Recent work of Fine and Ellis[12] indicates an interesting volume increase of ordered Fe-Co alloys. As indicated by one of us,[13] the increase of the average interatomic distance on ordering can be interpreted as an increase of repulsion between the partly filled 3*d*-shells. According to the theoretical explanation of the saturation moment in ferromagnetic alloys (Fig. 6), the 3*d*-shells consist of two parts, one of which is just filled up at the electron concentration corresponding to the maximum value of the magnetization. As we know from band theory of metals, full electronic shells produce strong repulsive forces. The apparent increase of repulsion between the iron and cobalt atoms can be interpreted as due to a more perfect filling of the lower part of the 3*d*-shells in both kinds of atoms in the state of perfect ordering. In the random state, as indicated before, only part of the atoms will have the proper neighborhood to assure the optimum local electron concentration, and the repulsion forces will be weaker.

[12] M. E. Fine and W. C. Ellis, to be published in Trans. A.I.M.E.
[13] R. Smoluchowski, discussion to paper in reference 12.

ACKNOWLEDGMENT

The authors are grateful to Dr. Sidney Siegel for discussions and for making the neutron diffraction studies, to Mr. R. K. McGeary for preparing the samples, to Mr. W. N. Johnson for capable assistance in making the measurements, and to the Westinghouse Research Laboratories for permission to publish these results.

Note added in proof: Preliminary calculations indicate that a similar mechanism might explain the rapid decrease of magnetostriction in Fe-Si alloys at the composition of about 12 at. percent Si, where ordering is known to take place. These results were reported by the authors at the National Research Council Conference on Phase Changes held at Cornell University in August, 1948.

PHYSICAL REVIEW VOLUME 75, NUMBER 1 JANUARY 1, 1949

Irreversible Magnetic Effects of Stress

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(Received June 8, 1948)

When Rayleigh's law for small hysteresis loops is interpreted in terms of the elementary process of domain wall displacement, its generalization to include effects of stress becomes possible. In this way theoretical formulas have been derived for the behavior of a soft iron or steel specimen which is first put into a state of normal magnetization at small magnetizing force H and is then subjected to a small tension cycle. According to the theory the magnetization should increase upon application of tension T by an amount $g(4.00\times 10^{-9}HT+9.86\times 10^{-18}T^2)$ in c.g.s. e.m.u., where $4\pi g$ is the slope of the normal permeability *vs.* H curve, and should remain at the new value upon removal of the tension. An experimental test of the theory verified these predictions within the precision claimed for them. On the basis of the theoretical model, certain effects of diminishing alternating fields and stresses are analyzed quantitatively; the results are used to explain qualitatively the observed long-time magnetic behavior of ships.

INTRODUCTION

THE changes of magnetization produced by stress are cause for concern whenever magnetic devices are subject to uncontrollable mechanical disturbances. Such changes are characterized by hysteresis and are, therefore, difficult to treat analytically. However, the stresses usually encountered are magnetically equivalent to rather small fields (as compared with those necessary to erase the previous history completely), and this suggests that it would be useful to extend the analytical description of small hysteresis loops, known as Rayleigh's law,[1] to cases in which the independent variable is stress rather than magnetizing force. The legitimacy of such an extension and the form that it must take both follow at once if one merely assumes that Rayleigh's law, with appropriate changes in the

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[1] R. Becker and W. Döring, *Ferromagnetismus* (Verlag Julius Springer, Berlin, 1939), p. 218 ff.

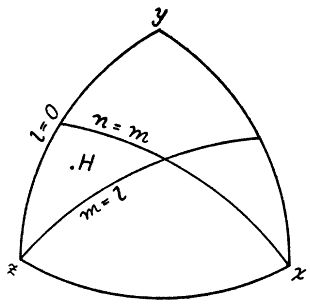

FIG. 1. Relation of field direction (H) to crystal axes. If labels (x, y, z) are assigned to the cubic axes of the individual crystals in a random manner, then with respect to the (x, y, z) axes the directions H will be distributed uniformly over the unit sphere. For present purposes the labels (x, y, z) are reassigned so that for half the crystals the relations are as shown here, with H somewhere in the indicated spherical triangle, and for the other half the axes are left-handed and the appropriate diagram is the mirror image of the one shown.

variables, applies to the elementary process of domain wall displacement[2] to which the macroscopic magnetization changes are due.

In 1945, at the Naval Ordnance Laboratory, the consequences of this assumption were developed theoretically and tested experimentally. The object was to arrive at a theoretical model capable of at least qualitative predictions, and therefore useful as a guide in planning experimental programs. From this point of view the theory was successful, for it predicted correctly the general behavior of the curves and the orders of magnitude of the constants deduced from them. Moreover, it did this on the basis of general crystal constants together with purely magnetic data on the particular specimen; no preliminary magnetomechanical data were required. Although the basic relations are simple, the detailed working out of their consequences is somewhat tedious, and therefore the results of this work—together with the experimental test of the theory—are of interest.

The theory presented here does not include an explanation of Rayleigh's law itself; that is a separate problem. For validity of the theory it is not necessary that Rayleigh's law hold for a single domain wall, provided it holds on the average for a group of many similar walls. The individual process is doubtless a discontinuous irreversible jump, but the jumps of different walls occur at different values of magnetizing force or stress. A derivation of Rayleigh's law, in the generalized form assumed here, therefore requires a statistical study of the behavior of domain walls. A derivation of this type has been given by Néel.[3]

THEORETICAL MODEL

The specimen is assumed to be a polycrystal wire or rod (of soft iron or steel) with field or tension or both applied along its axis. As is usual in domain theory, the magnetizing force and stress are assumed to be uniform. The polycrystalline specimen is furthermore assumed to consist of crystals whose axes are oriented at random. The results for it are therefore to be obtained by calculating for a single crystal in an arbitrary orientation and then averaging over all orientations.

In an iron crystal in which the internal stresses are not too high, each domain is (in the demagnetized state) magnetized to saturation along one of the "directions of easy magnetization," *viz.*, the [100] directions. Application of a magnetizing force or stress causes displacements of the walls separating adjacent domains, so that the more favorably magnetized domain grows at the expense of its neighbor. In the case of a magnetizing force, the more favorably magnetized domain is the one whose magnetization vector is closer, in direction and sense, to the magnetizing force; in the case of a tension, the more favorably magnetized domain is the one whose magnetization vector is closer in direction (without regard to sense) to the tension. Thus 90° walls, e.g. between [100] magnetization and [001], can be displaced either by magnetizing force or by tension; but 180° walls, e.g. between [100] and [$\bar{1}$00], are unaffected by tension. Furthermore, the 90° walls themselves may be grouped into two classes: those for which tension and positive magnetizing force produce displacements of the same sign and those for which they produce displacements of opposite signs.

In each crystal of the specimen, let the [100] axis closest to the positive field direction be labeled z, the one second closest y, and the one third closest x; then, for every crystal, the point on the unit sphere that represents the field direc-

[2] Reference 1, p. 105 ff.

[3] L. Néel, Cahiers de Physique, No. 12, p. 1 (Dec. 1942) and No. 13, p. 18 (Mar. 1943).

tion lies in the first octant and within (or on the boundary of) the spherical triangle bounded by the arcs $l=0$, $n=m$, and $m=l$, where (l, m, n) are the direction cosines of the field direction with respect to the (x, y, z) axes (Fig. 1). For an isotropic aggregate of crystals, the representative points are distributed at random over this spherical triangle; half the sets of axes are left-handed, but this is permissible in the present analysis.

An interdomain wall separates a domain magnetized along one of the directions z, y, x, $-x$, $-y$, $-z$ from one magnetized along another of these directions. Each possible pair of these symbols, with the above order preserved, represents a possible type of wall. Thus there are 15 types of wall:

	Class A		Class B		
z, y	z, x	$z, -x$	$z, -y$	$z, -z$	
	y, x	$y, -x$	$y, -y$	$y, -z$	
		$x, -x$	$x, -y$	$x, -z$	Class
			$-x, -y$	$-x, -z$	C
				$-y, -z$	

Of these, the ones labeled "Class A" undergo displacements of the same sign under positive magnetizing force and tension; the ones labeled "Class C" undergo displacements of opposite signs; the ones labeled "Class B" are not affected by tension.

Either magnetizing force or tension is thermodynamically equivalent to a hydrostatic pressure within each domain, acting on its domain walls. This pressure is $HJ_s \cos(J, H)+\frac{3}{2}T\lambda_{100}\cos^2(J, H)$ for the case of magnetizing force H and tension T along the same direction. Here J_s is the saturation magnetization, and λ_{100} is the saturation magnetostriction for a crystal magnetized along [100]. Subtraction of the pressures on opposite sides of a wall gives the net pressure on the wall; for instance, for a (z, y) wall the pressure (considered positive if it tends to increase the z-magnetized domain) is

$$p_{z,y}=HJ_s(n-m)+\tfrac{3}{2}T\lambda_{100}(n^2-m^2). \quad (1)$$

With the order of symbols used above, a positive field always favors the first member of the pair; a tension favors the first member of a class A pair and the second member of a class C pair.

It is usually assumed that small H and T produce a reversible displacement of each wall, proportional to the net pressure acting on it; and that the proportionality constant, averaged over all walls of a given type, has one value for the 12 types of 90° wall (classes A and C) and another value for the 3 types of 180° wall (class B). We now assume that in addition there is an irreversible displacement, governed, on the average for all walls of a given type, by a law similar to Rayleigh's; and that the constant in the irreversible term has one value for the 12 types of 90° wall and another value for the 3 types of 180° wall. Specifically, if v is the volume swept past by walls of any particular type (such as z, y) in unit volume of the crystal ($v=0$ in the demagnetized state) and if p is the net pressure determined by Eq. (1) or its appropriate modification, we assume:

(a) For a monotonic change from the demagnetized state, v has the sign of p, and

$$|v|=\alpha|p|+\beta p^2. \quad (2)$$

(b) For a subsequent change Δp of opposite sign to the change immediately preceding it, Δv has the sign of Δp, and

$$|\Delta v|=\alpha|\Delta p|+\tfrac{1}{2}\beta(\Delta p)^2, \quad (3)$$

provided $|\Delta p|$ is small enough so that the curve so calculated does not cross or touch the curve (2).

(c) For a change Δp larger than that specified in (b), (2) again holds.

(d) The constant α in the reversible term has one value α_1 for all wall types of classes A and C

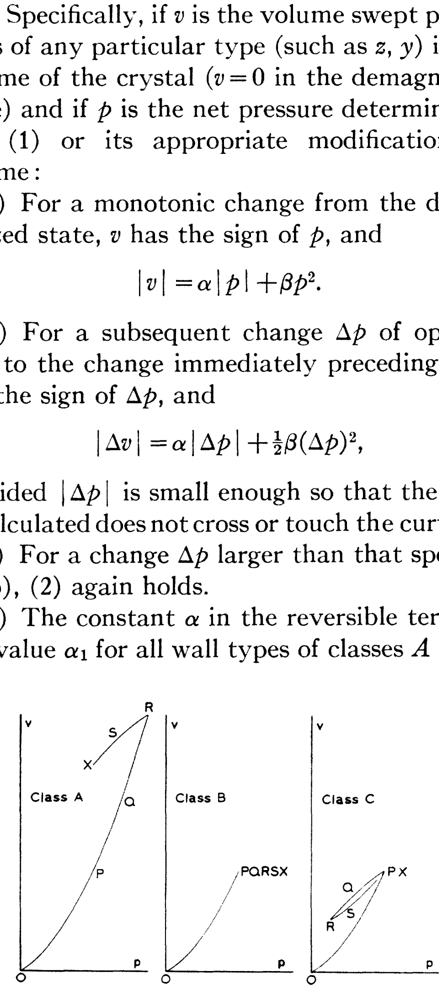

FIG. 2. Average behavior of a group of walls of one type. Class A, 90° walls for which tension is equivalent to positive magnetizing force; Class B, 180° walls; Class C, 90° walls for which tension is equivalent to negative magnetizing force. OP, magnetizing force applied; PQR, tension applied; RSX, tension removed. Abscissa p is hydrostatic pressure to which magnetizing force and tension are equivalent; ordinate v is volume swept out by walls of the specified type in unit volume of the specimen.

and another value α_2 for all wall types of class B. The constant β in the irreversible term has one value β_1 for all wall types of classes A and C and another value β_2 for all wall types of class B.

Assumptions (a) and (c) go somewhat beyond Rayleigh's law as it is usually stated, but they are at least approximately in accordance with experiment when the variables are magnetizing force and magnetization, and they also follow from Néel's theory. To the approximation adopted here, the normal and virgin states are identical and, more generally, any attainable pair of values (p, v) specifies a magnetic state uniquely.

Suppose, now, that an initially demagnetized specimen is subjected to the following treatment: (i) the magnetizing force is increased to a value H; (ii) a tension T is applied; (iii) the tension is removed. The behavior of each class of wall is illustrated in Fig. 2. In each of the three diagrams, curve OP corresponds to step (i), curve PQR to step (ii), and curve RSX to step (iii). The behavior of class B is simplest; Eq. (2) holds during step (i) and nothing happens thereafter. For class A, Eq. (2) holds during steps (i) and (ii), Eq. (3) during step (iii). For class C, Eq. (2) holds during step (i) and Eq. (3) during steps (ii) and (iii), which therefore cancel each other and return the walls to P after traversal of a Rayleigh loop. The net irreversible change of magnetization resulting from application and removal of the tension is entirely due to walls of class A.

Upon carrying out the details of the calculation and combining the contributions of all types of wall, we obtain the following formulas for the *irreversible* magnetization J_{ir} of a single crystal; the reversible part (determined by α_1 and α_2) is simply $\chi_0 H$, where χ_0 is the initial susceptibility.[4]

(i) H applied:

$$J_{ir} = J_s^3 H^2 (2\beta_1\phi_1 + \beta_2\phi_2) \tag{4}$$

with

$$\phi_1 = 2(n^3 + 3nm^2 + n^3 + 3nl^2 + m^3 + 3ml^2), \tag{5}$$

$$\phi_2 = 8(n^3 + m^3 + l^3). \tag{6}$$

(ii) T applied—ΔJ_{ir} measured from the state with H at its final value and T zero:

$$\Delta J_{ir} = \beta_1 (3 J_s^2 \lambda_{100} H T \psi_1 + (9/8) J_s \lambda_{100}^2 T^2 \psi_2) \tag{7}$$

with

$$\psi_1 = 4(n^4 - l^4), \tag{8}$$

$$\psi_2 = 2[n(n^2 - m^2)^2 + n(n^2 - l^2)^2 + m(m^2 - l^2)^2]. \tag{9}$$

(iii) T removed—ΔJ_{ir} measured from the state at the end of step (ii):

$$\Delta J_{ir} = 0. \tag{10}$$

The removal of tension causes no change of macroscopic magnetization because the displacements of walls of classes A and C produce equal and opposite changes of magnetization.

For an isotropic polycrystalline aggregate, the functions ϕ_1, ϕ_2, ψ_1, and ψ_2 must be averaged over the spherical triangle in Fig. 1. The averages are

$$\langle\phi_1\rangle = 3\sqrt{2} = 4.2426,$$
$$\langle\phi_2\rangle = 6 = 6.0000,$$
$$\langle\psi_1\rangle = 32/5\pi = 2.0372,$$
$$\langle\psi_2\rangle = \sqrt{2} = 1.4142.$$

I am indebted to Mr. Morton S. Raff for the carrying out of the integrations.

The variation of magnetization with tension during step (ii) is given, for the polycrystalline specimen, by

$$\Delta J = \gamma_1 H T + \gamma_2 T^2, \tag{11}$$

where

$$\gamma_1 = 3\beta_1 J_s^2 \lambda_{100} \langle\psi_1\rangle, \tag{12}$$

$$\gamma_2 = (9/8)\beta_1 J_s \lambda_{100}^2 \langle\psi_2\rangle. \tag{13}$$

The normal magnetization curve at low magnetizing force is given by the equation for step (i) with the reversible term added; consequently, the normal permeability as a function of magnetizing force is given (in accordance with the original form of Rayleigh's law) by

$$\mu = \mu_0 + cH, \tag{14}$$

where μ_0 is the initial permeability and

$$c/4\pi = J_s^3 (2\beta_1\langle\phi_1\rangle + \beta_2\langle\phi_2\rangle) \equiv g. \tag{15}$$

If the ratio $f = \beta_2/\beta_1$ were known, β_1 could be evaluated from the slope of the μ *vs.* H curve. For lack of better knowledge, we tentatively assume $f = 1$; this gives a tentative value for β_1,

$$\beta_1' = g / J_s^3 (2\langle\phi_1\rangle + \langle\phi_2\rangle). \tag{16}$$

The ratio of the (unknown) correct value to this tentative value is

$$\beta_1/\beta_1' = (2\langle\phi_1\rangle + \langle\phi_2\rangle)/(2\langle\phi_1\rangle + f\langle\phi_2\rangle)$$
$$= 1.707/(1 + 0.707 f). \tag{17}$$

[4] Reference 1, p. 148 ff.

If $f<1$, the correct value of β_1 must be between β_1' and $1.707\beta_1'$. If $f>1$, the correct value will be smaller than β_1', but of the same order of magnitude if f is of order of magnitude unity. The error introduced by assuming $f=1$ therefore is probably no more serious than other errors inherent in our simplified model, and it does not affect either the form of the laws or the order of magnitude of the constants.

The assumption $f=1$ implies equal importance of 90° and 180° walls in phenomena attributable to irreversible wall displacements. If only 90° walls were important ($\beta_2=0$), the results would not be much different; but if only 180° walls were important ($\beta_1=0$), the phenomena under discussion would, according to our theory, not occur.

By setting $f=1$ and inserting the numerical values of the averages, we get

$$\gamma_1/g=0.42190\lambda_{100}/J_s, \tag{18}$$

$$\gamma_2/g=0.10983(\lambda_{100}/J_s)^2. \tag{19}$$

With $J_s=1710$ e.m.u. and $\lambda_{100}=16.2\times10^{-6}$,

$$\gamma_1/g=4.00\times10^{-9} \text{ e.m.u.}, \tag{20}$$

$$\gamma_2/g=9.86\times10^{-18} \text{ e.m.u.} \tag{21}$$

The theoretical model leads not only to a definite formula for the variation of magnetization with tension, but to definite values of the constants in the formula, and to evaluate them one needs only a set of purely magnetic data on the particular specimen—the variation of normal permeability with small magnetizing force. At this point, therefore, a series of measurements was carried out to test the predictions of the theory.

For validity of formula (11), the tension must not exceed a certain critical value, namely, the value at which the condition for validity of Eq. (3) ceases to hold during step (ii). This critical value is

$$T_c=(4/3\sqrt{2})(J_s/\lambda_{100})H=0.945(J_s/\lambda_{100})H. \tag{22}$$

If the critical value is exceeded, some walls of some of the crystals will cease to obey Eq. (3) and will obey, instead, Eq. (2). The effect on the behavior of the specimen will be small if the critical value is not greatly exceeded, since only a small fraction of all the walls will violate the condition for validity of Eq. (3).

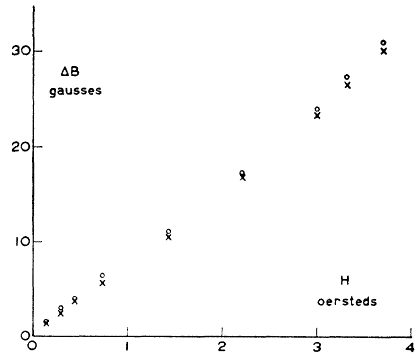

FIG. 3. Representative experimental data on irreversible change of flux density, ΔB, resulting from application and subsequent removal of tensile force 58.3 lb., with specimen initially in state of normal magnetization at magnetizing force H. Circles, change resulting from application of tension; crosses, net change resulting from application and subsequent removal of tension.

EXPERIMENTAL TEST

To test the theory, a steel specimen whose μ *vs.* H curve had been measured was put through the succession of steps analyzed in the preceding section, with various values of H and T. These measurements were carried out by Mr. E. L. Sanderson under the direction of Mr. J. R. Wright.

The specimen was a cylindrical rod 24″ long and 0.126″ in diameter. The correction for demagnetizing force is small and was neglected. For $H\leq4$ gauss, Eq. (14) holds with $\mu_0=75.2$ and $c=4.40$ gauss^{-1}. (The dimensional convention adopted here for e.m.u. is $[B]=[H]$.) The theoretical values of γ_1 and γ_2 are, therefore,

$$\gamma_1=1.40\times10^{-9} \text{ cm}^2 \text{ dyne}^{-1},$$
$$\gamma_2=3.45\times10^{-18} \text{ gauss cm}^4 \text{ dyne}^{-2}.$$

For the tension experiments, the magnetizing force was applied in the usual manner, with a solenoid. The ends of the sample were threaded into brass rods, one of which was connected to a weight and lever system for applying known tensions. The changes in flux density were measured by means of a 5000-turn, 252.6-ohm search coil connected to a fluxmeter (Leeds and Northrup galvanometer, Cat. No. 2290).

For each measurement, the specimen was demagnetized and put into a state of normal magnetization (the use of normal rather than virgin magnetization in step (i) makes Eq. (2) more certainly applicable). A known tension was applied and the flux change observed; then the tension was removed and the flux change observed. The entire procedure was repeated for each pair of values of magnetizing force and tension.

According to Eq. (11), the flux change upon application of tension should be given by

$$\Delta B = \Gamma_1 FH + \Gamma_2 F^2, \tag{23}$$

where F is the force applied; the Γ's are simply related to the γ's and to the area of the cross section of the specimen. The flux change upon removal of the tension should be zero. Figure 3 shows representative data on the change of flux density produced (a) by application of tension and (b) by application and subsequent removal of tension. There is some change upon removal of the tension, but it is small.

The data were plotted and analyzed and the experimental values of the constants evaluated by Mr. Morton S. Raff.

To test Eq. (23), ΔB was first plotted against H, for each value of F. The data for each value of F fall on a straight line within the precision of the measurements. The slopes give values of Γ_1 ranging from 0.114 lb.$^{-1}$ for $F=23.1$ lb. to 0.154 lb.$^{-1}$ for $F=71.5$ lb. The intercepts $\Gamma_2 F^2$ are too small to be determined by this method.

The requirement $T \leq T_c$ becomes in this case $F/H \leq 18.0$ lb./gauss. For the smallest F, 23.1 lb., this condition is satisfied for $H \geq 1.28$ gauss; for the largest F, 71.5 lb., it is satisfied only for $H \geq 3.97$ gauss. The progressive change in the slopes of the curves indicates a progressively increasing importance of processes not correctly described by Eq. (3), and the best value of Γ_1 for comparison with theory is that for the smallest F, 0.114 lb.$^{-1}$. This gives

$$\gamma_1 = 1.64 \times 10^{-9} \text{ cm}^2 \text{ dyne}^{-1},$$

in satisfactory agreement with the theoretical value 1.40×10^{-9}. In fact, the agreement is better than one could legitimately expect in view of the simplifications made in the theory, such as uniformity of magnetizing force and stress and equality of the two β's.

To determine Γ_2, $\Delta B/F$ was plotted against F for each value of H. With this method of plotting, the requirement $T \leq T_c$ is satisfied over no part of the range at the lowest H, 0.148 gauss, but over all of it at the highest, 4.062. The straight line drawn for $H=4.062$ gauss gives $\Gamma_1 = 0.107$ lb.$^{-1}$, which agrees with the previous value 0.114 within the precision of the straight line representation of the badly scattered points, and it gives $\Gamma_2 = 0.00309$ gauss/lb.^{2}, whence

$$\gamma_2 = 8.03 \times 10^{-18} \text{ gauss cm}^4 \text{ dyne}^{-2}.$$

This agrees with the theoretical value, 3.45 $\times 10^{-18}$, as well as could be expected. The theoretical value is affected by the approximations made in the derivation and by the uncertainty in the value of λ_{100} (which is squared here); the experimental value is deduced with difficulty from data which, plotted in this way, exhibit large scatter.

The experimental data confirm the predictions of the theory within the precision claimed for it.

DIMINISHING ALTERNATING FIELDS AND STRESSES

The same theoretical model can be used to calculate the behavior under other field and stress conditions. A few such results will be stated here for the case of a magnetic field or tension that undergoes alternations of sign with gradual diminution of amplitude, in the presence of a constant polarizing magnetizing force h. The results apply only to cases in which the instantaneous net magnetizing force or, more generally, the instantaneous "pressure" of Eq. (1) expressed as an equivalent magnetizing force is always considerably smaller than the coercive force, so that Rayleigh's law applies. Such a process can produce significant changes of magnetization, but it cannot bring about complete demagnetization in the case $h=0$ or establish the ideal magnetization corresponding to a value $h \neq 0$. Simple results are obtained only when h is small in comparison with the initial amplitude of the alternating component H; in view of the restrictions already placed on the net magnetizing force, the case $h \ll H$ may seem trivial, but actually it is not, for the presence of even a small biasing field can result in an appreciable magnetization when the process is of the type considered.

To a good approximation, the magnetization resulting from diminishing cycles of initial amplitude H in polarizing magnetizing force $h \ll H$ ("idealization in h by H") is

$$J = (\chi_0 + 2gH)h; \tag{24}$$

the susceptibility with respect to h is increased by the idealization process from χ_0 to χ_0+2gH. For diminishing tension cycles of initial amplitude T ("idealization in h by T"), gH is to be replaced by $\gamma_1 T$. If, as was assumed before, $\beta_2=\beta_1$, then an idealizing tension T is equivalent to an idealizing magnetizing force H given by

$$H/T=\gamma_1/g=0.4219(\lambda_{100}/J_s), \quad (25)$$

or 1 dyne/cm² is equivalent to 4.00×10^{-9} gauss.

The effect of successive idealizations in h by H and in h' by H' depends on which is larger, H or H'. If the second idealization is weaker than the first, it produces a change of magnetization (from the value after the first)

$$\Delta J=(\chi_0+2gH')(h'-h); \quad (26)$$

if it is stronger than the first, it erases it and produces a magnetization

$$J=(\chi_0+2gH')h'. \quad (27)$$

More generally, in a series of idealizations, each produces a magnetization differing by the amount (26) from that after the most recent stronger idealization; any intermediate weaker ones are erased. For successive idealizations in h by T and in h' by T', gH' is to be replaced by $\gamma_1 T'$. Idealizations first by field and then by stress, or *vice versa*, are more complicated in their effects; walls of class B are affected by fields but not by stresses, and so complete erasure of the former by the latter does not occur.

The relations summarized above are in qualitative agreement with the experimentally known behavior. For instance, according to our simplified theory the first of a series of equal shocks (magnetic or mechanical) should produce a change of magnetization, and the subsequent ones should have no effect; what actually happens is that 90 percent or more of the change is produced by the first shock. No quantitative data are available to test the theory in the small-field range where, if anywhere, it should hold precisely. Shock amplitudes of practical interest are likely to lie somewhat beyond the range of validity of the theory, so that its chief practical usefulness is for approximate rather than precise predictions.

The results quoted here for tension are subject to an error not present in the purely magnetic case. The condition analogous to $h\ll H$, when H is replaced by a tension, is never satisfied by every wall of every crystal because of the geometrical factors in Eq. (1). However, if $J_s h\ll\lambda_{100}T$, the condition will be satisfied by most of the walls.

The foregoing results may be used to calculate, roughly, the behavior of a complex body, such as a ship, when it is subjected to irregular mechanical disturbances of the sort that occur in its normal history.[5] For this purpose we use a much simplified model. We suppose that the body consists of a large number of identical and mechanically independent parts and that these parts are also magnetically independent, except for a common demagnetizing field $-NJ$ determined by the average magnetization J of the body and by a demagnetizing factor N characteristic of the body as a whole. We suppose that the probability of receiving a shock of specified amplitude, in any specified time interval, is the same for each part, and we interpret a "shock" as a process similar to the "idealization in h by T" considered above.

We consider first the case in which the shocks all have the same amplitude T, but the applied field varies in an arbitrary manner. Suppose at first that the magnetizing force is a given function of time, $h(t)$. Let pdt be the probability that a part will be shocked during time interval dt, and choose the time scale so that p is constant. During dt, a fraction pdt of the parts receives a shock and changes from the existing magnetization to a magnetization χh, where $\chi=\chi_0+2\gamma_1 T$. Since these parts were selected at random, their average magnetization before the shock was the same as that of the whole body, J, and therefore the change of magnetization caused by shocks is $\chi h-J$ when averaged over the shocked parts and $(\chi h-J)pdt$ for the whole body. Meanwhile, all but an infinitesimal fraction pdt of the parts have changed their magnetization reversibly by $\chi_0 dh$. The total change is $dJ=(\chi h-J)pdt+\chi_0 dh$, or

$$dJ/dt+pJ=p\chi h+\chi_0 dh/dt. \quad (28)$$

If the demagnetizing field is negligible, Eq. (28) shows that for constant applied field the magnetization approaches its final value according to

[5] The formulas to be given were derived by the author in 1942; the assumptions on which they rest had at that time only an experimental basis. A qualitative formulation of the theory, based on the same general picture and leading to similar conclusions, was arrived at independently at about the same time by Dr. S. Wolman and his collaborators at the Bureau of Ordnance.

an exponential law with a time constant $1/p$, which is easily seen to be the mean time that a part has to wait before receiving its first shock. If the demagnetizing field is not negligible, set $h=h_0-NJ$ in Eq. (28); the result is an equation of the same form, but with the susceptibilities replaced by the corresponding apparent susceptibilities and with p replaced by

$$p'=p(1+N\chi)/(1+N\chi_0). \tag{29}$$

Since $\chi>\chi_0$, the time constant is shortened by the demagnetizing field. For constant h_0 this may be interpreted as follows: the first changes occur at a larger h than is ultimately present; therefore, they take the average magnetization toward its ultimate value faster than if h were maintained constant and equal to its ultimate value.

We consider second the case in which the demagnetizing field is negligible and $h=h_0=$constant, with the body initially demagnetized, but the shocks have various amplitudes. Then in any part a shock produces an effect only if it is larger than every previous shock. We measure J from its initial value $\chi_0 h$ as zero, and we measure the shock amplitude by the J which it can produce. Let $p(J)dJdt$ be the probability that, in time dt, a part will receive a shock of amplitude between J and $J+dJ$. Then the probability per unit time of a shock greater than J is

$$F(J)=\int_J^{J_m} p(J)dJ, \tag{30}$$

where J_m corresponds to the largest shocks that ever occur. The fractional number of parts that escape shocks larger than J up to time t is, by familiar arguments, $e^{-F(J)t}$. The fractional number for which the largest shock experienced, up to time t, was one between J and $J+dJ$, is the differential of this with respect to J. The average magnetization at time t is therefore

$$\bar{J}=\int_0^{J_m} J(\partial/\partial J)[e^{-F(J)t}]dJ=Je^{-F(J)t}\Big|_0^{J_m}-\int_0^{J_m} e^{-F(J)t}dJ=J_m-\int_0^{J_m} e^{-F(J)t}dJ, \tag{31}$$

since $F(J_m)=0$.

For small t, Eq. (31) gives approximately

$$\bar{J}=t\int_0^{J_m} F(J)dJ. \tag{32}$$

Thus the initial rate of rise depends on the frequencies of shocks of all amplitudes. For large t, let $F(J)=u$; then the integrand in Eq. (31) may be written $e^{-ut}du/p(J)$, and if $p(J_m)\neq 0$ the result is approximately

$$\bar{J}=J_m-1/p(J_m)t. \tag{33}$$

Thus the asymptotic rate of approach to the final value depends only on the frequencies of the largest shocks: the smaller ones have already done all they can.

The results obtained here are formally very similar to those of Kittel's[6] phenomenological theory, but our interpretation is quite different from his. Kittel attributes the time constants to relaxation processes in the material, whereas in the present theory the time constants are properties not of the material but of its environment. Our calculation shows that magnetostrictive processes are capable of producing a behavior of the general type observed and therefore must be taken into account in any satisfactory theory of the phenomenon. On the other hand, slow mechanical and metallurgical changes in the material unquestionably do occur, and these may also contribute to the observed behavior; their effect would be primarily a change of magnetic *properties*, rather than merely of magnetic *state*.

The contributions made to this investigation by Messrs. Wright, Sanderson, and Raff have already been mentioned. I wish in addition to acknowledge my indebtedness to Dr. S. Wolman and to Mr. J. H. Sweer for many helpful discussions of this and related problems, and, finally, to Mr. J. A. Osborn for his careful carrying out of an earlier experimental program that formed the basis for many of the qualitative ideas presented here.

[6] Charles Kittel, Phys. Rev. 69, 640 (1946). The theory was first propounded in 1941.