On the Maximum β-Energy Release in Tritium*

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SOME time ago (1947) Konopinski emphasized that the then existing data on the half-life and the maximum β-energy release in H^3 implied a "degree of allowedness" for it much greater than that for the supposedly equally allowed He^6 spectrum:[1]

$$|M|^2 \text{ ft.} = 900 \text{ for } H^3, \quad |M|^2 \text{ ft.} = 5760 \text{ for } He^6.$$

More recently Bowers and Rosen[2] have pointed out that work by Curran *et al.*[3] and by Novick[4] (maximum β-energy release of 16.9 ± 0.3 kev; half-life of 12 years) greatly minimize the above discrepancy, while the yet unpublished maximum β-energy value of Pontecorvo obtained with a proportional counter, 18.5 kev (quoted in Seaborg),[5] practically removes it (see below).

We wish to remark in the present note that the accurate calorimetric determination of the average β-energy release in H^3, 5.69 ± 0.06 kev, just published by Jenks *et al.*[6] enables an equally accurate determination of the maximum β-energy release which, moreover, turns out to be in excellent agreement with Pontecorvo's. Thus, suppose the tritium spectrum is Fermi allowed; one then has:[7]

$$5.69\pm0.06 \text{ kev} = \text{average } \beta\text{-energy release} \equiv \langle(\epsilon-1)\rangle_{Av}$$

$$= \frac{\int_1^{\epsilon_{max}} \frac{2\pi Z}{137}\frac{\epsilon}{(\epsilon^2-1)^{\frac{1}{2}}}\left\{1-\exp\left(-\frac{2\pi Z}{137}\frac{\epsilon}{(\epsilon^2-1)^{\frac{1}{2}}}\right)\right\}^{-1}(\epsilon^2-1)^{\frac{1}{2}}\epsilon(\epsilon_{max}-\epsilon)^2(\epsilon-1)d\epsilon}{\int^{\epsilon_{max}} \frac{2\pi Z}{137}\frac{\epsilon}{(\epsilon^2-1)^{\frac{1}{2}}}\left\{1-\exp\left(-\frac{2\pi Z}{137}\frac{\epsilon}{(\epsilon^2-1)^{\frac{1}{2}}}\right)\right\}^{-1}(\epsilon^2-1)^{\frac{1}{2}}\epsilon(\epsilon_{max}-\epsilon)^2 d\epsilon} \quad (1)$$

where ϵ = kinetic + rest energy of the emitted β-particle (in units of its rest energy), ϵ_{max} = maximum kinetic + rest energy of the emitted β-particle, and Z = nuclear charge of the daughter element = 2. Numerical integration and interpolation in Eq. (1) gives (see Fig. 1):

$$\epsilon_{max} - 1 = (3.64\pm0.04)\times10^{-2} = 18.6\pm0.2 \text{ kev},$$

the discrepancy with Pontecorvo's value being well within experimental error. A comparison of the "degrees of allowedness" of H^3 and He^6 (calculated with the above $(\epsilon_{max})_{H^3}$, with an H^3 half-life of 12.46 years,[6] and with more recent values of the half-life and maximum β-energy release of He^6: 0.89 sec.[5], $(\epsilon_{max})_{He^6} = 3.5\pm0.6$ Mev[8]) yields:

$$|M|^2 \text{ ft.} = 3360\pm200 \text{ for } H^3,$$
$$|M|^2 \text{ ft.} = 6300\pm3000 \text{ for } He^6.$$

Complete consistency is thereby established between the last quoted H^3 and He^6 measurements and between the application of the Gamow-Teller selection rules to these two simplest of the β-active nuclei.

In conclusion it may be pointed out that if we had (incorrectly) used the value $Z=1$, appropriate to the parent nucleus, in the Coulomb factor of the β-energy distribution in the integral, we would have obtained the (incorrect) maximum β-energy release: 17.8 ± 0.2 kev. The error made in using $Z=1$ instead of $Z=2$ is also appreciable in the Kurie plot of the β-spectrum; below we append a Kurie plot of an H^3 β-distribution actually obeying the Fermi allowed shape with $Z=2$ (curve A, Fig. 2); the same distribution is then plotted with use of a Coulomb factor appropriate to $Z=1$ (curve B, Fig. 2).

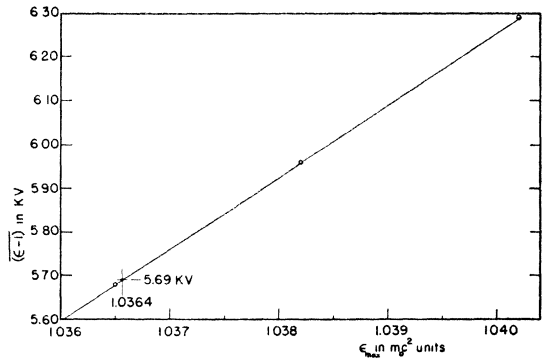

FIG. 1.

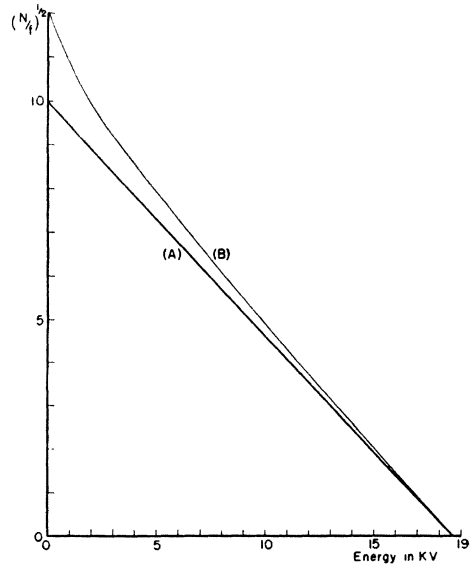

FIG. 2. A: Kurie plot for an H^3 β-distribution with $Z=2$. B: Plot of the same distribution with Coulomb factor appropriate to $Z=1$.

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[3] S. C. Curran, J. Angus, and A. L. Cockcroft, Nature **162**, 302 (1948).
[4] A. Novick, Phys. Rev. **72**, 972 (1947).
[5] G. T. Seaborg, Rev. Mod. Phys. **20**, 585 (1948).
[6] G. H. Jenks, J. A. Ghormley, and F. H. Sweeton, Phys. Rev. **75**, 701 (1949).
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Neutron and Proton Binding Energies in the Region of Lead*

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THE maxima in α-particle decay energies for mass numbers 210–215 recently emphasized by Perlman, Ghiorso, and Seaborg[1] can be looked at in terms of neutron

and proton energies and, when thus interpreted, reveal rather sharp discontinuities in these bindings at proton number 82 and neutron number 126. The numbers 82 and 126 are two of the "magic" numbers connected with marked nuclear stability on which attention has been focused by M. G. Mayer.[2] If one considers these "magic" numbers as numbers for which neutron or proton shells are closed, one would expect unusually high binding energies for the 82nd, 81st, etc., proton and for the 126th, 125th, etc., neutron and unusually small bindings for protons with number slightly greater than 82 and neutrons with number slightly greater than 126. As the new shells fill up, the binding energies should gradually return to "normal."

The binding energy of four neutrons to certain heavy nuclei can be found with a good deal of accuracy from known α- and β-decay energies. Thus the binding energy of four neutrons to U^{234} is equal to {mass (U^{234})+mass ($4n$'s) −mass (U^{238})} or {mass ($4n$'s)−mass (He^4)−$E_\alpha(U^{238})$ −$E_\beta(UX_1)$−$E_\beta(UX_2)$−E_{recoil}} which equals[3,4] {29.64−4.18 −0.20−2.32−0.07} or 22.9 Mev. The binding energy of four neutrons to Pb^{208} turns out by a similar calculation to be only 17.6 Mev using $E_\beta(\text{ThB})=0.88$ Mev, $E_\beta(\text{ThC})=2.20$ Mev, and $E_\alpha(\text{ThC}')=8.78$ Mev. The Bohr-Wheeler[5] liquid drop model with the semiempirical constants given by them gives 22.8 Mev for the binding of 4 neutrons to U^{234}. However, for the Pb^{208} value the Bohr-Wheeler model gives again 22.8 Mev in marked disagreement with the 17.6 Mev now found from decay energies.

If one neutron binding energy in each of the four naturally radioactive families is known, a whole series of additional binding energies can be calculated from the information now available on α- and β-decay energies. In the case of α-decay

$$B_n(A-4, Z-2)=B_n(A, Z)-E_{\alpha 1}-E_{\alpha 2},$$

where $B_n(A, Z)$ is the binding energy of a neutron to a nucleus with mass A and charge Z, and where $E_{\alpha 1}$ and $E_{\alpha 2}$ are the disintegration energies associated with the emission of α-particles from the nuclei (A, Z) and $(A+1, Z)$, respectively. A similar relation holds for β-decay.

Since no measurements of neutron binding energies in the very heavy region are available, the four individual values for U^{234}, U^{235}, U^{236}, and U^{237} given by the Bohr-Wheeler model have been taken as starting points. The sum of these four binding energies as already pointed out, agrees very well with the latest measurements of α- and β-decay energies and there seems at present no reason to believe that shell structure plays an important part in nuclear binding in the uranium region.

Proceeding in steps by means of decay energies the neutron and proton binding energies in the region of the two magic numbers 82 and 126 have been calculated. The results are in doubt not only because of inaccuracy in the binding energies taken as starting points but also because of uncertainties in some of the disintegration energies, especially those for β-decay where decay schemes are often in some doubt. Some checks were possible from stability considerations and cycle calculations but in some cases it was necessary to depend on cycle calculations entirely. Such a calculation is one in which a value is found by requiring disintegration energies leading from the same initial to the same final nucleus to be equal.

Table I shows the results. Here the value 7.1 in the first row is the binding energy in Mev of the 126th neutron to a nucleus containing 81 protons and 125 neutrons, while the value 7.8 in the second row gives the binding energy of the 82nd proton to the same nucleus. The values found support the shell picture since unusually high binding energies are observed just before and unusually small ones just after the completion of the shells at neutron number 126 and proton number 82. The nucleus Pb^{208} containing both 82 protons and 126 neutrons is seen to act like a core, additional neutrons and protons being bound to it by approximately equal amounts.

TABLE I. Neutron and proton binding energies.

N / Z	124	125	126	127	128	129	130	131
Tl_{81}		7.1	3.5	4.7	4.0(?)			
Pb_{82}	6.9	7.8	7.7	7 7	8.5(?)			
		7.0	3.5	5.5	4.0	4.6		
Bi_{83}		*	3.5	4.9	4.6	4.8		
			4.9	5.3	4.1	4.9	4.3	
Po_{84}			5.3	5.2	5.6	5.4	6.7	6.7
			4.8	5.7	4.0	6.2	4.3	5.3
At_{85}							4.9	5.5

* Experimental value of 7.45±0.2 just reported by McElhinney *et al.*, Phys. Rev. **75**, 542 (1949).

Results rather similar to those of Table I were found by Berthelot[6] in 1942 from data on the disintegration energies of Pb, Bi, and Po isotopes only.

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[1] I. Perlman, A. Ghiorso, and G. T. Seaborg, Phys. Rev. **74**, 1730 (1948).
[2] Maria G. Mayer, Phys. Rev. **74**, 235 (1948).
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[4] H. Bradt, Nucleonics (May 1948), Part 2.
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Erratum: On the Application of Heisenberg's Theory of S-Matrix to the Problems of Resonance Scattering and Reactions in Nuclear Physics

[Phys. Rev. **74**, 131 (1948)]

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IT should be pointed out, first, that the one-level formula (52) is not different from the one given in literature. The difference in form is only due to the fact that in (52) the true level W_S and true width γ_S are used, while in the usual formulation W_S and γ_S are defined differently. Second, the parameters α and β of Section 4 should in general cases be considered as functions of $k_A^{(0)}$ and $k_C^{(0)}$. No demand on the analytical nature of α and β can be made by the present treatment. In fact they are not analytic according to the treatment of Wigner and Eisenbud.[1] Third, the present author was not aware of several papers by Professor Wigner and others[1] which give many far reaching results. Therefore he is anxious to withdraw the last sentence in the Introduction of his paper.

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[1] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947); E. P. Wigner, Phys. Rev. **73**, 1002 (1948); G. Goertzel, Phys. Rev. **73**, 1463 (1948).

Erratum: Second-Order Stark Effect of Methyl Chloride

[Phys. Rev. **75**, 889 (1949)]

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BECAUSE of an error in reducing the data, the field strengths of Fig. 1 should all be reduced by the factor 0.91. Using the corrected field strengths, the value of the dipole moment for methyl chloride becomes $\mu_0=1.87\pm0.03$ Debye units, which is in agreement with 1.86D given by Smyth.[1]

[1] C. P. Smyth, *Dielectric Constant and Molecular Structure* (Reinhold Publishing Company, New York, 1931).