An Interpretation of the Resonance Scattering of Neutrons by Cobalt in Terms of the Breit-Wigner Equation*, †

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The neutron transmission of a Co foil (8.67 mg/cm^2) was measured for epi-thermal pile neutrons which were filtered through various thicknesses of B^{10}. The detector consisted of an annular BF_3 counter. As used for the transmission measurements this counter was set up to count only neutrons scattered by a second Co foil (9.55 mg/cm^2). The counter sensitivity as a function of neutron energy was studied. Effective total cross sections of 3100 b and over were calculated from observed transmissions. By means of B^{10} absorption, a resonance was located at 108±10 ev and this was associated with resonance scattering of the neutrons. The total resonance cross section σ_0 was estimated to be 12,500 ±1250 b and the magnitude of the resonance scattering, expressed by $\gamma\cdot\sigma_s(E_r)$, was estimated to be ~45,000 ev-b, here γ denotes the total level width and $\sigma_s(E_r)$ denotes the elastic scattering cross section at resonance. The neutron transmission was calculated from the single-level Breit-Wigner equation for the case where scattering is the predominant process. This equation was written in a conventional form involving, besides energy dependence, the parameters Γ, R, and j; where the neutron width is $\Gamma(E/E_r)^{\frac{1}{2}}$; where $4\pi R^2$ represents the scattering from the surface of the initial nucleus, and where j is the spin quantum number of the compound nucleus. Calculated transmissions agreed with experimental transmissions in two instances: (1) $\Gamma=2.0\pm0.1$ ev, $R=+0.93\times10^{-12}$ cm, $j=4$, and (2) $\Gamma=5.0\pm0.5$ ev, $R=+0.97\times10^{-12}$ cm, $j=3$. However, the value $\sigma_0=12{,}500$ ±1250 b is consistent only with $j=4$. A negative R will mean that the minimum of the "dispersion" curve $\sigma_s(E)$ *vs.* E occurs above the resonance energy E_r and such was found to be inconsistent with the data. Finally, mention is made of the similar scattering resonance which takes place in manganese, $E_r\approx300$ ev; here σ_0 lies between 4000 and 5000 b and $\Gamma\sim10$ ev.

I. INTRODUCTION

A STRONG scattering of neutrons by cobalt has been reported,[1] and this effect was attributed to a resonance interaction with neutrons of energies in the neighborhood of $E_r=115\pm5$ ev. The value of E_r has been determined at Columbia University by means of the modulated-cyclotron time-of-flight technique.[2]

Both the measurements of the total cross section of cobalt for neutrons of energies about 115 ev, and the measurements which demonstrate that the interaction is predominantly scattering, now have been extended. The purpose of extending these measurements was to obtain a sufficiently accurate value of the total resonance cross section to allow an estimate of the total angular momentum j of the resonance state of the compound nucleus. Moreover, it was desirable to see how well the data could be fitted to a single-level Breit-Wigner equation by adjusting the parameters Γ and R which denote the neutron width at E_r and the equivalent nuclear radius.

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† The calculations described in Section VII were carried out at Brookhaven National Laboratory under the auspices of the AEC.

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[1] S. P. Harris, A. S. Langsdorf, Jr., and F. G. P. Seidl, Phys. Rev. **72**, 866 (1947).

[2] C. S. Wu, L. J. Rainwater, and W. W. Havens, Jr., Phys. Rev. **71**, 174 (1947).

II. APPARATUS[3]

Figure 1 illustrates the experimental arrangement. The incident neutrons consisted of a collimated beam coming from a hole in the concrete shield of the Argonne heavy-water pile. Thermal neutrons were eliminated by passing the beam through a sheet of cadmium which was located near the iron shutter (see D, Fig. 1). The incident neutron beam travelled down the axis of the apparatus and through the central tube (I.D. $1\frac{7}{8}$ inches) of an annular BF_3 counter (AC, Fig. 1). When nothing was situated at the position S (see Fig. 1), the incident beam would pass through without affecting the counter. Neutrons were detected only after being scattered out of the incident beam into the neutron-sensitive region AC. The scattering was produced by a substance (e.g. a thin cobalt foil) placed at the center of the annular counter (S, Fig. 1). The neutrons most likely to be counted were those most strongly scattered out of the incident beam; consequently, the detector system, as composed of a thin cobalt scattering foil and the annular BF_3 counter plus its paraffin reflector, was especially sensitive to neutrons of resonance energy where the cobalt scattering cross section is very large.

The annular counter was constructed of 2S aluminum. Its sensitive region lay between the central tube of O.D. 2 inches and an outer tube of I.D. slightly under 4 inches. This region was enclosed by welding to each end an aluminum disk

[3] The design of the apparatus, particularly the annular counter, had been undertaken by A. S. Langsdorf, Jr. Its construction was carried out by S. P. Harris, G. E. Thomas, Jr., H. Schroeder, and the author.

having a central hole of $1\frac{7}{8}$-inch diameter which matched the hole through the inner tube and permitted an open path through the center of the counter. The collecting electrode consisted of a cylindrical grid of 2-mil Kovar wires running longitudinally, and it was supported between the inner and outer tube by an insulated aluminum frame. Within the counter all sharp edges were rounded and the surfaces were polished. The assembly was filled to 15 cm of mercury pressure with pure BF_3 of normal isotopic composition, and the counter was operated in the proportional region.

A two-inch thick paraffin neutron reflector (*P*, Fig. 1) surrounded the counter. The object of the reflector was to cause the response to be more nearly independent of neutron energy, and it also served to increase the sensitivity.

Monitoring of the incident neutron beam was accomplished by a uranium-235 fission counter placed at the region *M* in Fig. 1.

The electronic equipment was of standard design. The mechanical registers for the annular counter and the fission monitor were preceded by a scale-of-512 circuit.

The heavy borax-paraffin shield (*B*, Fig. 1) and the cadmium sheet which surrounded the paraffin reflector reduced the background due to stray neutrons. Another source of background would have been the scattering of neutrons from the incident beam by the air contained in the central tube of the counter; this was eliminated by sealing off each end of the central tube with thin aluminum foils (AL, Fig. 1) and evacuating the region. The last procedure itself reduced the total background by about a factor of ten. Moreover, the shield (*Sh*, Fig. 1) was introduced so that neutrons which were scattered through a small angle by a foil at *f* would not reach the annular counter *AC*.

The alignment of the apparatus was tested frequently during the investigation. This was accomplished by placing at *S* (Fig. 1), a graphite sector ($\sim\frac{1}{4}$ inch thick) which occupied about $\frac{1}{4}$ of the cross-sectional area of the central tube; this sector would only scatter into the annular counter neutrons travelling along that quarter of the central tube which it intercepted. By rotating the sector in quarter-revolution steps about the direction of the incident beam, and by observing the counting rate at each step, asymmetries in the alignment could be detected and suitable adjustments would be made.

III. THE ENERGY DEPENDENCE OF THE COUNTING RATE

The energy dependence of the counting rate was described by a function $\varphi(E)$, E being the incident neutron energy in ev. $\varphi(E)$ included both the energy dependence of the neutron flux and the energy variation of the sensitivity of the annular counter plus the paraffin reflector; it did not contain any energy dependence of the cross section of the scatterer. In order to determine an empirical expression for $\varphi(E)$, the cobalt scattering foil at S was replaced by a thin graphite disk, and a study was made of the scattering by graphite as the incident flux was changed in a known way. The manner in which the graphite scattering varied with the flux depended upon the form of $\varphi(E)$. Modifications of the flux were achieved by inserting various thicknesses of B^{10} in the incident beam[4] (*b*, Fig. 1).

Because of the action of the paraffin reflector, the counter response was expected to be more nearly constant than $1/v$, v being the incident neutron velocity. Therefore, the sensitivity of counter and reflector was first conjectured to behave as $1+a/E^{\frac{1}{2}}$, a being determinable by means of a best fit with the experimental data. The epi-thermal pile flux was known to vary as $1/E$; thus $\varphi(E)$, which included both the $1/E$ flux variation and the sensitivity, became

$$\varphi(E)=1/E+a/E^{\frac{3}{2}}. \qquad (1)$$

In the energy range from 0.01 ev to 1000 ev, the total B^{10} cross section has been found to be almost entirely due to capture which varies as $1/v$, and it is to be expected that the capture cross section will continue to vary as $1/v$ up to much higher energies.[5] Accordingly, the result of inserting boron in the incident beam was to multiply the intensity by the factor

$$\exp[-(0.294+37.3/E^{\frac{1}{2}})B], \qquad (2)$$

where B is the thickness of B^{10} in g/cm^2 and E is

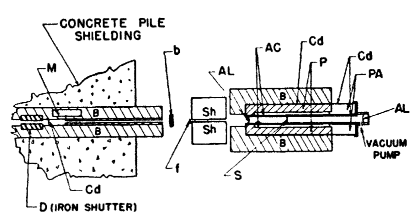

FIG. 1. Experimental arrangement. Legend:

AL—aluminum foil
S—scattering foil
D—approximate position of iron shutter
f—filter foil
Cd—cadmium sheet
b—boron—10 absorber
M—monitor
P—paraffin
AC—annular counter
PA—location of pre-amplifier
B—borax-paraffin neutron shield
Sh—"neutron shield"

[4] The B^{10} was in the form of disks made by compressing a mixture of fine-grain lead powder and crystalline boron powder which had been enriched to about 70 percent B^{10} atoms per mole of boron. The disks contained 71.3 percent by weight of lead and 28.7 percent of boron.

[5] H. H. Goldsmith, H. W. Ibser, and B. T. Feld, Rev. Mod. Phys. 19, 259 (1947). This paper contains a résumé of cross-section measurements for various elements and includes references to the original works.

TABLE I. Typical transmission data for a thin cobalt foil f.

Detector foil at S mg/cm²	Filter foil at f mg/cm²	B Thickness of B^{10} absorber g/cm²	Position of Cd filter	Counting rate per unit monitor count*
9.55 Co	none	none	in	0.2812
9.55 Co	8.67 Co	none	in	0.2316
9.55 Co	none	none	in	0.2848
9.55 Co	8.67 Co	none	in	0.2304
9.55 Co	8.67 Co	0.148	in	0.1415
9.55 Co	none	0.148	in	0.1728
9.55 Co	8.67 Co	0.148	in	0.1449
9.55 Co	none	0.148	in	0.1735
9.55 Co	8.67 Co	0.148	in	0.1440
9.55 Co	none	0.148	in	0.1741
9.55 Co	none	none	in	0.2837
9.55 Co	8.67 Co	none	in	0.2290
9.55 Co	none	0.635	in	0.0623
9.55 Co	8.67 Co	0.635	in	0.0594
9.55 Co	none	0.635	in	0.0630
9.55 Co	8.67 Co	0.635	in	0.0578
none	8.67 Co	none	in	0.0683
none	none	none	in	0.0698
none	none	0.148	in	0.0486
etc.	etc.	etc.	etc.	etc.

* Corrected for dead time according to $N_0 = N/(1-Nt)$, where N_0 is the true counting rate, N the observed rate, and t the dead time of the apparatus; in this case $t = 24$ microseconds.

expressed in ev. The quantity 0.294 pertains to the scattering by all the material in the boron disk.[6] The constant 37.3 expressing the absorption was calculated from cross-section values given in reference 5 and made use of the value 18.7 percent as the isotopic abundance of B^{10} in normally occurring boron.[7]

The effect of the cadmium was approximated by assuming the neutron flux to vanish abruptly for all energies below 0.35 ev. Above 0.35 ev the flux was supposed to be unaffected. Later (Section VII), in calculating the neutron transmission of a thin cobalt foil when $B=0$, it was necessary to consider the effect of the cadmium cut-off more carefully.

For pure graphite it is known that the neutron capture cross section is less than 0.1 percent of the scattering cross section. The latter is equal to 4.80 b and remains relatively independent of E up to about 0.1 Mev.[5] Consequently the energy dependence of the graphite scattering cross section was neglected, and the intensity scattered by a thin disk of graphite situated at S (see Fig. 1) was assumed to be proportional to

$$\exp[-0.294B]\int_{0.35\text{ ev}}^{E_M}(1/E+a/E^{\frac{3}{2}})\times\exp[-37.3B/E^{\frac{1}{2}}]dE+F(B), \quad (3)$$

where E_M is an equivalent maximum energy above which $\varphi(E)$ effectively vanishes. $F(B)$ represents a contribution in the incident beam due to the fission neutrons ($\gtrsim 1$ Mev).

A good match was obtained with experiment if

$$E_M=(10.0\pm0.5)\times10^4\text{ ev},\quad a\approx0.78,\quad F(B)\approx0. \quad (4)$$

The exact value of a was not critical, provided it was close to unity. For the above values expression (3) is plotted as a function of B in Fig. 2;†† the indicated points are experimental.

The results given in (4) show that the neutron sensitivity of the annular counter surrounded by a paraffin reflector is relatively independent of energy, especially in the region of 100 ev. Thus the response of the arrangement illustrated in Fig. 1 as a function of neutron energy must be controlled by the energy dependence of the scattering cross section of the scatterer located at S. When this scatterer is a thin cobalt foil which possesses a very large scattering cross section for neutrons of energies about 115 ev, the apparatus will be selectively responsive to the cobalt resonance-scattered neutrons.

$F(B)\approx0$ was further justified by means of certain measurements carried out at the Argonne Laboratory by G. E. Thomas, Jr. The total cross sections σ_t for lead, graphite, and beryllium showed a regular decrease with increasing B, and this effect was attributed to the fast neutrons which constituted a progressively larger fraction of the incident beam as B was increased. By subtracting intensity data obtained with larger B from those obtained with smaller B, the influence of the fast neutrons could be minimized and Thomas's results were brought into accord with other measurements.[8] The relative magnitude of $F(0)$ was estimated to be that required to cause the decrease of σ_t with B as observed by Thomas. Thus,

$$F(0)\Big/\int\varphi(E)dE\approx0.027\pm0.003. \quad (5)$$

IV. TRANSMISSION MEASUREMENTS

The neutron transmission of a thin cobalt foil (8.67 mg/cm²) located at f in Fig. 1 was measured

[6] The measurement of the scattering by a typical boron disk was made during a set of experiments previously described; cf. H. V. Lichtenberger, R. G. Nobles; G. D. Monk, H. Kubitschek, and S. M. Dancoff, Phys. Rev. **72**, 164A (1947); also the AEC declassified report AECD-1781.

[7] The isotopic composition of boron varies from sample to sample. Investigations by H. G. Thode, J. Macnamara, F. P. Lossing, and C. B. Collins, J. Am. Chem. Soc. **70**, 3008 (1948), show the ratio B^{11}/B^{10} to vary as much as from 4.27 to 4.42. The work of M. G. Inghram, Phys. Rev. **70**, 653 (1946), is in agreement, the mean result being that 18.8 percent of normally occurring boron atoms are B^{10}. The value used above, i.e., 18.7 percent, is a sort of average of these measurements.

†† Actually Eq. (3) multiplied by $\exp[-0.294B]$ is plotted in Fig. 2.

[8] For recent measurements of the cross section of lead cf. W. W. Havens, Jr., I. I. Rabi, and L. J. Rainwater, Phys. Rev. **72**, 634 (1947).

for various modifications of the incident flux. Again the energy distribution of the flux was varied by filtering the incident beam through various thicknesses of B^{10} (b, Fig. 1), and a cadmium foil was always placed in the beam to remove thermal neutrons. A second thin cobalt foil (9.55 mg/cm^2) was situated at S (Fig. 1); thus the detecting system was most sensitive to cobalt resonance-scattered neutrons.

The cobalt foils were prepared electrolytically from chemically pure materials. A spectrochemical analysis revealed that the most abundant impurities were 1 percent Fe, 0.1 percent Cu, 0.05 percent Al, and 0.05 percent Ni by weight, and other impurities appeared to be considerably less. The foils had the undesirable feature of being pitted with many small holes of diameters about 10^{-3} cm; however, it was estimated on the basis of relative areas that the error introduced into the transmission measurements because of the holes was negligible.

A small portion of data which illustrates the sequence of readings is set out in Table I.

Table II shows the results of the transmission measurements. $T(\delta, B)$ denotes the transmission of a foil of thickness δ atoms/cm^2 for epi-thermal pile neutrons which have penetrated B g/cm^2 of B^{10}. In this case δ was always the same, i.e. $8.86\times 10^{+19}$ atoms/cm^2 corresponding to 8.67 mg/cm^2. The counting rates per unit monitor count, corrected both for dead time and for background, are denoted by $I(\delta, B)$ and $I(0, B)$; the latter represents the intensity when the foil f was removed. The errors given for the $T(\delta, B)$ were calculated from the standard deviations of the means of the measurements of $I(\delta, B)$ and $I(0, B)$.

In order to minimize the effects of fast neutrons, the intensities were corrected for the scattering by the boron absorbers, and the resulting values obtained for larger B were subtracted from those obtained with smaller B. In Table III the subtractions are indicated for certain of the possible combinations; the corresponding transmissions and σ_{eff} follow. The errors stated in Table III were again based upon the standard deviations of the means of the $I(\delta, B)$ and $I(0, B)$.

V. THE RESONANCE ENERGY BY MEANS OF BORON ABSORPTION

Upon the passage of a monochromatic neutron beam through one or more of the boron disks of total B^{10} thickness equal to B g/cm^2, its intensity will be depleted by the factor (2). From the form of (2), the variation of intensity of the monochromatic beam traversing B g/cm^2 should be exponential with the absorption coefficient $\mu = 0.294 + 37.3/E_r^{\frac{1}{2}}$. Thus, if E_r is unknown, a measurement of the absorption coefficient μ will enable E_r to be calculated.

TABLE II. Neutron transmission of a thin cobalt foil f for epi-thermal pile neutrons which were filtered through various amounts of boron.

Thickness of B^{10} absorber B g/cm^2	Characteristic values of the flux distribution E_e* ev	E_m** ev	Transmission of foil f	Effective total cross section σ_{eff}† barns
0.000	—	—	0.759 ± 0.005	3100 ± 75
0.148	33	7.5	0.758 ± 0.006	3100 ± 90
0.157	36	8.4	0.759 ± 0.006	3100 ± 90
0.313	163	33.6	0.781 ± 0.007	2800 ± 100
0.635	835	138.0	0.852 ± 0.018	1800 ± 230

* A neutron of energy E_e has the probability $1/e$ of penetrating the corresponding B^{10} thickness.
** Assuming a $1/E$ epi-thermal incident neutron spectrum, E_m is the most probable energy of a neutron traversing the corresponding B^{10} thickness.
† $\sigma_{eff} = ln[1/T(\delta, B)]/\delta$.

With a thin cobalt foil at S, the apparatus of Fig. 1 would detect mostly cobalt resonance-scattered neutrons. Nevertheless, to a lesser degree it also detected neutrons of energies away from resonance; this was because the flux of neutrons about 115 ev constituted only a very small fraction of the total flux incident on the foil S, and in the off-resonance case the greater numbers of incident neutrons tended to counterbalance the much smaller cross section. Consequently, a rigorous procedure was introduced in order to obtain the effects of a strictly monochromatic beam. The manner of isolating the resonance neutrons will now be described.

Clearly a thin cobalt foil, when placed in the incident beam at f, will have little effect except on neutrons of energies for which the cobalt cross section is very large; e.g. in order that a cobalt foil of 8.67 mg/cm^2 may cause a decrease in intensity of a collimated neutron beam by 1 percent the

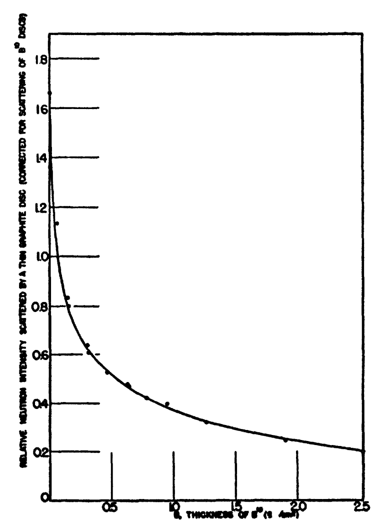

FIG. 2. Curve calculated from $\phi(E)$, the empirical expression for the energy variation of the product of flux and sensitivity. The points are experimental.

TABLE III. Neutron transmission of a thin cobalt foil f for pile neutrons which were filtered through B_1 g/cm² of B^{10} minus those filtered by B_2 g/cm² of B^{10}.

Combination of B^{10} absorbers† B_1-B_2 g/cm²	Characteristics of resulting flux Fb*	Fa**	Intensities $I(0, B_1)-I(0, B_2)$	$I(\delta, B_1)-I(\delta, B_2)$	Transmission $T(\delta)$	Effective total cross section σ_{eff} barns
0.000–0.148	0.84	0.16	0.0819±0.0009	0.06225±0.00110	0.760±0.016	3100±230
0.000–0.635	0.68	0.32	0.1772±0.0009	0.13120±0.0010	0.740±0.007	3400±100
0.148–0.313	0.49	0.51	0.0465±0.0006	0.03335±0.00080	0.717±0.020	3750±300
0.148–0.635	0.36	0.64	0.0953±0.0006	0.06895±0.00080	0.724±0.010	3660±150
0.157–0.313	0.48	0.52	0.0384±0.0006	0.02730±0.0008	0.711±0.024	3850±370
0.313–0.635	0.23	0.77	0.0488±0.0005	0.03560±0.0006	0.730±0.014	3560±220

* Fb denotes the fraction of the resulting flux distribution extending *below* the cobalt resonance at 115±5 ev.
** Fa denotes the fraction of flux *above* 115 ev.
† In all cases the neutrons were first filtered through cadmium.

average cross section per cobalt atom would have to be 114 b. Hence, if intensities measured with a thin cobalt foil (8.67 mg/cm²), located at f in Fig. 1, are subtracted from intensities measured under similar conditions but with the foil removed, there will be a mutual cancellation except for neutrons of energies in the neighborhood of resonance. Such measurements have been carried out for eight values of B with a 9.67-mg/cm² cobalt foil at S. The results are plotted in Fig. 3. Sufficient measurements were taken at each B to enable the standard deviations of the means to be calculated and these are shown in Fig. 3 as vertical lines extending above and below each point. From the slope of the semi-logarithmic plot (Fig. 3) of resonance neutron intensity *vs.* B, it was found that $E_r=108\pm10$ ev.

The scattering of the points beyond the indicated errors is ascribed to uncertainties in the values of B. However, the above value of E_r agrees with the results found at Columbia University[2] and shows that the values of B assigned to the several boron disks should not contain systematic errors greater than about 5 percent.

VI. AN ESTIMATE OF THE TOTAL RESONANCE CROSS SECTION AND THE MAGNITUDE OF THE SCATTERING RESONANCE

The results in this section depend upon an analysis into two components of the total neutron scattering by the cobalt foil at S: (1) a monochromatic or resonance component due only to neutrons of energies close to 115 ev, and (2) a component which behaves like the scattering by graphite, i.e., a potential-scattering part. Since the detecting system is especially sensitive to resonance-scattered neutrons and in light of the large values of σ_{eff} which are given in Table III, it might appear that the off-resonance scattering could be nelected in estimating σ_0 from those data taken with moderate thicknesses of boron; however, such a procedure would introduce into the estimate of σ_0 errors of 20 percent and greater (cf. the difference between $T(\delta)$ and T_0 which is discussed in the following paragraphs). Accordingly, the scattering by cobalt of epi-thermal neutrons was separated into two parts, r and p, which were distinguishable because they varied with the incident energy E in different ways.[9] The energy dependence was studied by changing B, the thickness of B^{10}, through which the incident beam was passed. r was associated with resonance neutrons and was *defined* to be that component of the total scattering which varied as $\exp[-(0.294+37.3/E_r^{\frac{1}{2}})B]$. The component of scattering which remained after subtracting r from the total was designated p.

The ratio r/p was measured in the following manner. $I(C_o, B)$ represented the average counting rate per unit monitor count when a 9.55-mg/cm² cobalt foil was placed at S, and $I(G, B)$ was the similar quantity when the cobalt at S was replaced by a graphite disk of thickness 150 mg/cm². Both these quantities were corrected for background and dead time. The experimental value for r/p was obtained from an analysis of $I(C_o, B)$ as a function of B. By definition

$$I(C_o, B)=r(B)+p(B), \tag{6}$$

and it was assumed that

$$p(B)=kI(G, B), \tag{7}$$

k being a constant. From (6) and (7)

$$r/p=r(0)/p(0)=I(C_o, 0)/kI(G, 0)-1. \tag{8}$$

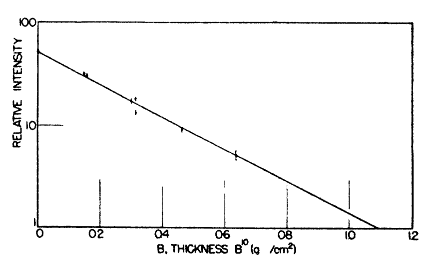

FIG. 3. Absorption in B^{10} of Co^{59} resonance neutrons (obtained by a differential experiment using a thin Co^{59} filter). From slope $E_r=108$ ev±10.

[9] The method of separating the total scattering into r and p was suggested by the somewhat similar treatment of absorption data by S. M. Dancoff; see also reference 6.

Equation (6) was written as

$$r(B)=I(C_o, B)-kI(G, B). \tag{9}$$

The factor k was determined empirically to be such a value that when the measured values of $I(C_o, B)$ and $I(G, B)$ were substituted in (9), the expression for $r(B)$ varied exponentially with B. In Fig. 4 $\ln[r(B)]$ is plotted as a function of B for $k=0.038$. The energy associated with $r(B)$ was found from the slope of the best straight line through the points in Fig. 4, and this energy came out to be 112 ev (see Section V). The agreement with other determinations of the resonance energy E_r lends justification to the above treatment. The errors indicated in Fig. 4 were based upon the standard deviations of the means of $I(C_o, B)$ and $I(G, B)$.

A sufficient number of individual measurements of $I(C_o, B)$ and $I(G, B)$ were made at each B so that the standard deviations of the mean $I(C_o, B)$ were always under 1 percent except for $B>0.5$ g/cm^2, and the standard deviations of the mean $I(G, B)$ were always less than 0.5 percent. The same values of $I(G, B)$ were used to determine $\varphi(E)$ (cf. Section III).

Using $k=0.038$ it was found that

$$r/p=2.42. \tag{10}$$

The error in k is probably not more than 5 percent.

The quantity p may be imagined to arise from an equivalent energy-independent scattering cross section σ_p which was calculated by the expression

$$k=\sigma_p\tau_{Co}/[1-e^{-(\sigma_s\tau)_G}], \tag{11}$$

which is the ratio of the scattering by $\sigma_p\tau_{Co}$ to that by the graphite disk. The subscripts "Co" and "G" respectively designate quantities pertaining to

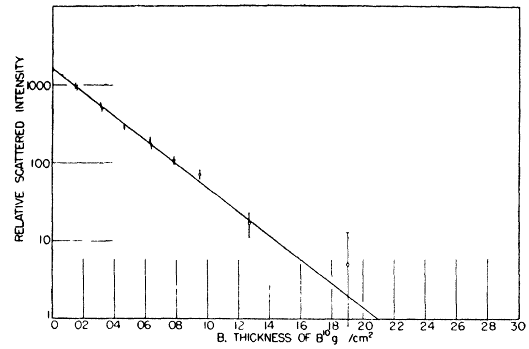

FIG. 4. Absorption in B^{10} of Co^{59} resonance neutrons (obtained by subtracting potential scattering). From slope $E_r=112$ ev.

cobalt and graphite, and τ denotes thickness in atoms/cm^2. If $k=0.038$, then $\sigma_p=13.7$ b.

The manner in which the total resonance cross section σ_0 was estimated is discussed below. The experimental transmission $T(\delta)$ was chosen for those cases where the off-resonance scattering should be least; i.e., for the two rows in Table III corresponding to $B_1-B_2=0.148-0.313$ and $0.157-0.313$. The average for the two cases is $T(\delta)=0.714\pm0.016$.

The data which are plotted in Fig. 3 show that the only resonance which has an appreciable effect upon the counting rate lies about 110 ev (see Section VII). Moreover, it was assumed that once a neutron has been scattered out of the incident beam by a thin cobalt foil at S, its probability of detection is independent of its initial energy E other than any such dependence contained in the function $\varphi(E)$. Then $T(\delta)$ was written

$$T(\delta)=\frac{P+R_0\int_{\text{res}}\sigma_s(E)e^{-\sigma_t(E)\cdot\delta}[1-e^{-\sigma_t(E)\cdot\tau}]dE/\sigma_t(E)}{P+R_0\int_{\text{res}}\sigma_s(E)[1-e^{-\sigma_t(E)\cdot\tau}]dE/\sigma_t(E)}, \tag{12}$$

where P denotes scattering of off-resonance neutrons and δ and τ respectively denote the thicknesses of foil f and foil S in number of atoms per cm^2. $\sigma_s(E)$ is the cross section for elastic scattering and $\sigma_t(E)$ is the total cross section. Here the foil f was so thin that its effect was not included in the P in the numerator of (12). Since the data had been corrected for scattering by the boron disks, it was unnecessary to consider terms of the type $\exp[-0.294B]$ and

$$P=\frac{p\int_{0.35\text{ ev}}^{10^5}(\exp[-37.3B_1/E^{1/2}]-\exp[-37.3B_2/E^{1/2}])(1/E+0.78/E^{3/2})dE}{\int_{0.35\text{ ev}}^{10^5}(1/E+0.78/E^{3/2})dE}, \tag{13}$$

$$R=R_0\int_{\text{res}}\sigma_s(E)[1-e^{-\sigma_t(E)\cdot\tau}]dE/\sigma_t(E), \tag{14}$$

$$R=r(\exp[-37.3B_1/E_r^{1/2}]-\exp[-37.3B_2/E_r^{1/2}]). \tag{15}$$

TABLE IV. Values of T_0 for several values of σ_0.

σ_0 barns	T_0 Eq. (20)
9,000	0.726
10,000	0.707
11,000	0.689
12,000	0.673
13,000	0.658

Using (10) and Eqs. (12) through (15)

$$T_0=\frac{\int_{\text{res}}\sigma_s(E)e^{-\sigma_t(E)\cdot\delta}[1-e^{-\sigma_t(E)\cdot\tau}]dE/\sigma_t(E)}{\int_{\text{res}}\sigma_s(E)[1-e^{-\sigma_t(E)\cdot\tau}]dE/\sigma_t(E)}=0.666\pm0.019. \quad (16)$$

The error stated for T_0 in (16) was calculated from the relation (17);

$$\Delta T_0/T_0=(1.569[\Delta T(\delta)/T(\delta)]^2+1.050\times10^{-2}(\Delta k/k)^2)^{\frac{1}{2}}. \quad (17)$$

It is important to note that the error in T_0 does not depend very strongly upon the error in k and depends even less on the error in B which has not been included in (17). However, the effect of the errors in values of B may be included as an addition to Δk.

By neglecting the potential scattering and the interference between potential and resonance scattering $\sigma_t(E)$ may be approximated in the resonance region by

$$\sigma_t(E)=\sigma_0/(1+\zeta^2), \quad (18)$$

where

$$\zeta=2(E-E_r)/\gamma, \quad (19)$$

E being the incident neutron energy, and γ the total level width. Thus

$$T_0=\frac{\int_{-\infty}^{+\infty}e^{-\sigma_0\delta/(1+\zeta^2)}[1-e^{-\sigma_0\tau/(1+\zeta^2)}]d\zeta}{\int_{-\infty}^{+\infty}[1-e^{-\sigma_0\tau/(1+\zeta^2)}]d\zeta}. \quad (20)$$

The integrals may now be evaluated in terms of Bessel functions of imaginary arguments;[10] e.g.

$$\int_{-\infty}^{+\infty}[1-e^{-\sigma_0\tau/(1+\zeta^2)}]d\zeta=\pi\sigma_0\tau e^{-\sigma_0\tau/2}[J_0(i\sigma_0\tau/2)-iJ_1(i\sigma_0\tau/2)]. \quad (21)$$

The values of T_0 corresponding to several values of σ_0 are set out in Table IV for the case of $\delta=0.886\times10^{20}$ atoms/cm² and $\tau=0.976\times10^{20}$ atoms/cm².

[10] This was pointed out to the author by A. S. Langsdorf, Jr.; see also M. Born, *Optik* (Verlag. Julius Springer, Berlin, 1933), Eqs. (7) and (11), p. 489.

TABLE V. Maximum possible resonance cross sections *vs.* E_r.

Max. σ_0 if $j=3$	E_r ev	Max. σ_0 if $j=4$
12,700 b	90	16,300 b
11,400	100	14,600
9900	115	12,700
9500	120	12,200
8100	140	10,500

Interpolating between the values for $\sigma_0=12{,}000$ b and 13,000 b, we find that (16) corresponds to

$$\sigma_0=12{,}500\pm1250\text{ b}. \quad (22)$$

Calculations similar to the one above also have been made for $B_1-B_2=0.000-0.148$ and $0.148-0.635$ g/cm². In these cases $\sigma_0=14{,}800\pm2000$ b and $11{,}700\pm1000$ b, respectively.

According to the Breit-Wigner equation (see Section VII) the total cross section at resonance cannot exceed $4\pi G\lambda\!\!\!^{-}_r{}^2$ where $2\pi\lambda\!\!\!^{-}_r$ is the resonance wave length of the incident neutron in the center of mass reference frame. G is a statistical factor equal to $(2j+1)/(2i+1)(2s+1)$ where j is the total angular momentum quantum number of the compound nucleus, and i and s are respectively the spin quantum numbers of the initial nucleus[11] and incident particle. In order that $\sigma_t(E_r)$ approach $4\pi G\lambda\!\!\!^{-}_r{}^2$, it is necessary for elastic scattering to be the predominate resonance process; i.e. $\sigma_t(E_r)\approx\sigma_s(E_r)$. Values of max. $\sigma_0=4\pi G\lambda\!\!\!^{-}_r{}^2$ were calculated for several values of E_r and for the two possible spins of the compound nucleus; i.e. $j=3$ and $j=4$. These results are given in Table V. It is of considerable interest to note that *within the statistical error the estimated resonance cross section σ_0 is consistent only with $j=4$.*

The magnitude of the cobalt scattering resonance was expressed as $\gamma\cdot\sigma_s(E_r)$ where γ represents the total level width. An estimate of $\gamma\cdot\sigma_s(E_r)$ was obtained from a relation between this quantity and r/p. The relation was derived by assuming that the total scattering by a thin cobalt foil could be *completely* described by two components: (1) a component r due solely to neutrons of energies very close to E_r, and (2) a component p which arises entirely from an energy-independent scattering cross section σ_p. As will be pointed out later the total scattering can only approximately be described in this way. Expressions for r and p were taken to be

$$r=S_0\int_{\text{res}}\varphi(E)\sigma_s(E)[1-e^{-\sigma_t(E)\cdot\tau}]dE/\sigma_t(E), \quad (23)$$

$$p=S_0\sigma_p\tau\int_{0.35\text{ ev}}^{10^5}\varphi(E)dE, \quad (24)$$

[11] For Co^{59} $i=7/2$, cf. J. Mattauch, *Nuclear Physics Tables* (Interscience Publishers, Inc., New York, 1946), p. 112. Co^{59} constitutes 100 percent of normally occurring cobalt.

where S_0 is a constant depending upon the absolute values of the incident neutron flux and detector sensitivity, and $\varphi(E)$ is given by (1) and (4). But it was assumed that r was due only to resonance neutrons; hence,

$$r \approx [S_0\varphi(E_r)\gamma\sigma_s(E_r)/2\sigma_0] \times \int_{-\infty}^{+\infty} [1-e^{-\sigma_0\tau/(1+\zeta^2)}]d\zeta. \quad (25)$$

From (24) and (25) it was possible to show that

$$\gamma\cdot\sigma_s(E_r) \approx \frac{1037\sigma_p}{f(\sigma_0\tau)}\cdot\frac{r}{p}. \quad (26)$$

The constant 1037 came from an evaluation of

$$[2/\pi\varphi(E_r)]\int_{0.35\text{ ev}}^{10^5} \varphi(E)dE$$

using (1) and (4) and $E_r = 115$ ev. Moreover, the definition of $f(\sigma_0\tau)$ is

$$f(\sigma_0\tau) = e^{-\sigma_0\tau/2}[J_0(i\sigma_0\tau/2) - iJ_1(i\sigma_0\tau/2)]. \quad (27)$$

The function (27) is not very sensitive to σ_0 since, if σ_0 is varied from 8000 b to 13,000 b, $f(\sigma_0\tau)$ will only change from 0.836 to 0.761. According to (10) and (22) and using $\sigma_p = 13.7$ b,

$$\gamma\cdot\sigma_s(E_r) \approx 44{,}700 \text{ ev-b}, \quad (28)$$

or

$$\gamma \sim 3.5 \text{ ev}. \quad (29)$$

In general because of the thermal motion of the target nuclei, the observed width will be increased over the intrinsic γ (i.e. Doppler broadening). The magnitude of this effect has been estimated in terms of the quantity

$$\Delta = E - \tfrac{1}{2}M_n v^2 \approx M_n v u_v, \quad (30)$$

where M_n and v denote the neutron mass and velocity, and u_v represents the component parallel to v of the velocity of the cobalt nucleus. The magnitude of u_v was obtained from

$$\tfrac{1}{2}M_A u_v^2 \approx \epsilon,$$

where, excluding the zero-point energy, ϵ denotes the average energy per degree of freedom in a cobalt crystal and M_A is the mass of the cobalt atom. ϵ was evaluated from the Debye theory of specific heats.[12]

Thus,

$$\Delta \approx 0.38 \text{ ev}. \quad (31)$$

Since, the approximate Doppler width Δ is considerably less than the value (29), the Doppler effect was neglected.

Another measurement of r/p was undertaken by a different method; i.e. the scattering of neutrons by cobalt and graphite were compared when the incident beam was and was not passed through a cadmium foil. The cobalt and graphite were located alternatively at S (see Fig. 1) and their respective thicknesses were 9.55 mg/cm^2 and 150 mg/cm^2. It was possible to estimate r/p from the relative scattering of the two substances and the result is:

$$r/p = 6.2. \quad (32)$$

Equation (32) contains a correction for the small amount by which the cadmium foil depleted the epi-thermal beam and also for the fact that the cadmium cut-off energy was not sharp. The above p corresponds to a scattering cross section $\sigma_p = 6.6 \pm 0.1$ b. According to (26), (22), and (32)

$$\gamma\cdot\sigma_s(E_r) \approx 55{,}000 \text{ ev-b}. \quad (33)$$

Neither (28) nor (33) are exact. In this regard it may be mentioned that, if the cobalt resonance scattering arises from a single level, then according to (28) and (33) the width of this level must be exceptionally large. However, when γ is so large, Eqs. (24) and (25) together cannot be an exact description of the total scattering; because either r is defined as the monochromatic component for the resonance energy E_r and some of the energy-dependent scattering will overlap into p, or p is strictly associated with an energy-independent scattering cross section and r will require a more complicated expression which involves some dependence upon B. The estimate given by (28) was obtained by rigorously defining r as a monochromatic component; while that of (33) was based upon the strict association of p with an energy-independent cross section σ_p which behaved like the graphite scattering cross section. Note that in the former case $\sigma_p = 13.7$ b and in the latter $\sigma_p = 6.6$ b.

In the case of the resonance interaction between neutrons and cobalt, the value of σ_0 given in (22) placed the spin quantum number of the compound nucleus at $j = 4$ within the statistical errors. However, difficulties have appeared in the attempt to calculate $\gamma\cdot\sigma_s(E_r)$, cf. the difference between (28) and (33), and these depreciate the method used to obtain σ_0. Consequently, the next attempt was to relate the experimentally observed transmissions $T(\delta)$ and the single-level Breit-Wigner equation more correctly than was done by (12) and (22). A description of the more exact treatment will be found in Section VII.

[12] See J. E. Mayer and M. G. Mayer, *Statistical Mechanics* (John Wiley & Sons, New York, 1941), p. 251. The Debye temperature of cobalt was taken to be $\Theta_D = 385$°K, cf. Landolt-Börnstein, *Physikalische Chemie Tabellen* (Verlag. Julius Springer, Berlin, 1931), Eg **II b**, p. 1233.

VII. FITTING OF THE BREIT-WIGNER EQUATION TO THE EXPERIMENTAL TRANSMISSIONS BY NUMERICAL INTEGRATIONS

According to the Breit-Wigner theory,[13] the energy dependence of the cross section exhibits certain peaks or resonances which are associated with energy levels of the compound nucleus. For light nuclei and small incident kinetic energies E the resonances are found to be spaced relatively far apart. In this case and when the orbital angular momentum of the incident particle relative to the initial nucleus is zero, the cross sections for the various possible processes which may occur after the absorption of a neutron of energy in the neighborhood of a resonance are expressed by the "single-level" equations given below. The equations are simplest if the incident particles are neutrons which do not have to overcome an electrostatic repulsion before entering a nucleus.

$$\sigma_s(E)=\pi G\lambda_r\Gamma\frac{\lambda_r\Gamma+4R\cdot(E-E_r)}{(E-E_r)^2+\gamma^2/4}+4\pi R^2; \quad (34)$$

$$\sigma_a(E)=\pi G\lambda_r\Gamma\frac{\lambda\gamma_a}{(E-E_r)^2+\gamma^2/4}. \quad (35)$$

$\sigma_s(E)$ denotes the neutron elastic scattering cross section. $\sigma_a(E)$ includes the neutron inelastic scattering as well as the cross section for neutron absorption with the emission of a different kind of particle or of a γ-ray. γ is the total width and very closely equals the width of the resonance peak at half maximum. Moreover,

$$\gamma=\Gamma(E/E_r)^{\frac{1}{2}}+\gamma_a, \quad (36)$$

where $\Gamma(E/E_r)^{\frac{1}{2}}$ is the neutron width, and γ_a is the partial width corresponding to processes other than elastic scattering. $2\pi\lambda$ denotes the wave length of the incident neutron; at resonance $\lambda=\lambda_r$. G is a statistical factor equal to $(2j+1)/(2i+1)(2s+1)$. The probability that the incident neutron may be reflected at the surface of the nucleus without ever combining with the latter is included in the elastic scattering by the quantity $4\pi R^2$. The contributions from more than one resonance are not contained in expressions (34) and (35).

Provided λ_r and λ are always expressed in the center-of-mass frame, the forms of Eqs. (34) and (35) will be the same regardless of whether the energies and widths are written in terms of the center-of-mass frame or the laboratory frame.[14] In what follows, the energies and widths shall be expressed in the laboratory frame and the λ shall be expressed in the center-of-mass frame; i.e. E will be the incident kinetic energy of the neutron relative to the cobalt nucleus.

According to (28) and (33) $\gamma\gtrsim 3$ ev; however, it is doubtful that γ_a can be much larger than ~ 0.1 ev. Furthermore, the estimate (22) of the total resonance cross section σ_0 shows it to be the order of $4\pi G\lambda_r^2$, and σ_0 can approach this value only if elastic scattering is the predominant resonance process. Therefore, it was written that

$$\gamma\approx\Gamma(E/E_r)^{\frac{1}{2}}\gg\gamma_a. \quad (37)$$

The transmission $T(\delta, B)$ of the thin cobalt foil f of thickness δ atoms/cm² for epi-thermal neutrons which were filtered through B g/cm² of B^{10} was written as

$$T(\delta,B)=\frac{\int_E c(E)\varphi(E)\exp[-37.3B/E^{\frac{1}{2}}]dEe^{-\sigma_t(E)\cdot\delta}\sigma_s(E)[1-e^{-\sigma_t(E)\cdot\tau}]/\sigma_t(E)}{\int_E c(E)\varphi(E)\exp[-37.3B/E^{\frac{1}{2}}]dE\sigma_s(E)[1-e^{-\sigma_t(E)\cdot\tau}]/\sigma_t(E)}, \quad (38)$$

where

$$\varphi(E)=1/E+a/E^{\frac{3}{2}}, \quad (39)$$

and

$$c(E)=\exp\{-50/[1+300(E-0.176)^2]\}. \quad (40)$$

In setting up (38) it was assumed that, once a neutron was scattered, its probability of being counted was independent of its incident energy E other than any such dependence included in the function $\varphi(E)$.

The factor $c(E)$ represented the effect of the cadmium foil which had been introduced in the incident beam to eliminate the thermal neutrons. The constants in (40) were taken from E_r and γ of the cadmium resonance.[5] However, for all cases except $B=0$ it was shown to be sufficient to set $c(E)=1$ and to use $E_m=5$ ev as the inferior limit of integration.

The object in calculating $T(\delta, B)$ was to find one or more sets of values for Γ, R, G which would

[13] G. Breit and E. P. Wigner, Phys. Rev. **49**, 519 (1936); H. Bethe and G. Placzek, Phys. Rev. **51**, 450 (1937); P. L. Kapur and R. Peierls, Proc. Roy. Soc. **A166**, 277 (1938); H. Feshbach, D. C. Peaslee and V. F. Weisskopf, Phys. Rev. **71**, 145 (1947); E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947); etc.

[14] If quantities in the $c-m$ frame are primed and quantities in the laboratory frame remain unmarked, then

$$E'=M_AE/(M_A+M_n) \quad\text{and}\quad \gamma'=M_A\gamma/(M_A+M_n),$$

M_A and M_n being respectively the masses of initial nucleus and incident neutron. For the neutron-cobalt interaction $(M_A+M_n)/M_A=1.017$.

TABLE VI. Illustrates the agreement between the experimental transmissions and those calculated from the Breit-Wigner equation using the indicated values of Γ, R and j.

Γ ev	R cm ×10^{-12}	j†	B g/cm²	$T(\delta, B)$ Calculated	$T(\delta, B)$ Experimental
2.0±0.1	+0.93	4	0.000*	0.757	0.759±0.005
			0.148	0.757	0.758±0.006
			0.157	0.758	0.759±0.006
			0.313	0.786	0.781±0.007
			0.635	0.858	0.852±0.018
1.8±0.1	−0.56	3	0.000*	0.778	0.759±0.005
			0.148	0.757	0.758±0.006
			0.635	0.820	0.852±0.018
5.0±0.5	+0.97	3	0.000*	0.753	0.759±0.005
			0.148	0.754	0.758±0.006
			0.313	0.774	0.781±0.007
			0.635	0.831	0.852±0.018

* Referring to Eq. (39), it was found to be more correct to use $a=1.00$ in these cases than $a=0.78$ as reported in Section III; this difference arose from expression (40) which describes the effect of the Cd more accurately and which was not used in the analysis described in Section III.
† j is the spin quantum number of the compound nucleus.

cause the calculated $T(\delta, B)$ as a function of B to agree with the experimental values given in Table II. Only two values of G are possible, i.e. $\frac{9}{16}$ or $\frac{7}{16}$ corresponding to the cases where the incident neutron spin is aligned parallel or anti-parallel to that of the cobalt nucleus. Furthermore, an energy-independent component of the cobalt cross section equal to 6.7±0.3 b was found from the experiments by Wu, Rainwater, and Havens[2] over the energy range from ~5 ev down to 0.1 ev. This last fact was used to limit the arbitrariness of Γ and R by assuming that $\sigma_s(E)=6.7$ b for, say, $E\sim4$ ev. The assumption that the value 6.7 b constituted the scattering cross section for $E\sim4$ ev was checked approximately by means of the apparatus shown in Fig. 1.[15]

In Eq. (38) it was desired to set $\sigma_t(E)=\sigma_s(E)$ for E in the neighborhood of E_r and to express $\sigma_s(E)$ by the single-level Eq. (34). However, at Columbia University recent measurements on the neutron transmission of cobalt as a function of E indicated the existence of one or more higher resonances in the neighborhood of 4000 ev.[16] Consequently, it was necessary to consider what effect this would have on the scattered intensity as measured by the apparatus of Fig. 1. Hence, an attempt was made to fit the intensity data of Fig. 3, which had been corrected for scattering by the boron disks, to the expression $A_1\exp[-3.55B]+A_2\exp[-0.585B]$. The absorption coefficients 3.55 and 0.585 were computed respectively for resonance energies of 115 ev and 4000 ev. The ratio A_2/A_1 was found to be -0.004 ± 0.025, where 0.025 is the standard deviation of the mean. It was then estimated from the standard deviation that the errors in the calculated transmissions which arose because of neglecting the higher resonances should be less than 1 percent. Thus, it was possible to use the one-level Eq. (34) for $\sigma_s(E)$ and the calculations were carried out accordingly.

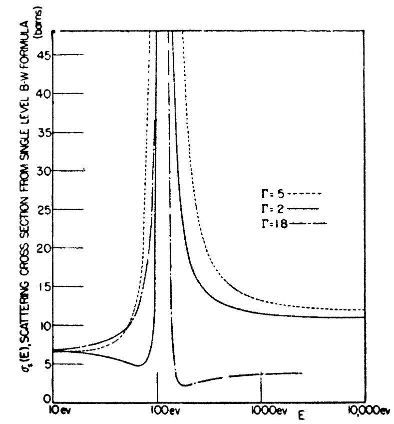

FIG. 5. Curves showing the variation with incident neutron energy of the scattering cross section calculated from the single-level Breit-Wigner formula for three pairs of values for Γ and R. The curve for Γ=1.8 ev is also for $R<0$. In the cases of Γ=2 ev and Γ=5 ev, $R>0$.

The first calculations of $T(\delta, B)$ were made for the case of $B=0.148$ g/cm² of B^{10}. After obtaining three sets of values for Γ and R which brought about a match between the calculated and experimental $T(\delta, 0.148)$, the corresponding $T(\delta, B)$ were

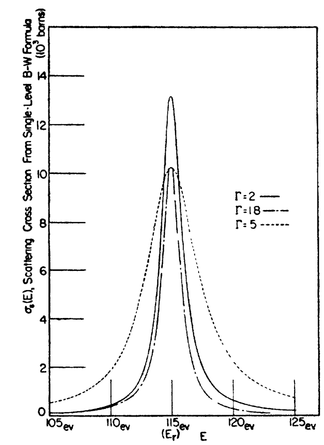

FIG. 6. The above curves are continuations of those in Fig. 5 up to the peak values of $\sigma_s(E_r)$ at $E=E_r=115$ ev.

[15] A comparison was made of the scattering of 4.8-ev neutrons by a cobalt foil situated at S (Fig. 1) to that by a graphite disk situated in place of the cobalt. Neutrons of energies near 4.8 ev were isolated by means of their differential absorption in a 1½-mil gold foil which was moved in and out of the incident beam at f, cf. isolation of cobalt resonance neutrons, Section V. The ratio of cobalt scattering to graphite scattering was denoted by k and $k=[\sigma_s(1-\exp(-\sigma_t\tau))/\sigma_t]_{Co}/[1-\exp(-\sigma_s\tau)]_G$. The best measurement on the apparatus of Fig. 1 gave $k=0.92\pm0.10$; while, as calculated from values obtained by the Columbia group and using $\sigma_s=4.80$ b for graphite, $k=0.94\pm0.05$. The isolation of 4.8-ev neutrons by differential absorption in gold is, of course, only possible because of the strong gold resonance for 4.8-ev neutrons.

[16] The author is grateful to W. W. Havens, Jr., and L. J. Rainwater for the use of their data prior to publication.

calculated for the other B. The results are given in Table VI. The "errors" quoted for the Γ are such that, if a particular Γ were to be varied by an amount outside these "errors" and if the corresponding R were adjusted so as to maintain the relation $\sigma_s(4 \text{ ev})=6.7$ b, then the resulting calculated $T(\delta, B)$ would be changed by more than the error of the experimental $T(\delta, B)$. All the numerical integrations of (38) were carefully checked and these should not contain errors greater than 1 percent. However, the effects of any higher resonances were neglected and, as has been mentioned, they could introduce into the calculated values of $T(\delta, B)$ an error of 1 percent.

Equation (34) is plotted in Figs. 5 and 6 for the three sets of values Γ, R, and j set out in Table VI.

VIII. CONCLUSION

According to Table VI the variation of $T(\delta, B)$ with B may be accounted for by a single scattering resonance in cobalt at $E_r=115$ ev. The observed transmissions are fitted best if $\Gamma=2.0\pm0.1$ ev, $R=+0.93\times10^{-12}$ cm, and $j=4$, where $\Gamma(E/E_r)^{\frac{1}{2}}$ is the neutron width, R is an equivalent nuclear radius and j is the total angular momentum quantum number of the compound nucleus. These numbers are in agreement with the value of the total resonance cross section σ_0 which was obtained from a less exact treatment; i.e. $\sigma_0=12{,}500\pm1250$ b. Nevertheless, the set $\Gamma=5.0\pm0.5$ ev, $R=+0.97\times10^{-12}$ cm, and $j=3$ cannot be entirely excluded in light of possible errors. As concerns Table VI, this last fit would be better if Γ were increased slightly, e.g. $\Gamma=5.5$ or 6 ev. The case of $\Gamma=2$ and $j=4$ corresponds to $\gamma\cdot\sigma_s(E_r)\approx26{,}000$ ev-b, and that of $\Gamma=5$ and $j=3$ to $\gamma\cdot\sigma_s(E_r)\approx50{,}000$ ev-b. The last is closer to the values of $\gamma\cdot\sigma_s(E_r)$ given by (28) and (33); but these estimates are not accurate enough to decide which value of Γ and therefore which j is more appropriate. In short the present analysis can only be taken as a strong indication that for the 115-ev resonance state in cobalt, the spin quantum number of the compound nucleus is $j=4$.

The values for negative R (i.e. $\Gamma=1.8$ ev, $R=-0.56\times10^{-12}$ cm, $j=3$) are unlikely because as is evident from Table VI the calculated $T(\delta, B)$ do not vary with B in the same manner as the experimental $T(\delta, B)$. Besides the magnitude of $\gamma\cdot\sigma_s(E_r)$ derived from this set is 18,000 ev-b which is much less than (28) and (33). Another possibility of negative R exists for $\Gamma<1$ ev and $j=4$; however, this case has not been computed for two reasons: (1) the Γ would be comparable in size to the Doppler width, see Eq. (31), and this would introduce considerable difficulties into the calculations; (2) such a small Γ is incompatible beside the large value of $\gamma\cdot\sigma_s(E_r)$, see Eqs. (28) and (33).

IX. APPENDED NOTE ON THE RESONANCE SCATTERING BY MANGANESE

Corresponding to the resonance scattering by cobalt, a similar phenomenon has been observed in manganese for neutrons of about 300 ev.[1,17–19] Since the first publications, additional data on the manganese resonance have been obtained at the Argonne National Laboratory by S. P. Harris and the author. These data were taken before the completion of the apparatus shown in Fig. 1, and at that time the neutrons were detected by means of the resonance activation of thin ($\sim$11 mg/cm^2) manganese foils. The transmission $T(\delta, B)$ of thin layers of $MnSO_4$ dissolved in D_2O was investigated as a function of B, the thickness of B^{10} in the incident beam. The measurements were made on manganese layers ranging in thickness from $\delta=0.2\times10^{+20}$ atoms per cm^2 to $1.5\times10^{+20}$ atoms per cm^2. An analysis of these transmission data indicated that for manganese σ_0 lies between 4000 b and 5000 b and that $\Gamma\sim10$ ev. Again as for cobalt, the peak cross section of this resonance appears to approach the theoretical upper limit of $4\pi G\lambda\!\!\!^{-}{}_r{}^2$.

X. ACKNOWLEDGMENT

The author wishes to thank Dr. A. S. Langsdorf, Jr., of Argonne National Laboratory for his stimulating discussions and to reiterate that the apparatus in Fig. 1 was largely designed by him. The author also expresses his gratitude to Dr. Hartland S. Snyder of Brookhaven National Laboratory for several valuable suggestions regarding the calculations of $T(\delta, B)$, and to Miss Jean H. Snover who was responsible for carrying out the numerical integrations.

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[17] M. Goldhaber and A. A. Yalow, Phys. Rev. **69**, 47A (1946); N. H. Barbre and M. Goldhaber, Phys. Rev. **71**, 141A (1947).

[18] F. G. P. Seidl, S. P. Harris, and A. S. Langsdorf, Jr., Phys. Rev. **72**, 168A (1947).

[19] For the determination of E_r for manganese, see L. J. Rainwater, W. W. Havens, Jr., C. S. Wu, and J. R. Dunning, Phys. Rev. **71**, 65 (1947).