha}^{(1)}\cdot\mathbf{k})(\boldsymbol{\alpha}^{(2)}\cdot\mathbf{k})+k^2\beta^{(1)}\beta^{(2)}]\\+i(g_1g_2/\kappa)[((\boldsymbol{\gamma}^{(1)}-\boldsymbol{\gamma}^{(2)})\cdot\mathbf{k})(1-(\boldsymbol{\alpha}^{(1)}\cdot\boldsymbol{\alpha}^{(2)}))-i\beta^{(1)}(\boldsymbol{\alpha}^{(2)}\cdot\mathbf{k})+i\beta^{(2)}(\boldsymbol{\alpha}^{(1)}\cdot\mathbf{k})]\}\chi_{m_1}{}^{(1)}\chi_{m_2}{}^{(2)}\Phi_{m_1m_2}.\end{aligned}\tag{24}$$

$\mathbf{k}$ is defined as $\mathbf{p}_{m_1}-\mathbf{p}_{n_1}$. By means of (11) and of the properties of Dirac matrices, the expression (24) can be transformed. The main points of the calculation can be found in the appendix. The final result is

$$(H_2{}^v)_{\Psi\leftarrow\Phi}=(-H_{\text{dir.}}{}^v+W_1{}^v+W_2{}^v+W_3{}^v)_{\Psi\leftarrow\Phi},\tag{25}$$

$$(W_1{}^v)_{\Psi\leftarrow\Phi}=-\sum\Psi_{n_1n_2}{}^*\chi_{n_1}{}^{(1)*}\chi_{n_2}{}^{(2)*}\frac{g_3{}^2[(\boldsymbol{\alpha}^{(1)}\cdot\boldsymbol{\alpha}^{(2)})-1-\beta^{(1)}\beta^{(2)}]+(g_1{}^2-g_2{}^2)\beta^{(1)}\beta^{(2)}}{\epsilon_k{}^2-(E_{m_1}-E_{n_1})^2}\chi_{m_1}{}^{(1)}\chi_{m_2}{}^{(2)}\Phi_{m_1m_2},\tag{26}$$

$$(W_2{}^v)_{\Psi\leftarrow\Phi}=(g_2g_3/\kappa)\sum\Psi_{n_1n_2}{}^*\chi_{n_1}{}^{(1)*}\chi_{n_2}{}^{(2)*}\frac{(E_{m_1}-E_{n_1})(\boldsymbol{\alpha}^{(1)}\cdot\boldsymbol{\alpha}^{(2)})-2(E_{m_1}+E_{n_1})}{\epsilon_k{}^2-(E_{m_1}-E_{n_1})^2}\times(\beta^{(1)}+\beta^{(2)})\chi_{m_1}{}^{(1)}\chi_{m_2}{}^{(2)}\Phi_{m_1m_2},\tag{27}$$

$$(W_3{}^v)_{\Psi\leftarrow\Phi}=-(g_2/\kappa)^2\sum\Psi_{n_1n_2}{}^*\chi_{n_1}{}^{(1)*}\chi_{n_2}{}^{(2)*}\left[1-\frac{8E_{m_1}E_{n_1}}{\epsilon_k{}^2-(E_{m_1}-E_{n_1})^2}\right]\beta^{(1)}\beta^{(2)}\chi_{m_1}{}^{(1)}\chi_{m_2}{}^{(2)}\Phi_{m_1m_2},\tag{28}$$

with $g_3=g_1+(2M/\kappa)g_2$, and the total second-order interaction is given by $W^v=W_1{}^v+W_2{}^v+W_3{}^v$.

For large relative momentum p or small relative distance r, the expectation values $\langle W_1{}^v\rangle$, $\langle W_2{}^v\rangle$, and $\langle W_3{}^v\rangle$ behave, respectively, as $p\sim r^{-1}$, $p^2\sim r^{-2}$, and $p^3\sim r^{-3}$. All three expressions have non-vanishing diagonal terms and consequently are of physical importance in second-order approximation.

VI. DISCUSSION OF THE RESULTS

Although obviously the second-order interactions W and W^v deduced in the foregoing sections differ from the usual expressions[1,2] only by some contact interaction terms and by quantities negligible in the approximations involved in the usual derivations, the expressions we have obtained look very different

[6] H. J. Bhabha, Proc. Roy. Soc. **A166**, 501 (1938).

from the usual ones. Starting with our expressions, which are exact in second order, it is possible to get the static part of the interaction between nucleons, by making from Eqs. (19) and (20) for the pseudoscalar field and (26) to (28) for the vector field:

$$E_{m_1} \sim E_{n_1} \sim M, \quad \rho_1 \sim 0, \quad \rho_2 \sim 0, \quad \boldsymbol{\alpha} \sim 0, \quad \beta \sim 1. \tag{29}$$

For the pseudoscalar field, the second-order interaction vanishes completely in this static approximation, whereas for the vector field one gets the pure Yukawa potential $e^{-\kappa r}/r$ and a contact interaction, i.e., an expression essentially different from the usual one. This shows how much the expression of the static interaction depends on the way of deriving it, although, of course, the difference, apart from some contact interaction terms, must vanish for vanishing nucleon velocities. By thinking, for example, of Kemmer's derivation of the usual static interaction,[2] one could perhaps say that it corresponds to the static approximation for infinitely heavy nucleons: $M \gg \kappa$, whereas when putting (29) in our Eqs. (19), (20), and (26) to (28), we assume a given and finite value for M/κ (this ratio comes in the coupling constants f_3 and g_3).

As regards the deuteron problem, a first question is whether one can get reliable information about the ground state of the deuteron by putting the interactions W or W^v derived above in a two-particle Dirac equation. This seems doubtful, for, as shown in Sections III and IV for the pseudoscalar case, the non-diagonal elements of second-order interaction are not uniquely defined in the treatment using canonical transformations (W can be made equal to W_1+W_2 or to W_1), and these elements strongly influence the eigenvalues of the wave equation. By instance, in the pseudoscalar case, the r^{-1} singularity of W_1 is low enough to account for the existence of a ground state,[7] whereas W_1+W_2, with its r^{-2} singularity, excludes the possibility of any ground state with finite binding energy. The procedure, so successful in electrodynamics, of putting the static interaction in the wave equation and treating non-static effects as perturbations, here also meets with difficulties: one has to choose a definite expression for the static interaction, and the latter may happen to vanish, as we have observed for the pseudoscalar case. Accordingly, for a satisfactory discussion of the deuteron problem, a new line of approach seems desirable.[8]

Nevertheless, a few general conclusions can be attained. For the pseudoscalar meson, apart from the W_2 term given in (20) and deprived of physical effects of second order, both types of coupling (in f_1 and f_2) give the same second-order interaction. This is in agreement with Nelson[5] and Dyson's[3] conclusion. For the vector meson, g_1 and g_2 couplings give essentially different interactions. For small separations r of the nucleons, the second-order interaction has a r^{-3} singularity for the vector meson, whereas for the pseudoscalar field the singularity is not higher than r^{-2} and probably in r^{-1}. This makes it hopeless to discard the inadmissible singularities in the relativistic region by mixing vector and pseudoscalar fields, as was done by Møller and Rosenfeld[1] for the static interaction as usually defined. The same conclusion was reached by Hu[9] for the approximate expressions obtained when neglecting recoil energies and disregarding contact interaction terms.

ACKNOWLEDGMENT

The author's thanks are due the Belgian "Centre Interuniversitaire de Physique Nucléaire," who awarded the grant enabling him to stay in Copenhagen. He is greatly indebted to Professor N. Bohr for his kind hospitality, and to him as well as to Professor W. Pauli, Professor C. Møller, Dr. N. Hu, Dr. A. Bohr, and Dr. J. M. Luttinger for many stimulating discussions and suggestions.

APPENDIX

The following equations connect quantities which may be replaced by each other in the curly bracket of (24). The quantities are equal only when introduced into (24). The deduction uses Eqs. (11) and (17) and relation $\mathbf{k}=\mathbf{p}_{m_1}-\mathbf{p}_{n_1}$, and runs exactly as the deduction of (12).

$$(\boldsymbol{\alpha}^{(1)}\cdot\mathbf{k})=E_{m_1}-E_{n_1}; \quad (\boldsymbol{\alpha}^{(2)}\cdot\mathbf{k})=E_{n_1}-E_{m_1};$$

$$(\boldsymbol{\gamma}^{(1)}\cdot\mathbf{k})=2iM-i(E_{m_1}+E_{n_1})\beta^{(1)};$$

$$(\boldsymbol{\gamma}^{(2)}\cdot\mathbf{k})=-2iM+i(E_{m_1}+E_{n_1})\beta^{(2)};$$

$$\begin{aligned}(\boldsymbol{\gamma}^{(1)}\cdot\boldsymbol{\gamma}^{(2)})(\boldsymbol{\alpha}^{(1)}\cdot\mathbf{k})(\boldsymbol{\alpha}^{(2)}\cdot\mathbf{k}) &= (\boldsymbol{\gamma}^{(1)}\cdot\boldsymbol{\gamma}^{(2)})[(\boldsymbol{\alpha}^{(1)}\cdot\mathbf{p}_{m_1})(\boldsymbol{\alpha}^{(2)}\cdot\mathbf{p}_{m_1})+(\boldsymbol{\alpha}^{(1)}\cdot\mathbf{p}_{n_1})(\boldsymbol{\alpha}^{(2)}\cdot\mathbf{p}_{n_1})\\ &\quad -(\boldsymbol{\alpha}^{(1)}\cdot\mathbf{p}_{m_1})(\boldsymbol{\alpha}^{(2)}\cdot\mathbf{p}_{n_1})-(\boldsymbol{\alpha}^{(1)}\cdot\mathbf{p}_{n_1})(\boldsymbol{\alpha}^{(2)}\cdot\mathbf{p}_{m_1})];\end{aligned}$$

$$(\boldsymbol{\gamma}^{(1)}\cdot\boldsymbol{\gamma}^{(2)})(\boldsymbol{\alpha}^{(1)}\cdot\mathbf{p}_{m_1})(\boldsymbol{\alpha}^{(2)}\cdot\mathbf{p}_{m_1}) = -(\boldsymbol{\gamma}^{(1)}\cdot\boldsymbol{\gamma}^{(2)})(E_{m_1}-M\beta^{(1)})(E_{m_1}-M\beta^{(2)});$$

$$\begin{aligned}(\boldsymbol{\gamma}^{(1)}\cdot\boldsymbol{\gamma}^{(2)})(\boldsymbol{\alpha}^{(1)}\cdot\mathbf{p}_{n_1})(\boldsymbol{\alpha}^{(2)}\cdot\mathbf{p}_{n_1}) &= (\boldsymbol{\alpha}^{(1)}\cdot\mathbf{p}_{n_1})(\boldsymbol{\alpha}^{(2)}\cdot\mathbf{p}_{n_1})(\boldsymbol{\gamma}^{(1)}\cdot\boldsymbol{\gamma}^{(2)})\\ &= -(E_{n_1}-M\beta^{(1)})(E_{n_1}-M\beta^{(2)})(\boldsymbol{\gamma}^{(1)}\cdot\boldsymbol{\gamma}^{(2)});\end{aligned}$$

$$\begin{aligned}-(\boldsymbol{\gamma}^{(1)}\cdot\boldsymbol{\gamma}^{(2)})&[(\boldsymbol{\alpha}^{(1)}\cdot\mathbf{p}_{m_1})(\boldsymbol{\alpha}^{(2)}\cdot\mathbf{p}_{n_1})+(\boldsymbol{\alpha}^{(1)}\cdot\mathbf{p}_{n_1})(\boldsymbol{\alpha}^{(2)}\cdot\mathbf{p}_{m_1})]\\ &= (\boldsymbol{\alpha}^{(1)}\cdot\boldsymbol{\alpha}^{(2)})[-(\boldsymbol{\alpha}^{(2)}\cdot\mathbf{p}_{n_1})(E_{m_1}-M\beta^{(1)})\\ &\qquad +(\boldsymbol{\alpha}^{(1)}\cdot\mathbf{p}_{n_1})(E_{m_1}-M\beta^{(2)})]\beta^{(1)}\beta^{(2)}\\ &= \{[-2(\boldsymbol{\alpha}^{(1)}\cdot\mathbf{p}_{n_1})+(\boldsymbol{\alpha}^{(2)}\cdot\mathbf{p}_{n_1})(\boldsymbol{\alpha}^{(1)}\cdot\boldsymbol{\alpha}^{(2)})](E_{m_1}-M\beta^{(1)})\\ &\qquad +[2(\boldsymbol{\alpha}^{(2)}\cdot\mathbf{p}_{n_1})-(\boldsymbol{\alpha}^{(1)}\cdot\mathbf{p}_{n_1})(\boldsymbol{\alpha}^{(1)}\cdot\boldsymbol{\alpha}^{(2)})](E_{m_1}-M\beta^{(2)})\}\beta^{(1)}\beta^{(2)}\\ &= \{[2(M\beta^{(1)}-E_{n_1})+(M\beta^{(2)}-E_{n_1})(\boldsymbol{\alpha}^{(1)}\cdot\boldsymbol{\alpha}^{(2)})](E_{m_1}-M\beta^{(1)})\\ &\qquad +[2(M\beta^{(2)}-E_{n_1})+(M\beta^{(1)}-E_{n_1})(\boldsymbol{\alpha}^{(1)}\cdot\boldsymbol{\alpha}^{(2)})]\\ &\qquad\qquad \times(E_{m_1}-M\beta^{(2)})\}\beta^{(1)}\beta^{(2)};\end{aligned}$$

whence

$$\begin{aligned}(\boldsymbol{\gamma}^{(1)}\cdot\boldsymbol{\gamma}^{(2)})(\boldsymbol{\alpha}^{(1)}\cdot\mathbf{k})(\boldsymbol{\alpha}^{(2)}\cdot\mathbf{k}) &= -(E_{m_1}-E_{n_1})^2(\boldsymbol{\gamma}^{(1)}\cdot\boldsymbol{\gamma}^{(2)})\\ &\quad -4E_{m_1}E_{n_1}\beta^{(1)}\beta^{(2)}+2M[(E_{n_1}-E_{m_1})(\boldsymbol{\alpha}^{(1)}\cdot\boldsymbol{\alpha}^{(2)})\\ &\quad +(E_{m_1}+E_{n_1})](\beta^{(1)}+\beta^{(2)})+4M^2[(\boldsymbol{\alpha}^{(1)}\cdot\boldsymbol{\alpha}^{(2)})-\beta^{(1)}\beta^{(2)}].\end{aligned}$$

Introducing into (24), one readily obtains Eqs. (25) to (28).

[7] This does not mean that an acceptable ground state effectively exists for the W_1 interaction.

[8] The author is indebted to Professor N. Bohr, Dr. A. Bohr, and also to Dr. J. M. Luttinger for an illuminating discussion of this rather delicate point.

[9] Ning Hu, Phys. Rev. 67, 339 (1945).