

## The Magnetic Moment of the Proton\*

H. TAUB

*Columbia University and The City College of New York, New York*

AND

P. KUSCH

*Columbia University, New York, New York*

(Received January 20, 1949)

The molecular beam magnetic resonance method has been employed to measure, in the same magnetic field, the frequency corresponding to a reorientation of the proton in the molecule NaOH and the frequency corresponding to a transition between certain of the h.f.s. levels of the ground state of both the atoms Cs<sup>133</sup> and In<sup>115</sup>. From these data are calculated the ratio of the  $g$  factor of the proton,  $g_H$ , to the  $g$  factor of total electronic angular momentum,  $g_J$ , with the result  $g_H/g_J(\text{Cs}, {}^2S_{1/2}) = 15.1911 \times 10^{-4}$  and  $g_H/g_J(\text{In}, {}^2P_{3/2}) = 45.6877 \times 10^{-4}$ . From a knowledge of the ratios  $g_J(\text{Cs}^{133})/g_J(\text{Na}^{23})$  and  $g_J(\text{In}^{115})/g_J(\text{Na}^{23})$  two entirely independent values of the ratio  $g_H/g_J(\text{Na}^{23})$  have been obtained:

$$\begin{aligned} &\text{From indium} \\ &g_H/g_J(\text{Na}) = 15.1923 \times 10^{-4}. \\ &\text{From caesium} \\ &g_H/g_J(\text{Na}) = 15.1931 \times 10^{-4}. \end{aligned}$$

From the known value of  $g_s/g_l = 2(1.00116)$ , where  $g_s$  and  $g_l$  are, respectively, the  $g$  factors of electron spin and of orbital angular momentum, and on the basis of the assumptions that  $g_s = g_l$  and that  $g_l = 1$ , we find,

$$g_H = 30.4206 \times 10^{-4} \pm 0.005 \text{ percent.}$$

Including a small diamagnetic correction, the magnetic moment of the proton,  $\mu_H$ , is

$$\mu_H = (15.2106 \times 10^{-4} \pm 0.005 \text{ percent}) \text{ Bohr magneton.}$$

### INTRODUCTION

THIS paper describes a determination, with high precision, of the magnetic moment of the proton. The data of this experiment yield directly the ratio of the gyromagnetic ratio of the proton,  $g_H$ , to the  $g_J$  ( $g$  of total electronic angular momentum) of indium in the ground state ( ${}^2P_{3/2}$ ) and to the  $g_J$  of caesium in the ground state ( ${}^2S_{1/2}$ ). The ratios ( $g_J/g_l$ ), where  $g_l$  is the  $g$  of orbital angular momentum, are known for both indium and caesium from the experiments of Kusch and Foley<sup>1</sup> and Kusch and Taub.<sup>2</sup> Accordingly, the ratio  $g_H/g_l$  may be found, and, on the assumption that  $g_l$  is identically unity, the value  $g_H$  and the ratio  $\mu_H/\mu_0$  may be calculated, where  $\mu_H$  is the magnetic moment of the proton and  $\mu_0$  the Bohr magneton.

The magnetic moments of nuclei generally, and in particular the magnetic moments of the proton, neutron and deuteron, serve as an important guide to theoretical considerations concerning the forces between fundamental nuclear particles. Aside from the obvious importance of a precise knowledge of the proton moment as a fundamental physical constant, a value of the proton moment permits a precise determination of the neutron, deuteron and triton moments  $\mu_N$ ,  $\mu_D$  and  $\mu_T$  from the precisely

known ratios  $\mu_N/\mu_H$ ,<sup>3</sup>  $\mu_D/\mu_H$ <sup>4</sup> and  $\mu_T/\mu_H$ .<sup>5</sup> Further, the application to nuclear spectroscopy of the methods of nuclear absorption and nuclear induction have yielded precise values of the ratios of magnetic moments of a number of nuclei from which the value of the magnetic moments of the nuclei may be determined with the knowledge of the magnetic moment of the proton. In addition, a measurement of the proton moment permits a calculation of the fine structure constant,  $\alpha$ , to be made with the aid of calculations of Fermi,<sup>6</sup> Breit<sup>7</sup> and Meyerott<sup>8</sup> and from a knowledge of the h.f.s. constant of hydrogen, measured by Nafe and Nelson.<sup>9</sup>

The experiment described in this paper is, in principle, identical to an earlier experiment of Millman and Kusch<sup>10</sup> in which the value  $g_H$  was compared to the  $g_J$  values of the atoms Na<sup>23</sup>, Rb<sup>85</sup> and Cs<sup>133</sup>, all in the ground state, ( ${}^2S_{1/2}$ ). In the present experiment, by the application of improved techniques, by the more propitious selection of atoms, and from a more extensive body of data, the ratio  $\mu_H/\mu_0$  has been determined with a precision

<sup>3</sup> Bloch, Nicodemus, and Staub, Phys. Rev. **74**, 1025 (1948).

<sup>4</sup> Bloch, Levinthal, and Packard, Phys. Rev. **72**, 1125 (1947).

<sup>5</sup> Bloch, Graves, Packard, and Spence, Phys. Rev. **71**, 551 (1947).

<sup>6</sup> E. Fermi, Zeits. f. Physik **60**, 320 (1930).

<sup>7</sup> G. Breit, Phys. Rev. **35**, 1447 (1930).

<sup>8</sup> G. Breit and E. R. Meyerott, Phys. Rev. **72**, 1023 (1947).

<sup>9</sup> J. E. Nafe and E. B. Nelson, Phys. Rev. **73**, 718 (1948).

<sup>10</sup> S. Millman and P. Kusch, Phys. Rev. **72**, 1023 (1947).

\* Publication assisted by the Ernest Kempton Adams Fund for Physical Research of Columbia University.

<sup>1</sup> P. Kusch and H. M. Foley, Phys. Rev. **74**, 250 (1948).

<sup>2</sup> P. Kusch and H. Taub, Phys. Rev. **75**, 1477 (1949).

far exceeding the precision of the earlier experiment. Further, present knowledge concerning the  $g_J$  ratios of alkali atoms<sup>2</sup> avoids the error of the previous assumption that the  $g_J$  value of the ground state of all the alkalis is exactly equal to  $g_s$ . In addition, present knowledge of the spin gyromagnetic ratio of the electron in terms of the orbital gyromagnetic ratio of the electron permits a determination of the  $g$  value of the proton in terms of the fundamental quantity, the orbital gyromagnetic ratio of the electron.

A direct determination of the magnetic moment of the proton,  $\mu_H$ , in terms of absolute standards is to be preferred to the present determination which is indirect and which serves to determine the ratio  $\mu_H/\mu_0$  rather than the absolute value of  $\mu_H$ . However, a precise determination of the absolute value of  $\mu_H$  is not feasible, with currently available techniques, on account of the difficulty of making an absolute determination of the intensity of a magnetic field. The ratios of  $g$  values may, on the other hand, be determined with a precision comparable to the precision with which one can measure the frequencies of spectral lines. Nevertheless, it is to be noted that in certain calculations, such as in the calculation of the fine structure constant,  $\alpha$ , the ratio  $\mu_H/\mu_0$  is precisely the quantity which is required, and, in addition, of course, the present determination of  $\mu_H/\mu_0$  yields the absolute value of  $\mu_H$  with the same precision with which  $\mu_0$  is known.

A direct measurement of  $g_H$  in terms of the  $g_J$  of sodium or lithium would yield the most direct determination of  $g_H$ , since for these atoms  $g_J = g_s$ . However, technical reasons make such a determination extremely difficult and subject to large experimental error. The procedure employed in the present experiments, is to measure  $g_H$  in terms of the  $g_J$  of caesium and indium. The precise values of these  $g_J$  values in terms of the more fundamental quantities  $g_s$  and  $g_I$  is not known. The value of  $g_H$  in terms of  $g_s$  is found, however, by a determination of the auxiliary ratios of the  $g_J$  of indium to that of sodium and of the  $g_J$  of caesium to that of sodium. All the required ratios of  $g$  values can be measured to a high degree of precision and the net result is of considerably greater precision than could be obtained by a comparison of  $g_H$  to the  $g_J$  of sodium directly. The details of the procedure and the reasons which dictate the choice of atoms are discussed below.

#### THE RATIO $g_H/g_J$

In a sufficiently strong magnetic field the nuclear spins in a molecule will be decoupled from one another and also from the molecular rotation. In this case, if the net electronic angular momentum is zero, the energy levels due to the interaction of a nuclear magnetic moment with the applied field,  $H$ ,

will be  $W = m_I g_N \mu_0 H$ , in which  $m_I$  and  $g_N$  are the magnetic quantum number and the nuclear  $g$  factor respectively. An oscillating magnetic field perpendicular to the fixed field will induce transitions for which  $\Delta m_I = \pm 1$ . Accordingly, in a molecule which contains hydrogen, a nuclear transition due to the proton may be observed at a frequency,

$$f_H = g_H \mu_0 H / h. \quad (1)$$

The energy levels in a magnetic field of the hyperfine structure of an atom for which the quantum number of total orbital angular momentum  $J$  is equal to  $\frac{1}{2}$  are described by the Breit-Rabi<sup>11</sup> formula:

$$W_{I \pm \frac{1}{2}, m_F} = -h\Delta\nu/2(2I+1) + [h\Delta\nu/(g_J/g_I - 1)]m_F x \pm (h\Delta\nu/2)[1 + (4m_F x)/(2I+1) + x^2]^{\frac{1}{2}}, \quad (2)$$

in which  $I$  is the nuclear spin,  $\Delta\nu$  is the separation, in frequency units, at zero field, of the two levels which are characterized by the values of the total angular momentum  $F = I \pm J = I \pm \frac{1}{2}$ ,  $m_F$  is the projection in the direction of the magnetic field of the total angular momentum,  $g_J$  and  $g_I$  are respectively the Landé  $g$  factors of orbital angular momentum of the atom and of the nuclear spin, and  $x$  is defined by

$$x = g_J [1 - g_I/g_J] \mu_0 H / h\Delta\nu. \quad (3)$$

If, at a particular magnetic field, a measurement is made of the frequency of a line which results from a transition between levels described by the Breit-Rabi formula, a value of  $x$  may be calculated from Eq. (2) in terms of  $\Delta\nu$  and the ratio  $g_J/g_I$ . A determination, in the same magnetic field, of the frequency,  $f_H$ , of a proton resonance permits the determination of the ratio  $g_H/g_J$  since, from Eqs. (1) and (3),

$$g_H/g_J = f_H [1 - g_I/g_J] / x\Delta\nu. \quad (4)$$

Since the quantities  $\Delta\nu$  and  $g_J/g_I$  may be determined with no assumptions whatsoever concerning the value of  $g_J$ , the ratio  $g_H/g_J$  is likewise independent of assumptions concerning  $g_J$ .

#### GENERAL EXPERIMENTAL CONSIDERATIONS

In order that it be possible to observe a nuclear resonance in a molecular beam apparatus, it is necessary that the deflection of the molecular beam, in the inhomogeneous fields of the deflecting magnets, be at least of the order of magnitude of the widths of the slits and detector wire which define the beam. However, since the ratio of the magnetic moment of an atom to the magnetic moment of a nucleus is of the order of the ratio of the Bohr to the nuclear magneton, deflecting fields, adequate to insure sufficient deflection of molecules, will, in

<sup>11</sup> G. Breit and I. I. Rabi, Phys. Rev. **38**, 2082 (1931).

general, cause atoms to strike the pole faces of the magnets. Since a readjustment of the deflecting fields may cause an appreciable change in the value of the homogeneous transition field, an adjustment of the deflecting fields to permit in turn an examination of nuclear and atomic resonances, may introduce large systematic errors in the results of an experiment in which a constant transition field is assumed. If, however, one takes advantage of the fact that at characteristic values of the magnetic field, certain states in the hyperfine structure of atoms may have zero magnetic moment,<sup>12</sup> one may, by the appropriate choice of atom and state, use deflecting fields adequate for the molecules and not excessive for the atoms.

It is desirable that comparison of the proton resonance frequency and the atomic resonance frequency be made in as high a magnetic field as possible. For, since the width of the proton resonance is constant and independent of the field, the ratio of the uncertainty in the proton resonance frequency to the frequency itself will decrease as the frequency increases. The maximum field, however, which may be employed is dictated by the maximum field at which the calibrating atom provides a line sufficiently field sensitive to determine with precision the value of the quantity  $x$  and which originates in a state which has a zero moment.

Of the states which exhibit zero moments at a particular value of the field, not all are available for the purpose at hand, since states whose moment increases with increasing magnetic field are, in principle, non-focusing states.<sup>13</sup> That is, atoms in such states have the property, that if they deviate from the line along which they have zero moment, the induced moment in the atoms is such as to occasion forces which will cause still further deviations.

All of the lines which have been measured in this experiment result from transitions in which  $\Delta m_J = 0$  and  $\Delta m_I = \pm 1$ , first, because transitions of the type  $\Delta m_J = \pm 1$  lie for the most part in a frequency range which is neither conveniently attained nor measured, and secondly, because a considerable part of the advantage which might accrue from the high field sensitivity of the frequencies of such lines is canceled by the fact that slight deviations from homogeneity in the transition magnet cause these lines to have excessive widths.

Altogether then, the choice of atom to whose  $g_J$  value  $g_H$  is to be compared is dictated by the consideration that the atom provide the most field sensitive lines at the highest possible field, that the frequency involved shall not lie in an excessively inconvenient range, and that at least one of the

terminal states of the transition shall have a zero moment at a field adequate to insure reasonable deflections of the molecular beam. In strong fields, the energies of the h.f.s. states may be written as  $W_{m_J, m_I} = m_I g_I \mu_0 H + m_J g_J \mu_0 H + (\Delta W / I + \frac{1}{2}) m_I m_J$ , so that transitions of the type  $\Delta m_J = 0$ ,  $\Delta m_I = \pm 1$  approach asymptotically the frequencies  $\nu$  (strong field)  $= (\Delta \nu / 2I + 1) \pm g_I \mu_0 H$  as the magnetic field increases. The field sensitivity of these lines decreases, in general, with increasing field, being a maximum at zero field. The character of the field dependence of the line frequencies is determined largely by the extent to which the coupling between the magnetic moment of the nucleus and the magnetic field produced by electrons is broken down by the externally applied magnetic field. At weak fields, the energy of the states may be written  $W_{F, m_F} = W_F + m_F g_F \mu_0 H$ , so that lines corresponding to transitions  $\Delta F = 0$ ,  $\Delta m_F = \pm 1$  have frequencies given by  $\nu$  (weak field)  $= g_F \mu_0 H$ , where  $g_F$  is the  $g$  factor for a state of quantum number  $F = I \pm J$ . Since  $|g_F| \sim g_J / 2I + 1$ ,  $\nu$  (weak field)  $= g_J \mu_0 H / (2I + 1)$ . The ratio of the field sensitivity of these lines at low and high fields is then equal to  $(g_J / g_I (2I + 1))$ . At intermediate fields the field sensitivity will be intermediate between the two values. A quantity which measures the extent of the decoupling of nuclear and orbital magnetic moment is precisely the quantity  $x$  which is introduced as a dimensionless parameter in the Breit-Rabi formula. From these considerations, it can be seen that indium is an atom ideally suited for the purpose of providing highly field dependent lines at strong fields. The ground state of  $\text{In}^{115}$  is a  $^2P_{\frac{1}{2}}$  state for which the nominal value of  $g_J$  is  $\frac{3}{2}$ . Accordingly, from the definition of  $x$  it appears that a value of  $x$  sufficiently small to assure the existence of field sensitive lines will correspond to a higher absolute value of the magnetic field in the case of  $\text{In}^{115}$  in the  $^2P_{\frac{1}{2}}$  state than will be the case for atoms in the  $^2S_{\frac{1}{2}}$  state of comparable  $\Delta \nu$  for which the nominal  $g_J$  value is 2. In addition, since the  $\Delta \nu$  of  $\text{In}$  is large (greater than that of any of the alkali atoms) the field to be obtained for any  $x$  is correspondingly large. It is to be noted, however, that the maximum field at which an atomic line is suitable for use in the present application is limited not only by the fact that the field sensitivity decreases with increasing field but also by the consideration that at strong fields the sensitivity of such lines depends to a marked extent on the ratio  $g_J / g_I$ , which is, in most cases, not known with sufficient precision to permit its use as an important factor in the determination of  $x$ .

The  $^2P_{\frac{1}{2}}$  state of indium offers another advantage over  $^2S_{\frac{1}{2}}$  states in that it permits the use of large deflecting fields. For an atom whose hyperfine structure is described by the Breit-Rabi formula,

<sup>12</sup> V. W. Cohen, Phys. Rev. **46**, 713 (1934).

<sup>13</sup> I. I. Rabi, Phys. Rev. **49**, 324 (1936).

zero moments occur nominally at values of  $x$  given by  $x = -(2m_F/2I+1)$  for negative values of  $m_F$ . Since the occurrence of zero moments is determined by the quantity  $x$  rather than by the magnetic field directly, the  $^2P_{3/2}$  state provides atoms of zero moment at much higher fields than is the case with the  $^2S_{3/2}$  state. As an example, the zero moment peaks for  $\text{In}^{115}$ , in the  $^2P_{3/2}$  state,  $I=9/2$ ,  $\Delta\nu \sim 11,000 \times 10^6 \text{ sec.}^{-1}$ , occur,<sup>14</sup> for  $m = -4, -3, -2, -1$  at  $H \sim 10,000, 7500, 5000, 2500$  gauss respectively, while for  $\text{Cs}^{133}$  in the  $^2S_{3/2}$  state,  $I=7/2$ ,  $\Delta\nu \sim 9000 \times 10^6 \text{ sec.}^{-1}$ , the zero moment peaks for  $m = -3, -2, -1$  occur<sup>15</sup> at  $H \sim 2300, 1500$  and  $750$  gauss respectively.

$\text{In}^{115}$  in the  $^2P_{3/2}$  state offers, then, the best choice of atom among those available for study by current techniques for a precise determination of  $g_H$  in terms of an atomic  $g_J$ , while  $\text{Cs}^{133}$  in the  $^2S_{3/2}$  state offers a somewhat less attractive choice. We have, however, used the Cs atom as a second comparison atom, principally to ascertain from the internal consistency of the results, the general validity of the procedures employed in this experiment.

#### THE DETERMINATION FROM ATOMIC SPECTRA OF THE SIGN OF $g_J/g_I$

An error in the sign of the quantity  $g_J/g_I$  will obviously introduce a large systematic error in the determination of the ratio  $g_H/g_J$ . The sign of  $g_J/g_I$  for  $\text{Cs}^{133}$  is known to be negative from the results of two independent experiments.<sup>15,16</sup> No previous reliable determination of the sign of  $g_J/g_I$  for  $\text{In}^{115}$

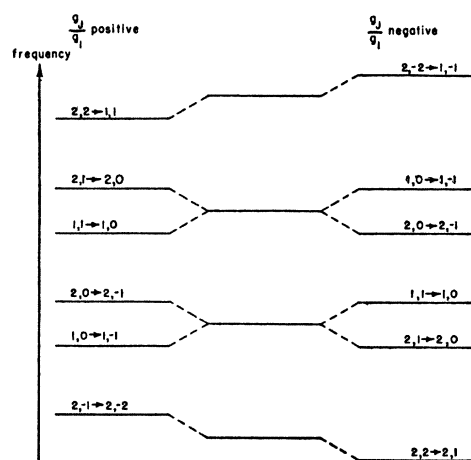


FIG. 1. The frequencies of the lines (not drawn to scale) in the h.f.s. spectrum in an intermediate magnetic field of an atom for which  $J=3/2$ , and  $I=3/2$ . The lines result from the transitions  $\Delta m_I = \pm 1$ ,  $\Delta m_J = 0$  and are labeled by the weak field quantum numbers  $F, m_F$ . In the center of the figure the doublet structure has been neglected while on the sides the complete spectrum is indicated for an assumed sign of  $g_J/g_I$ .

<sup>14</sup> Millman, Rabi, and Zacharias, Phys. Rev. **53**, 384 (1938).

<sup>15</sup> S. Millman and J. R. Zacharias, Phys. Rev. **51**, 1049 (1937).

<sup>16</sup> Kusch, Millman, and Rabi, Phys. Rev. **55**, 1176 (1939).

is available. The following considerations, however, provide a means by which the sign of  $g_J/g_I$  may be determined from an examination of the h.f.s. spectrum.

If the ratio  $g_J/g_I$  is negative, the zero field energy of the state  $F=I+J$  must be higher than the energy of the state  $F=I-J$ . In this case, the quantity  $\Delta\nu$  and the parameter  $x$ , in the Breit-Rabi formula, must both be taken as positive. If, however,  $g_J/g_I$  is positive, the zero field energy of the state  $F=I-J$  is higher than that of the state  $F=I+J$  and both  $\Delta\nu$  and  $x$  become negative.

The energy levels of an atom for which  $g_J/g_I$  is negative together with the spectrum resulting from transitions of the type  $\Delta m_J = 0$ ,  $\Delta m_I = \pm 1$  have been described in several papers<sup>17,18</sup> on h.f.s. spectroscopy by the atomic beam magnetic resonance method. All the lines are components of doublets with the exception of lines which involve the state  $F=I+J$ ,  $m_F = \pm(I+J)$ . The doublet separation arises from the term  $[\hbar\Delta\nu/(g_J/g_I-1)]m_Fx = m_Fg_I\mu_0H$  in the Breit-Rabi formula, and the components of the doublets are displaced symmetrically by an amount  $g_I\mu_0H$  from the position they would occupy if this term were neglected.

In Fig. 1 are shown schematically the lines resulting from the transitions  $\Delta m_J = 0$ ,  $\Delta m_I = \pm 1$ , in which, for convenience, the nuclear spin has been taken to be  $3/2$ . The lines in the center of the figure indicate the spectrum when the term  $g_I\mu_0H$  is ignored. The lines on the right indicate the spectrum when a negative  $g_J/g_I$  is assumed, while the lines on the left indicate the spectrum for a positive  $g_J/g_I$ . The components of the doublets have the same frequencies in both cases for a fixed  $x$  value, although the magnetic field which corresponds to the  $x$  will be somewhat different in the two cases. From observations on lines which are components of doublets, it is possible to find  $\Delta\nu$ ,  $x$  and the absolute value of  $g_J/g_I$ . The positions of the single lines may then be predicted assuming either a positive or negative sign for  $g_J/g_I$  and the observed positions of the single lines will reveal which of the assumptions is correct.

For  $g_J/g_I$  negative, the states for which  $F=I+J$  and  $m_F$  is negative will be the focusing states while for  $g_J/g_I$  positive, the states of  $F=I-J$  and  $m_F$  positive will be the focusing states. However, in both cases a  $\Delta m_I = \pm 1$  transition from a focusing state will be the lower frequency component of a doublet. If it is therefore not possible as Hardy and Millman<sup>19</sup> claim to have done, to determine the sign of  $g_J/g_I$  by noting which of the components of a doublet is more intense under an experimental condition in which the deflecting fields are adjusted

<sup>17</sup> Kusch, Millman, and Rabi, Phys. Rev. **57**, 765 (1940).

<sup>18</sup> S. Millman and P. Kusch, Phys. Rev. **58**, 438 (1940).

<sup>19</sup> T. C. Hardy and S. Millman, Phys. Rev. **61**, 459 (1942).

so that one of the states involved in the transition has zero moment.

Measurements have been made at a nominal  $x \sim 0.65$  of the frequencies, at fixed field, of the set of indium lines  $(4, -3 \leftrightarrow 4, -4)$ ,  $(4, -1 \leftrightarrow 4, -2)$ ,  $(5, -1 \leftrightarrow 5, -2)$ , and  $(5, 5 \leftrightarrow 5, 4)$ , the first three lines being components of doublets and the last being a single line. The doublet components permit a calculation of  $\Delta\nu$  independently of any assumption concerning the sign or magnitude of  $g_J/g_I$  and the relative position of the single line then permits a determination of the sign of  $g_J/g_I$ . It is found that  $g_J/g_I$  in  $\text{In}^{115}$  is indeed negative as previously stated.<sup>19</sup>

#### THE CALIBRATING ATOMS

The value of the  $\Delta\nu$  and the ratio  $g_J/g_I$  for  $\text{Cs}^{133}$  used in the calculations involved in this experiment are  $\Delta\nu = 9192.76 \pm 0.10$  mc and  $g_J/g_I = -5018 \pm 3.0$ . These values are discussed in a previous paper<sup>2</sup> and the experiment there discussed was motivated in part by the requirements of the present experiment.

A redetermination has been made of the  $\Delta\nu$  of  $\text{In}^{115}$  using the method described by Kusch and Taub.<sup>2</sup> Observations were made of the frequencies of both components of a doublet in a region of field at which the mean frequency of a doublet attains a maximum. For this purpose, the doublet  $(F, -3 \leftrightarrow F, -4)$  has been employed. The maximum frequency of the mean of this doublet occurs at an  $x = 1.4$  and has a value  $\Delta\nu\sqrt{2}/10$ . The value of the magnetic field corresponding to this value of  $x$  is about 17,000 gauss, while the line frequency is about 1600 mc. Ten observations on each of the doublet components over a range of  $x$  from 1.35 to 1.48 gives a mean  $\Delta\nu$  of 11,409.50 mc with an average deviation of 0.10 mc. The ratio  $g_J/g_I$ , on the other hand, has been determined by three independent observations of the separation of the components of two other doublets in a region of field where the line frequency is markedly field dependent. The mean frequency of the doublet gives the value of  $x$  and the doublet separation at once determines  $g_J/g_I$ . At  $x \sim 0.65$ , the doublet  $(F, 4 \leftrightarrow F, 3)$  yields  $g_J/g_I = -1005.2$  and the doublet  $(F, 3 \leftrightarrow F, 2)$  yields  $g_J/g_I = -1004.7$ ; at  $x \sim 0.6$ , the doublet  $(F, 3 \leftrightarrow F, 2)$  gives  $g_J/g_I = -1005.4$ . We take  $\Delta\nu = (11,409.50 \pm 0.10)$  mc and  $g_J/g_I = -1005.1 \pm 0.5$ ; the precision measures assigned are a conservative estimate of the possible error. It is to be noted, that if an incorrect value of  $g_J/g_I$  is assumed, the calculated  $\Delta\nu$  from measurements on the doublet components of the line  $(F, -3 \leftrightarrow F, -4)$  will exhibit an increase or a decrease with field. The ten measured values, however, show no such field dependence, but only a statistical fluctuation.

An extensive series of observations has been made on the lines of indium,  $\Delta F = 0$ ,  $\Delta m_F = \pm 1$  at values of  $x$  of 0.6 and 0.65, for the purpose of

providing an independent determination of  $\Delta\nu$ . Since the ratio  $g_J/g_I$  has been found by independent observation, it is sufficient to measure the frequencies of two lines, not components of the same doublet, to determine both  $\Delta\nu$  and  $x$ . The parameter  $x$  is determined from the ratio of the line frequencies, and it is desired, for maximum precision, that the ratio be as field dependent as possible. Evidently, certain pairs of lines will give a better precision than others. Even for the best choice of available lines, the fractional uncertainty in the calculated  $\Delta\nu$  may be as much as three times the fractional uncertainty in the observed line frequency. Evidence has appeared as a result of the analysis of the data that the measured frequencies of the most field sensitive lines, which are also the lines of highest frequency, are systematically too low. The effect is not evident from a single run, but appears as a result of statistical analysis. That is, a single run yields a value of  $\Delta\nu$  which differs from the more exactly determined value of  $\Delta\nu$  by an amount well within the uncertainty of the determinations. The  $\Delta\nu$  is, however, consistently higher than the accurately known value. Analysis of a large body of data shows that the effect depends markedly on the field dependence of the line frequency. It is believed that the effect arises from inhomogeneities in the magnetic field which cause an unsymmetrical broadening of lines. An estimate of the maximum error which this effect can introduce into determinations of  $x$  from observations made on a single line is about one part in 25,000. Such an error in  $x$  would be directly reflected in the determination of the ratio  $g_H/g_J$ . Our observations on the ratio  $g_H/g_J$  have included measurements of a number of atomic lines of In at several different magnetic fields and the field dependence of the line frequency varied by a factor of 3 over the range of observation. The systematic error resulting from the shift is, in most of these cases, certainly less than 1 part in 25,000. The high degree of internal consistency among the observations leads to the conclusion that the effect is entirely negligible compared to the over-all precision of our result.

#### APPARATUS AND PROCEDURE

The apparatus used in this experiment to determine the ratio of  $g_H$  to the  $g_J$  of indium is identically the apparatus used by Kusch and Foley<sup>1</sup> in their determination of the intrinsic magnetic moment of the electron, while the experiment on the determination of the ratio of  $g_H$  to  $g_J$  of caesium was performed after the original homogeneous transition magnet had been replaced by a new magnet. The original magnet was made of Armco iron and had tapered Permendur pole pieces, which served to increase the maximum attainable field in the gap to about 21,000 gauss. Since it was felt that

TABLE I. Results of the individual determinations of the ratio  $g_H/g_J(\text{In}^{115}, ^2P_{3/2})$  and the circumstances under which the individual determinations were made. The atomic transitions are designated by the weak field quantum numbers  $F, m_F$ .

Run	Nominal $x$	Nominal magnetic field (gauss)	$\text{In}^{115}$ transition	$g_H/g_J(\text{In}^{115}, ^2P_{3/2}) \times 10^4$ Field direction $N$	Field direction $R$
1	0.65	8000	(5, -2) (5, -3)	45.6921	45.6967
2	0.65	8000	(5, -3) (5, -4)	45.6867	45.6871
3	0.85	10,000	(5, -2) (5, -3)	45.6862	45.6913
4	0.85	10,000	(5, -1) (5, -2)	45.6870	45.6846
5	1.0	12,000	(5, -1) (5, -2)	45.6853	45.6860
6	0.65	8000	(5, -3) (5, -4)	45.6875	45.6823
7	0.65	8000	(5, -2) (5, -3)	45.6864	45.6879
Average $N$				45.6873	
Average $R$					45.6880
Over-all average				45.6877	
Statistical probable error				0.0007	

inhomogeneities in the Permendur contributed in some measure to the lack of perfect homogeneity of the field, the new magnet, which has essentially the same geometry as the original one, is made entirely of Armco iron. That a worth while improvement has been made in the field homogeneity by the magnet replacement is evidenced by the reduced width, with the new magnet, of field sensitive lines.

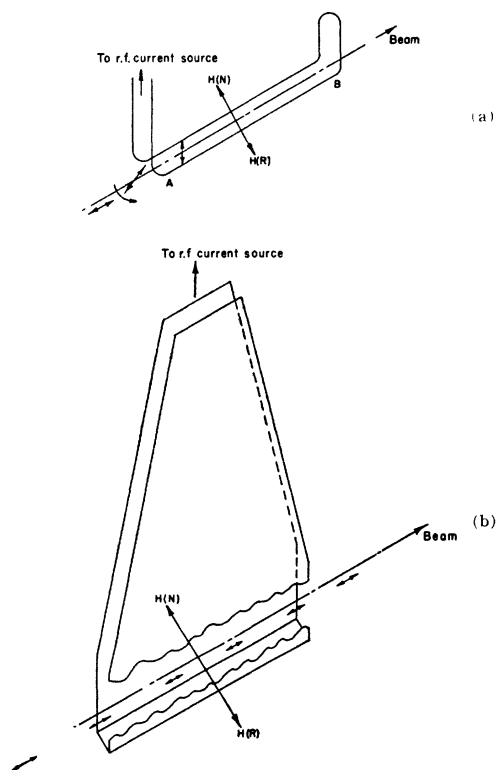


FIG. 2. Arrangements for superimposing an oscillating magnetic field on the fixed magnetic field. The orientations of the fixed and alternating fields are indicated by the arrows. (a) The two wire arrangement which exhibits marked Millman effect. (b) The parallel plane arrangement in which the Millman effect is suppressed.

An iron oven loaded with a mixture of powdered calcium and caesium chloride provided a caesium beam. An indium beam was obtained by evaporating indium from a graphite lined molybdenum oven, while a silver oven loaded with NaOH provided a beam in which a proton resonance could be observed. For the purpose of permitting consecutive measurements in rapid sequence of the proton resonance and a line of the spectrum of the atom employed, the double oven arrangement previously described<sup>1</sup> was again employed.

All of the data of this experiment consist of the frequency in a fixed magnetic field of a line in the h.f.s. spectrum of an atom and the frequency in the same magnetic field of a proton resonance. No precise knowledge of the magnitude of the magnetic field is required, but, as a matter of practice, data on the current through the magnetizing coil serves to correct for the drift of the magnetic field during the course of a run.

The frequency of any line measured in this experiment was determined by an observation of the frequency at which a maximum decrease in beam intensity occurred, or by an observation of the mean of the frequencies of two points on either side of the minimum which were symmetrical with respect to reduction in beam intensity. It was not feasible to make a detailed plot of beam intensity as a function of frequency in taking the data because the time consumed in making the necessary observations would seriously prejudice the essential requirement that atomic and molecular line be observed in the same magnetic field. To permit corrections to be made for field drift, the current through the coils of the magnet which produces the homogeneous field was carefully monitored with the aid of a Leeds & Northrup Type K potentiometer and a precision shunt. The errors in determining the frequency of a line occasioned by line width and beam unsteadiness are statistical in character, and the error may then be materially reduced by numerous repetitions of observations. All frequency measurements were made on a General Radio heterodyne frequency meter, type 620A. This instrument may be used to measure a particular frequency with an accuracy of about 1 part in 50,000. Since the limitation on accuracy imposed by line widths exceeded in general the limitation imposed by the wave meter, there was no advantage to be gained from the use of better frequency measuring equipment.

The data from which  $g_H/g_J$  is found, were obtained in the following manner. From four to eight separate and individual observations were taken of the frequency of the appropriate atomic line. At the instant one observer read the line frequency, a second observer read the corresponding value of the magnet current. The ovens having

been interchanged, the same procedure was followed in reading the frequency of the proton resonance. The series of observations were then repeated alternately on the atomic and molecular lines until at least three sets of observations had been made on the atomic line and at least two on the molecular line. The drift of the magnetic field, while in principle undesirable, nevertheless required that a line be read at a slightly different frequency on each occasion, and therefore served the useful purpose of making each individual line reading independent of the others, and of eliminating prejudice on the part of the observer, who was not aware of the nature of the drift. Further, since the errors made in reading the wave meter are statistical in character, the mean of a set of frequencies which require the use of differing parts of the scale will be of an accuracy appreciably better than the accuracy of an individual reading.

It has been noted experimentally that, in general, for small amplitudes of the radio frequency current which provides the oscillating magnetic field, the intensity of a line increases with increasing current and the width of the line remains approximately constant. For large r-f currents, on the other hand, the intensity remains approximately constant while the line increases in width with increasing current and may on occasion exhibit asymmetries. The exact adjustment of the current to a predetermined value is at best a difficult procedure. We have, however, in all cases, reduced the r-f current to a point where further reduction would have resulted in a marked decrease in line intensity.

The data taken in the manner described was interpreted on the assumption that the magnetic field varied linearly with magnet current. That this assumption is reasonable is supported by the fact that even at the largest fields employed, where the drift was most pronounced, the range of field over which the assumption of linearity was required to hold rarely exceeded three parts in one thousand. At lower fields, the drift was very much less and in some cases negligible. The validity of this assumption is further supported by the precision with which the points of a plot of magnetic field *versus* field current fall on a straight line, the magnetic field being determined by the measured frequency of a field sensitive atomic line. And finally, the validity of the assumption is borne out by the consistency of the data generally.

In the previous experiment<sup>10</sup> on the determination of the proton moment, the oscillating magnetic field which produces transitions was produced by the two wire arrangement illustrated in Fig. 2a. However, such an arrangement, on account of end effects will produce in a spectral line a spurious auxiliary line which may or may not be resolved

TABLE II. Results of the individual determinations of the ratio  $g_H/g_J(\text{Cs}^{133}, \text{}^2\text{S}_1)$  and the circumstances under which the individual determinations were made. In every case the atomic transition  $(4, -1) \leftrightarrow (4, -2)$  was employed.

Run	Nominal $\alpha$	Nominal magnetic field (gauss)	$g_H/g_J(\text{Cs}^{133}, \text{}^2\text{S}_1)$	
			Field direction $N$	Field direction $R$
1	1.0	3300	15.1910	
2	1.0	3300	15.1927	
3	1.0	3300	15.1908	15.1914
4	1.0	3300		15.1888
5	1.0	3300	15.1900	15.1911
6	1.2	4000	15.1902	
7	1.2	4000	15.1919	15.1929
8	1.2	4000	15.1915	
		Average $N$	15.1912	
		Average $R$		15.1911
		Over-all average		15.1911
		Statistical probable error		0.0003

from the main line. This is the so-called Millman<sup>20</sup> effect. The direction and magnitude of the displacement of the auxiliary line and its relative intensity will depend on the velocity of the beam, the amplitude of the oscillating current and the geometry of the wires. The shift may be reversed by a reversal of the fixed field. If the auxiliary line is not resolved its presence will nevertheless serve to broaden the line and shift it from the position it would occupy in the absence of the Millman effect. The magnitude of the Millman effect is unaffected by the frequency of the line. With the wire arrangement of Fig. 2a, described by Millman, for which the distance  $AB$  was of the order 6 cm and for which the end effects were confined to about  $\frac{1}{2}$  cm at each end, displacements of the order of  $10^4$  cycles per second had been observed. A frequency of  $10^4$  c.p.s. is an appreciable fraction of the frequencies encountered in molecular spectroscopy. It is therefore necessary that the correct frequency of a line be determined by taking the average of the frequencies measured for each of the two possible directions of the fixed magnetic field since, all other experimental conditions remaining fixed, the direction of the line shift reverses with reversal of the field. In the earlier experiment on the proton moment, measurements of  $g_H$  were made with the field in each of the two directions and the average of the two values was accepted as the correct value. In that experiment the proton moment as calculated from data taken for each of the field directions differs by amounts of the order of 1 part in 1000. In the present experiment it was not evident that the cancellation of Millman effect error accomplished by taking an average would eliminate the error to the precision which was contemplated for these experiments. Accordingly, the arrangement used in the present experiment for superimposing an oscillating field on the fixed field is that illus-

<sup>20</sup> S. Millman, Phys. Rev. 55, 628 (1939).



trated in Fig. 2b. The Millman effect occurs because the magnetic field produced by the oscillating current is such that a moving molecule in the beam sees a field in the neighborhood of the ends of the "hairpin," which rotates about an axis parallel to the main field. A comparison of figures indicates that the Millman effect disappears for a perfect symmetry of the hairpin of Fig. 2b and for a symmetrical approach of the beam to the hairpin and is, in any event, appreciably smaller for the arrangement of Fig. 2b than for the arrangement of Fig. 2a. Most importantly, however, the data presented below exhibits no Millman effect whatsoever so that no uncertainties arise on this account.

### RESULTS

In the experiment on the determination of the ratio of  $g_H$  to the  $g_J$  of  $\text{In}^{115}$  in the  $^2P_{1/2}$  state measurements were made under the conditions indicated in Table I for each of seven runs. It is to be noted that measurements have been made on three lines at three different magnetic fields. In each case, the choice of line was dictated by the considerations discussed above. The inhomogeneous, deflecting fields were set to refocus the state which had a zero moment at the higher field, that is, the state with the  $m_F$  number of larger absolute magnitude. The data taken with the field direction  $N$  and the data with the field  $R$ , where  $N$  and  $R$  are arbitrary designations of field direction, give the same value of  $g_H/g_J$  to better than 1 part in 50,000 and the discrepancy is too small to be meaningful. Since no Millman effect occurs we may consider each separate entry in Table I as an independent experimental value of  $g_H/g_J$ . The over-all average of these fourteen independent observations is  $g_H/g_J(\text{In}) = 45.6877 \times 10^{-4}$  and the probable error, computed by the usual statistical procedures, is  $0.0007 \times 10^{-4}$ , or about 1 part in 65,000.

In the comparison of  $g_H$  to the  $g_J$  of  $\text{Cs}^{133}$  in the ground state, measurements were made under the conditions noted in Table II. In every case, an attempt was made to have a complete run consist, as was the case with indium, of a determination of  $g_H$  with the fields in each of the two directions. The blank spaces in Table II do not represent rejected data, but are rather the result of experimental difficulties which required the termination of a run before completion. The parenthetical comment should be made that an experiment was successful only upon the simultaneous satisfactory operation of a large number of elements. The fact that all runs involving indium are complete is the result of an extraordinarily long run of good luck. Here again the difference between normal and reverse field directions is too small to be meaningful and provides further evidence of the absence of Millman effect. The over-all average of eleven independent deter-

minations gives  $g_H/g_J(\text{Cs}) = 15.1911 \times 10^{-4}$  with a probable error of  $0.0003 \times 10^{-4}$ , which is about 1 part in 50,000.

### $g_H$ IN TERMS OF $g_J(\text{Na})$

The most direct results of the present experiment are the quantities  $g_H/g_J(\text{In}) = 45.6877 \times 10^{-4}$  and  $g_H/g_J(\text{Cs}) = 15.1911 \times 10^{-4}$ . It is to be noted that these two experimental results are entirely independent. The essential part of the apparatus, namely the transition field, was different in the two determinations and the range of magnetic field used in the two determinations differed widely. To reduce the results of the two sets of observations to a common standard we use the ratio  $g_J(\text{Cs})/g_J(\text{Na}) = 1.000134$  determined by Kusch and Taub,<sup>2</sup> and the ratio  $g_J(\text{Na})/g_J(\text{In}) = 3(1.00243)$  determined by Kusch and Foley.<sup>1</sup> These ratios lead to the results:

From comparison with In:

$$g_H/g_J(\text{Na}) = 15.1923 \times 10^{-4};$$

From comparison with Cs:

$$g_H/g_J(\text{Na}) = 15.1931 \times 10^{-4}.$$

The discrepancy between the two measured quantities arrived at by two independent procedures is one part in 19,000. This high degree of consistency is of the greatest significance in establishing the general validity of the experimental procedures and of the interpretation of the data. Equal weight is given to each of the separate determinations, so that

$$g_H/g_J(\text{Na}) = 15.1927 \times 10^{-4} \pm 0.005 \text{ percent},$$

the precision measure, which is twice as large as that required to include both the individual determinations, will be justified in a following section.

### THE MAGNETIC MOMENT OF THE PROTON

The experiment discussed in the previous paper<sup>2</sup> leads to the result that  $g_J(\text{Na}) = g_s$ , the spin gyromagnetic ratio of the electron. The experimental value of the quantity  $g_s$  is  $2(1.00119)$ ;<sup>1</sup> this value is in agreement, within experimental error, with the result calculated by Schwinger,<sup>21</sup> that  $g_s = 2(1 + \alpha/2\pi) = 2(1.00116)$ . No sufficient grounds exist on the basis of which a choice can be made between the two values. Purely as a matter of convention the theoretical value of  $g_s$  is adopted. A simple application of this factor leads to the value of the gyromagnetic ratio of the proton:

$$g_H = 30.4206 \times 10^{-4} \pm 0.005 \text{ percent}.$$

As will be pointed out later a small diamagnetic correction of 0.002 percent increases the value of

<sup>21</sup> J. Schwinger, Phys. Rev. 73, 416 (1948).



the moment obtained from the experimentally observed  $g$  value. Accordingly,

$$\mu_H = (15.2106 \times 10^{-4} \pm 0.005 \text{ percent}) \text{ Bohr magnetons.}$$

It is to be emphasized that the precision measure depends on the data of the present series of experiments alone. No attempt is made to include in this measure any uncertainties arising from the interpretation of  $g_J(\text{Na})$  as  $g_s$  or any uncertainties in the value of  $g_s$  itself.

It is common practice to express the values of nuclear magnetic moments in terms of nuclear magnetons. This is, from the point of view of the present experiments, an artificial procedure since it demands the insertion of the quantity  $M/m$  into a result which is independent of any of the constants of atomic physics except the value of  $g_s$ . If we use  $M/m = 1836.57^{22}$  we find

$$\mu_H = 2.7935 \text{ nuclear magnetons.}$$

In an experiment in which  $\mu_H$  is determined by the observation of the resonant frequency of the proton in a known magnetic field it is not of importance whether the moment is expressed in Bohr or nuclear magnetons, since in one case the experimental quantity  $e/mc$  must be applied and in the other case the quantity  $e/Mc$ . At present no obvious choice of either value can be made.

Using for the absolute value of the Bohr magneton the value  $\mu_0 = (0.92731 \pm 0.00017) \times 10^{-20}$  erg/gauss, the absolute value of the magnetic moment of the proton is found to be

$$\mu_H = (1.4105 \times 10^{-23} \pm 0.02 \text{ percent}) \text{ erg/gauss.}$$

From the data of the present experiments, no information as to the sign of  $\mu_H$  is obtained. However,  $\mu_H$  is known to be positive from several previously reported lines of evidence. The value of  $g_H$  discussed in this paper is the absolute value of the quantity.

#### PRECISION

The statistical probable error in  $g_H/g_J(\text{Cs})$  is, as stated above, 1 part in 50,000, while the p.e. in  $g_J(\text{Cs})/g_J(\text{Na})$ , as given by Kusch and Taub is 1 part in 150,000. The p.e., then, in  $g_H/g_J(\text{Na})$  is about 1 part in 50,000. A review of the published data of Kusch and Foley<sup>1</sup> from which the quantity  $g_J(\text{In})/g_J(\text{Na})$  is derived reveals a p.e. of about 1 part in 80,000. The published precision measure for this ratio, 1 part in 10,000, is clearly pessimistic and this extremely pessimistic view does not appear to be justified, especially in view of the subsequent history of the apparatus used

in that experiment. It is to be noted further, that if the  $g_J$  values of atoms are characteristic of the atomic state rather than of the particular atom, then it is to be expected that  $g_J(\text{In } ^2P_{3/2})/g_J(\text{Na } ^2S_{1/2}) = g_J(\text{Ga } ^2P_{3/2})/g_J(\text{Na } ^2S_{1/2})$ . The remarkable agreement between these measured quantities (Kusch and Foley give  $g_J(\text{Na})/g_J(\text{In}) = 3(1.00243)$ ;  $g_J(\text{Na})/g_J(\text{Ga}) = 3(1.00242)$ ) is strong evidence to the effect that the precision of determination of these ratios is appreciably in excess of the stated precision. If we accept the p.e. of 1 part in 80,000 in  $g_J(\text{In})/g_J(\text{Na})$  then the p.e. of  $g_H/g_J(\text{Na})$  obtained from the In data, is, as in the case of the Cs data, about 1 part in 50,000. The p.e. of the mean is, then, 1 part in 70,000 and the precision measure of 1 part in 20,000, given above, is not only twice as large as is required to include both individual determinations, but is also 3.5 times as large as the p.e.

A straightforward numerical calculation based on Eqs. (1) and (2) reveals that the uncertainties in the  $\Delta\nu$ 's and the ratios  $g_J/g_I$  for In and Cs introduce no error in the value  $g_H/g_J$ , in each case, of magnitude sufficient to effect the proton moment to the precision here contemplated. The largest error in the ratio  $g_H/g_J(\text{In})$  arising from an error in the  $\Delta\nu$  and  $g_J/g_I$  of In occurs for the case  $x \sim 1.0$  where the line  $(5, -1 \leftrightarrow 5, -2)$  was used. Here, the assumption of an error of 0.10 mc in  $\Delta\nu$  gives rise to an error of about 1 part in 100,000 in  $g_H/g_J(\text{In})$ , while an error of 0.5 in  $g_J/g_I$  causes an error of 1 part in 80,000. For all other  $x$  values and lines employed the resultant error is very much smaller. Similarly, an assumed error of 0.10 mc in the  $\Delta\nu$  of Cs causes an error in the value  $g_H/g_J(\text{Cs})$  calculated at  $x \sim 1.0$  of 1 part in 90,000 and, at  $x = 1.2$ , an error of 1 part in 50,000. Further, an error of 3 in  $(g_J/g_I)\text{Cs}$  produces an error of only 1 part in 500,000 at  $x \sim 1.0$  and 1 part in 300,000 at  $x \sim 1.2$ .

All the errors discussed above are purely statistical errors, and the possibility of systematic errors must be considered. Since the present experiment consists entirely of the measurement of spectral lines in a fixed magnetic field, the only sources of systematic error which suggest themselves are errors in the frequency measuring equipment, errors occasioned by some circumstance associated with part of the apparatus which determines the frequencies of the observed lines, that is, the homogeneous transition magnet and the possibility that the center of the observed molecular line does not, in fact, correspond to the frequency  $g_I\mu_0 H/h$  because of interactions which give an unsymmetrical broadening of the line. The frequency measuring equipment had been periodically checked against the standard signals broadcast by WWV and the instability of the equipment is not great enough to introduce significant error. However, inhomogenei-

<sup>22</sup> J. W. M. DuMond and E. R. Cohen, Rev. Mod. Phys. 20, 82 (1948).

ties in the transition field, which have a marked effect on line width might conceivably result in an unsymmetrical broadening of the lines. That such effects occur has been previously noted but sufficient evidence exists from the measurements of the  $\Delta\nu$  of indium that the effects play a negligible part in the apparent value of  $g_H/g_J$ . The possibility of error from such sources, is, in addition, quite remote, in view of the wide variety of circumstances, with respect to the transition magnet, under which data has been taken in the present experiment and in view of the history generally of the apparatus employed. The data for the determination of the ratio  $g_H/g_J(\text{In})$  was taken using the original transition magnet of the apparatus, while all other data, including the data for the determination of the  $g_J$  ratios for the alkali atoms, described in a previous paper,<sup>2</sup> was taken after the replacement of the original magnet with the new one. It is to be noted that the dependence of the data on the characteristics of the transition magnet is so critical that the replacement of this magnet constitutes the creation of an almost entirely new apparatus. Second, the agreement between the ratio  $g_H/g_J$  from data taken for each of the two directions of the magnetic fields is convincing evidence that no systematic shift of line frequencies occurs, since it seems unlikely that such a reversal of fields would not affect the magnitude of a systematic error occasioned by field inhomogeneities and the disposition of the hairpin in the magnet. Most importantly, the extremely good agreement between the values  $g_H/g_J$  obtained through completely separate channels, that is, through the combinations of four completely independent experiments on the determination of  $g_H/g_J(\text{In})$ ,  $g_J(\text{In})/g_J(\text{Na})$ ,  $g_H/g_J(\text{Cs})$  and  $g_J(\text{Cs})/g_J(\text{Na})$ , militates against the occurrence of systematic error.

A further source of systematic error which suggests itself is the possibility of a perturbation of the  $^2P_{3/2}$  levels of indium by the  $^2P_{3/2}$  state, separated from the lower state by about  $2000\text{ cm}^{-1}$ . Such a perturbation might arise on account of the existence of matrix elements  $(J|\mu_N \cdot H_e|J+1)$  between the two states,  $J=\frac{1}{2}$ ,  $J=\frac{3}{2}$ , of the interaction between the nuclear moment and the field at the nucleus due to the electron. Mann and Kusch<sup>23</sup> of this laboratory have recently measured the intervals, at zero field, of the four levels of the  $^2P_{3/2}$  state of indium. They have found that to a precision of 1 part in 100,000 the frequencies of the three zero field lines agree with the frequencies calculated using only the elements diagonal in  $J$  of the interaction  $\mu_N \cdot H_e$  and the interaction due to the quadrupole moment of the nucleus. The matrix elements, non-diagonal in  $J$ , which are proportional to the

magnetic field serve at any particular field to shift all the levels by exactly the same amount and hence have no effect either on line frequencies or on the determination of magnetic field from the measured frequency of a line.

Finally we must inquire whether the proton resonance curve observed in NaOH is entirely symmetrical about the frequency  $f_H = g_H \mu_0 H$ . The natural width of the resonance under the circumstances which prevailed in this experiment is estimated to be of the order of 10 kc. The observed half width is of the order of 40 kc and this broadening is caused by interactions internal to the molecule. The only perturbing, internal interactions which are presently contemplated in a molecule in which the net electronic angular momentum is zero are the interactions between the nuclear spins, the interaction between the nuclear spins and the magnetic field which arises from molecular rotation and the interaction of a nuclear quadrupole moment with an electric field gradient at the position of the nucleus. Since the oxygen nucleus has no magnetic moment, the Hamiltonian from which the energy levels may be calculated is entirely similar to the Hamiltonian which has been applied to the molecule HD.<sup>24</sup> In both cases one nucleus is the proton which has a magnetic moment but no quadrupole moment, while the other nucleus has both magnetic moment and electric quadrupole moment.

The result of a first order calculation using this Hamiltonian is that the single line which would result from the reorientation of the magnetic moment of the proton is split on account of the internal interactions into a set of lines  $(2J+1)(2I+1)$  in number.  $J$  is the quantum number of molecular rotation and  $I$  is the spin of the other nucleus. What is important for our purpose is to note that the multiplet of lines which results are disposed entirely symmetrically about the frequency  $f_H = g_H \mu_0 H$ . The individual lines are not observed in the present experiment because the interval between lines is very much smaller than the resolution of the apparatus.

In second order, small asymmetries begin to appear, but second order corrections are completely negligible in the present case on account of the large fields which were employed. Proton resonances were observed at frequencies varying from 15 to 50 mc. The width of the line is constant and of the order of 40 kc. Accordingly, first order perturbation energy terms are smaller than the zero order energies, even in the worst case, by a factor of  $15\text{ mc}/40\text{ kc} \sim 400$ . And second order corrections, since they result from the same interactions as are responsible for first order terms, are less than the zero order energy by a factor  $(400)^2$ .

<sup>23</sup> A. K. Mann and P. Kusch (private communication).

<sup>24</sup> Kellogg, Rabi, and Ramsey, Phys. Rev. 57, 677 (1940).

These considerations, above the fact that the proton resonance as observed experimentally exhibits no asymmetry, and the consistency of results of measurements made over a range of magnetic field from 3000 to 12,000 gauss leads to the conclusion that the proton resonance curve is entirely symmetrical about the frequency  $f_H = g_H \mu_0 H$ .

### THE FINE STRUCTURE CONSTANT

The hyperfine splitting of a  $^2S_{1/2}$  state which results from the interaction of the nuclear moment with the perturbing magnetic field at the nucleus due to the electron is given by Fermi<sup>6</sup> as

$$\Delta\nu = (8\pi/3h)(2I+1/I)\mu_e\mu_N\psi^2(0),$$

where  $I$  is the nuclear spin,  $\mu_e$  and  $\mu_N$  are respectively the electronic and nuclear magnetic moments in absolute units and  $\psi(0)$  is the Schrödinger wave function evaluated at  $r=0$ . In the Fermi formula the assumption is made that the mass of the nucleus is infinite. Breit and Meyerott<sup>8</sup> have calculated that the finite mass of the nucleus may properly be taken into account by including a factor  $(m_r/m_0)^3$ , in which  $m_r$  is the reduced mass and  $m_0$  is the rest mass of the electron. It has also been shown by Breit<sup>7</sup> that the Fermi formula must be modified by the inclusion of a relativistic correction factor  $(1+\frac{3}{2}\alpha^2)$ . Including these factors, writing  $\mu_e = \mu_0(\mu_e/\mu_0)$ , and replacing  $\mu_0$  and  $\psi(0)$  by equivalent combinations of physical constants, we have finally for  $\Delta\nu$  of hydrogen,

$$\Delta\nu_H = 4/3(2I+1/I)cR_\infty(m_r/m_0)^3 \times (\mu_H/\mu_0)(\mu_e/\mu_0)\alpha^2(1+3\alpha^2/2).$$

In this expression  $\alpha$  is the fine structure constant,  $c$  is the velocity of light, and  $R_\infty$  is the Rydberg constant for infinite mass.

Using the Nafe-Nelson<sup>6</sup> value  $\Delta\nu_H = (1420.410 \pm 0.006) \times 10^6 \text{ sec.}^{-1}$ , the values  $\mu_e/\mu_0 = 1.00116$ ,  $\mu_H/\mu_0 = (15.2106 \times 10^{-4}) \pm 0.005$  percent and the values of the physical constants recommended by DuMond and Cohen,<sup>22</sup>  $M/m_0 = 1836.57 \pm 0.20$ ;  $R_\infty = (109,737.30 \pm 0.05) \text{ cm}^{-1}$ ;  $c = (2.99776 \pm 0.00004) \times 10^{10} \text{ cm sec.}^{-1}$ , we have,  $\alpha^2 = 5.3246 \times 10^{-5}$  or,

$$1/\alpha = 137.043 \pm 0.003$$

in which the precision measure assigned includes only the uncertainty in the measured ratio  $\mu_H/\mu_0$ . This value is to be compared to the values  $1/\alpha = 137.021 \pm 0.0007$  given by DuMond and Cohen and  $137.030 \pm 0.016$  given by Birge.<sup>25</sup>

### THE GYROMAGNETIC RATIOS AND MAGNETIC MOMENTS OF OTHER NUCLEI

The ratio of two nuclear  $g$ 's may, in general, be determined more conveniently and to a much

TABLE III.\* Nuclear gyromagnetic ratios and magnetic moments in terms of the Bohr magneton (without diamagnetic correction) and the magnetic moments in units of the nuclear magneton (with diamagnetic correction).

Nu- cleus	Spin	$ g  = \frac{\mu/\mu_0}{I}$ $\times 10^4$ (obs.)	$\mu/\mu_0 \times 10^4$ (obs.)	Diamag- netic correc- tion (%)	$\mu/\mu_N$ (corr.)	Preci- sion (%)	Refer- ence**
<sup>1</sup> N <sup>14</sup>	1/2	20.8381	-10.4191		-1.91354	0.0066	3
<sup>1</sup> H <sup>1</sup>	1/2	30.4206	15.2103	0.0018	2.79353	0.0050	
<sup>1</sup> H <sup>2</sup>	1	4.66975	4.66975	0.0018	0.857648	0.0050	4
<sup>1</sup> H <sup>3</sup>	1/2	32.4477	16.2239	0.0018	2.97968	0.0051	5
<sup>2</sup> He <sup>4</sup>	1/2	23.2	11.6	0.0046	2.13	0.92	a
<sup>3</sup> Li <sup>6</sup>	1	4.4769	4.4769	0.0081	0.82228	0.011	2, b
<sup>3</sup> Li <sup>7</sup>	3/2	11.8222	17.7333	0.0081	3.25711	0.011	b
<sup>4</sup> Be <sup>9</sup>	3/2	4.27	-6.41	0.0122	-1.177	0.38	c
<sup>10</sup> B <sup>10</sup>	3	3.269	9.806	0.0168	1.8012	0.036	b
<sup>10</sup> B <sup>11</sup>	3/2	9.7604	14.641	0.0168	2.6893	0.021	b
<sup>12</sup> C <sup>12</sup>	1/2	7.6487	3.8243	0.0219	0.7025	0.021	b
<sup>7</sup> N <sup>14</sup>	1	2.20	2.20	0.0275	0.404	0.50	10
<sup>7</sup> N <sup>15</sup>	1/2	3.05	±1.53	0.0275	±0.280	1.0	d
<sup>9</sup> F <sup>19</sup>	1/2	28.619	14.309	0.0401	2.6291	0.011	b
<sup>11</sup> Na <sup>23</sup>	3	3.1679	9.5038	0.0546	1.7464	0.012	2, e
<sup>11</sup> Na <sup>20</sup>	3/2	8.0462	12.0694	0.0546	2.21784	0.011	b
<sup>14</sup> Al <sup>27</sup>	5/2	7.9264	19.816	0.0709	3.6419	0.011	b
<sup>16</sup> P <sup>31</sup>	1/2	12.315	6.1573	0.0891	1.1318	0.011	b
<sup>17</sup> Cl <sup>35</sup>	3/2	2.9812	4.472	0.109	0.8222	0.032	b
<sup>17</sup> Cl <sup>37</sup>	3/2	2.48	3.72	0.109	0.683	0.44	f
<sup>18</sup> K <sup>39</sup>	3/2	1.418	2.13	0.131	0.391	0.3	16
<sup>18</sup> K <sup>40</sup>	4	1.755	-7.02	0.131	-1.291	0.3	2, k
<sup>18</sup> K <sup>41</sup>	3/2	0.780	1.171	0.131	0.215	0.3	2, 17
<sup>29</sup> Cu <sup>63</sup>	3/2	8.063	12.095	0.239	2.2266	0.021	b
<sup>29</sup> Cu <sup>65</sup>	3/2	8.637	12.955	0.239	2.3850	0.021	b
<sup>29</sup> Ga <sup>69</sup>	3/2	7.294	10.940	0.262	2.0145	0.036	i
<sup>31</sup> Ga <sup>69</sup>	3/2	7.25	10.87	0.262	2.001	0.25	j
<sup>31</sup> Ga <sup>71</sup>	3/2	9.264	13.895	0.262	2.559	0.048	i
<sup>31</sup> Ga <sup>71</sup>	3/2	9.21	13.81	0.262	2.543	0.20	j
<sup>36</sup> Br <sup>79</sup>	3/2	7.622	11.432	0.309	2.1061	0.043	b, g
<sup>36</sup> Br <sup>81</sup>	3/2	8.214	12.322	0.309	2.2700	0.032	b
<sup>37</sup> Rb <sup>85</sup>	5/2	2.9378	7.345	0.333	1.3534	0.032	b
<sup>37</sup> Rb <sup>87</sup>	3/2	9.953	14.930	0.333	2.7510	0.021	b
<sup>40</sup> In <sup>113</sup>	9/2	6.610	29.74	0.489	5.489	0.051	19
<sup>40</sup> In <sup>115</sup>	9/2	6.624	29.81	0.489	5.502	0.05	1
<sup>51</sup> I <sup>127</sup>	5/2	6.088	15.220	0.545	2.8105	0.029	i
<sup>56</sup> Cs <sup>133</sup>	9/2	3.989	17.951	0.573	3.316	0.032	b
<sup>56</sup> Ba <sup>135</sup>	3/2	3.018	4.527	0.587	0.8364	0.29	h
<sup>56</sup> Ba <sup>137</sup>	3/2	3.376	5.064	0.587	0.9354	0.29	h
<sup>81</sup> Tl <sup>203</sup>	1/2	17.385	8.6927	0.98	1.6121	0.011	b
<sup>81</sup> Tl <sup>205</sup>	1/2	17.557	8.7784	0.98	1.6280	0.011	b

\* Most of the  $g$  values listed in this table have been calculated from the measured ratios of  $g$  values. Exceptions occur in the following instances.

<sup>15</sup>N<sup>15</sup>: The  $g$  of <sup>15</sup>N<sup>15</sup> has been determined by the direct calibration of a magnetic field in terms of absolute standards.

<sup>14</sup>Li, <sup>40</sup>K<sup>41</sup>, <sup>113</sup>In: The  $g$  values of these nuclei have been calculated from the relationship  $g_1/g_2 = (2I_2+1)/(2I_1+1)(\Delta\nu_1/\Delta\nu_2)$  in terms of the measured  $g$  values of the more abundant isotopes, the spins and the appropriate hyperfine structure interaction constants,  $\Delta\nu$ .

<sup>115</sup>In: The  $g$  value of <sup>115</sup>In has been calculated from

$$g_I(\text{In}^{115}) = \frac{g_I(\text{In}^{113})}{g_J(\text{In}^{113}, ^2P_{3/2})} \cdot \frac{g_J(\text{In}^{115}, ^2P_{3/2})}{g_J(\text{Na}, ^2S_{1/2})} \cdot g_J(\text{Na}, ^2S_{1/2}),$$

assuming  $g_J(\text{Na}) = g_e = 2(1.00116)$ .

<sup>69</sup>Ga, <sup>71</sup>Ga: The  $g$  values of these isotopes were calculated on the one hand from the ratios  $g(\text{Ga}^{71})/g(\text{Ga}^{69})$  and  $g(\text{Ga}^{71})/g(\text{Na}^{23})$ , measured by Pound (see reference i) and on the other hand from the ratios  $g_I(\text{Ga}^{69,71})/g_J(\text{Ga}^{69,71}, ^2P_{3/2})$ , measured by Becker and Kusch (see reference j). Since the  $g$  values calculated from these two sets of measurements disagree by an amount which is larger than the sum of the assigned errors, both sets of values have been listed. The results obtained by Pound by the use of the nuclear absorption method are more precise than those obtained by Becker and Kusch by the application of the method of atomic h.f.s. spectroscopy. The ratios of the  $g$  values of the two isotopes as determined by the two methods are very nearly the same. Unpublished results of Kusch and Foley yield for  $g_I(\text{Ga}^{69})$  a value very close to that obtained by Becker and Kusch, and it is believed that the indicated discrepancy is real and does not arise from an excessively optimistic estimate of the precision of the spectroscopic result.

\*\* Numbered references refer to footnotes in text.

<sup>a</sup> H. L. Anderson and A. Nowick, Phys. Rev. **73**, 919 (1948).

<sup>b</sup> F. Bitter, Phys. Rev. **75**, 1326 (1949).

<sup>c</sup> Kusch, Millman, and Rabi, Phys. Rev. **55**, 666 (1939).

<sup>d</sup> J. R. Zacharias and J. M. B. Kellogg, Phys. Rev. **57**, 570 (1940).

<sup>e</sup> L. Davis, Phys. Rev. **74**, 193 (1948).

<sup>f</sup> P. Kusch and S. Millman, Phys. Rev. **56**, 527 (1939).

<sup>g</sup> R. V. Pound, Phys. Rev. **72**, 1273 (1947).

<sup>h</sup> R. H. Fay, Phys. Rev. **60**, 75 (1941).

<sup>i</sup> R. V. Pound, Phys. Rev. **73**, 1112 (1948).

<sup>j</sup> G. Becker and P. Kusch, Phys. Rev. **73**, 584 (1948).

<sup>k</sup> J. R. Zacharias, Phys. Rev. **61**, 270 (1942).

higher order of precision than the absolute  $g$  value of a particular nucleus. Accordingly, a large body of data is available in which the  $g$  value of a

<sup>25</sup> R. T. Birge, Rev. Mod. Phys. **13**, 233 (1941).

particular nucleus has been measured in terms of a  $g$  value which is well known, notably  $g(\text{H}^1)$ ,  $g(\text{Li}^7)$  and  $g(\text{Na}^{23})$ . In view of the present determination, with high precision of the ratio of the magnetic moment of the proton to the Bohr magneton, it seems desirable, whenever possible, to calculate, in terms of the Bohr magneton, the  $g$  values and magnetic moments of other nuclei. In Table III are tabulated the values of the gyromagnetic ratios and the magnetic moments of all nuclei for which data is currently available from molecular and atomic beam experiments and from experiments using the techniques of nuclear induction and absorption.

The first and second columns of Table III list the nuclei (and neutron) and their spins. In each case, except  $\text{He}^3$ , the spins have been determined by reliable experimental procedures, while in the case of  $\text{He}^3$  there seems to be little doubt, on theoretical grounds, that the spin is  $\frac{1}{2}$ . The third column lists the magnitude of the  $g$  values or the absolute value of the ratio of the magnetic moment in units of the Bohr magneton to the spin. The fourth column lists the magnetic moment in terms of the Bohr magneton. The signs of the moments are known except in the case of  $\text{N}^{15}$ .

The fifth column gives the correction which must be applied because of the diamagnetic effect of the electrons of the atom. This correction serves in every case to increase the  $g$  value and the moment. Lamb<sup>26</sup> has calculated the magnitude of this effect and finds that

$$g(\text{corrected}) = g(\text{measured})[1 + (e/3 mc^2)v(0)] \\ \cong g(\text{measured})(1 + 0.319 \times 10^{-4} Z^{4/3}),$$

in which  $v(0)$  is the electrostatic potential at the nucleus which is produced by the electrons. For hydrogen  $v(0)$  may be calculated exactly with the result that the factor 0.319 is replaced by 0.178 while in the cases  $Z = 19, 20, 26, 29, 37, 55, 74$  and 81 the value of  $v(0)$  which has been given explicitly

<sup>26</sup> W. E. Lamb, Jr., Phys. Rev. **60**, 817 (1941).

by Hartree and others<sup>27</sup> requires the replacement of the factor 0.319 by the factors 0.259, 0.259, 0.263, 0.268, 0.270, 0.274, 0.277 and 0.280 respectively. The diamagnetic corrections listed in Table III are calculated by linear interpolations between these values. In each case, of course, the accuracy of the diamagnetic correction may be improved by using the exact value of  $v(0)$  and by making more detailed considerations concerning the contribution to the diamagnetic effect of the outermost electrons. On account of the uncertainties in the diamagnetic corrections, and in order to keep experimental numbers distinct from theoretical calculations, the diamagnetic corrections have not been included in the  $g$  values and moments listed in the third and fourth columns of Table III. It is to be noted that in the calculation of the frequencies at which lines occur in the h.f.s. spectrum of atoms and molecules the uncorrected  $g$  values are precisely the values which are required.

The sixth column lists the moments in units of the nuclear magneton. The diamagnetic correction has been applied to these moments and for the conversion factor from Bohr to nuclear magnetons we have used the ratio  $M/m = 1836.57$  given by DuMond and Cohen.<sup>22</sup> The seventh column lists the precision measure which is the square root of the sum of the squares of the precision measures which have been assigned to the individual numbers or ratios from which the  $g$  values and moments have been calculated. The uncertainty in the ratio  $M/m$  and the uncertainties in the diamagnetic corrections have not been included in the precision measure.

We are pleased to acknowledge our indebtedness to Professor H. M. Foley for several clarifying discussions concerning perturbations of atomic states. We are grateful also to Miss Zelda Marblestone who performed a considerable part of the numerical computation involved in this experiment.

<sup>27</sup> The references are listed in F. Seitz, *Modern Theory of Solids* (McGraw-Hill Book Company, Inc., New York, 1940), p. 251, except for the case  $Z = 26$ , which is given by M. Manning and L. Goldberg, Phys. Rev. **53**, 662 (1938).