the two statistics becomes discernible at about 1.7 to 1.8°K. The vapor pressure of $He^4$ is then, however, of the order of 1 cm Hg or less and it becomes extremely difficult to measure $B$ accurately. Kistemaker[3] has made measurements down to 1.8°K but the uncertainty of his $B$ at the lowest temperature at which he measured is too high so that it is impossible to make an experimental verification of the fact that $He^4$ obeys Bose-Einstein statistics.

For $He^3$ the situation is, however, much more favorable. Due to its smaller mass, quantum effects will be more pronounced and its vapor pressure will be much higher. Indeed, as far as the vapor pressure is concerned, the theoretical predictions of J. de Boer and Lunbeck[4] were completely borne out by the experiments of Sydoriak, Grilly and Hammel.[5] At 1.2°K the vapor pressure of $He^3$ is still 2 cm Hg and it should therefore be possible to measure $B$ down to about 1.2°K. The amount of $He^3$ needed for these measurements is not too large, even not in view of the scarcity of the light helium isotope in a pure state. Kistemaker[3] used about 200 cc S.T.P. which is about ten times as much as the 20 cc S.T.P. used by Sydoriak, Grilly and Hammel[5] in their experiments.

In order to get a rough check as to the influence of the smaller mass on the behavior of the $BT^{\frac{1}{2}}$ curve, we have calculated $B$ as a function of $T$ for low temperatures, using the hard sphere model.[1, 6] Using the same radius for mass 3 and for mass 4, and assuming both times Bose-Einstein statistic it turns out that the maximum in the curve for mass 3 was shifted to a temperature about one third higher than in the case of mass 4. Calculations using the correct field of force are now in progress, but it seems safe to conclude that, indeed, for the case of $He^3$ the second virial coefficient can show us whether or not $He^3$ really obeys Fermi-Dirac statistics.

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[4] J. de Boer and R. J. Lunbeck, Physica **14**, 510 (1948).
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[6] G. E. Uhlenbeck and E. Beth, Physica **3**, 729 (1936).

## Determination of the Potential from the Asymptotic Phase

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As V. Bargmann has shown,[1] problems concerning the determination of the potential from the asymptotic phase require a mathematically rigorous treatment since heuristic processes lacking in rigor have led to erroneous results. By methods developed in a paper to appear shortly in the Det Kgl Danske Videnskabernes Selskab (where the case $l=0$ is taken up) it is possible to show that the solution $y(x, \lambda)$ of

$$y''+\left(\lambda^2-\frac{l(l+1)}{x^2}-V(x)\right)y=0$$

with boundary condition $y(x, \lambda)/x^{l+1}\rightarrow 1$ as $x\rightarrow +0$, exists if $\int_0^\infty x|V(x)|dx<\infty$. Moreover for any $\lambda>0$, as $x\rightarrow\infty$, it is the case that

$$y(x, \lambda)\sim(A(\lambda)/\lambda^{l+1})\sin(\lambda x-\tfrac{1}{2}l\pi-\phi(\lambda)).$$

The function $\phi(\lambda)$ is called the asymptotic phase. If $l(l+1)/x^2+V(x)\geqq 0$, then $\phi(\lambda)$ determines $V(x)$ uniquely. It is also the case that $\log A(\lambda)$ and $\phi(\lambda)$ are conjugate functions.

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## On the Beta-Particle Spectrum from the Decay of Radium $E$*

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Several investigations have now been made of that portion of the energy-distribution of beta-particles from the decay of Ra$E$($_{83}Bi^{210}$), which lies at quite small energies. H. O. W. Richardson seems to have made the first[1] by observing tracks from active sources on various thin supports in a cloud chamber, and making calculated allowances for back-scattering and absorption. The points plotted in Fig. 1 show the differential distribution (here called ($A$)) obtained[2] by Richardson. Somewhat later C. B. Madsen made an investigation[3] of this spectrum, using active deposits on collodium supports (0.2$\mu$ thick), and giving careful attention to the reduction of scattering in the analyzer; to correct for the effects of the source-support, he used an empirical relation suggested by the measurements of Flammersfeld:[4]

$$\log I_{\text{support}}=a-bE,$$

where $a$ and $b$ are constants. Dr. Madsen writes that his results, shown by curve ($B$) in Fig. 1, seem to be in good agreement with our earlier observations.[5]

Curve ($C$) in Fig. 1 shows the distribution which we have recently obtained from observations of beta-particle tracks from a vapor of $Bi^{210}(CH_3)_3$ in a cloud chamber. Professor Richardson writes that Mrs. A. Leigh-Smith and he considered this method some years ago. In our observations 131 tracks were photographed non-stereoscopically and measured by the method of Petrova.[6] Corrections were applied to reduce the errors of projections and to allow for the change of effective solid angle with track-length. The conversion to energy as the independent variable, was effected with the best available composite relations of energy and of stopping power to range for particles of low energy.

There is a significant measure of general agreement between the three sets of data shown in Fig. 1, possibly strengthened by the fact (based on the coherence of curve ($B$) with our earlier results) that data ($A$) and curve ($B$) may be under-corrected for the effects of the source-supports. Thus it appears: (1) that the maximum of intensity in the observed spectrum of beta-particle energies from Ra$E$ lies at a rather low energy (~30 kev, more or less); and (2) that the intensity

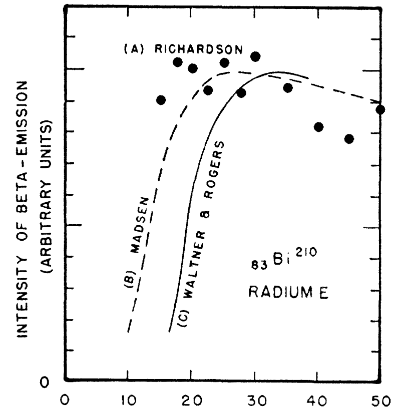


FIG. 1. Three determinations of the beta-decay spectrum of Ra$E$ at very low energies, normalized to approximately the same maximum intensity. See text for references.

in this observed spectrum drops off sharply at lower energies, apparently to quite small values. It is impossible to overemphasize the need for further investigations of this spectrum, for these conclusions appear to stand in contradiction of the modern theories of beta-decay.

* The new data reported here, shown as curve (*C*) in Fig. 1, were obtained in 1948 in the research laboratories of The University of North Carolina in Chapel Hill.

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[5] Arthur Waltner and F. T. Rogers, Jr., Phys. Rev. **74**, 699 (1948).
[6] J. Petrova, Zeits. f. Physik **55**, 628 (1929).

## Measurement of Particle Energies with Scintillation Counters

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In a recent paper Kallmann[1] has shown that single crystals of cadmium sulphide, used with a photo-multiplier to detect single $\alpha$-particles, give output pulses of fairly uniform size from $\alpha$-particles of a given energy. Such crystals emit red luminescence which matches the spectral response of the photo-multiplier used by Kallmann. We have found that similar uniformity of pulse heights is obtained when small ($1\times 1\times 1.5$ mm) crystals of natural, transparent scheelite are used as $\alpha$-particle detectors in conjunction with photo-multipliers having an antimony-caesium cathode. Figure 1 gives the "integrated bias" curve for pulses caused by $\alpha$-particles of 4.5-Mev energy, and also the pulse height distribution curve derived from it. A considerable spread of pulse heights is shown which is not accounted for by scattering and straggling of the incident particles. Removal of the spherical concave mirror, used in the system for collecting a large amount of the emitted light, does not alter the form of the "bias" curve to any marked extent. Similar "bias" curves were obtained for the scintillations produced in small single crystals of naphthalene activated by anthracene or by stilbene. We find that the average pulse height is approximately proportional to the energy of the incident particles. The rise in counting rate shown at low bias values is the result of multiple counting and is not present for crystals having a much more rapid decay of luminescence than scheelite (3 $\mu$sec.).

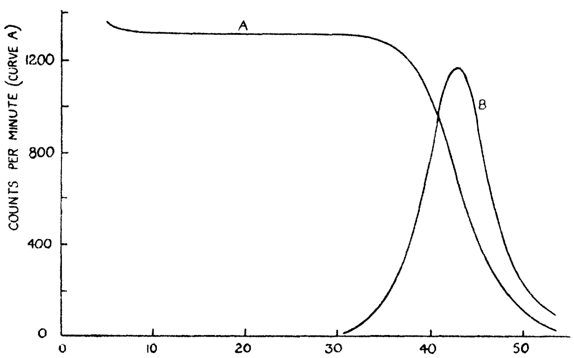


Fig. 1. Bias curves for multiplier output pulses due to scintillations in scheelite crystal bombarded by $\alpha$-particles of 4.5-Mev energy. *A* is integrated bias curve; *B* is pulse height distribution curve derived from *A*.

Other observations would appear to be of interest here. We have found that stilbene functions as an efficient activator in naphthalene crystals in a similar manner to anthracene, but gives output pulses of about twice the magnitude of those from anthracene-activated crystals. We find also that for both these phosphors, as well as for such inorganic phosphors as zinc oxide, silver-activated zinc sulphide, lead-activated barium sulphate, and synthetic and natural scheelite, there are no marked differences in maximum pulse heights due to $\alpha$-particles and $\gamma$-rays, if these are of equivalent energies. These results are contrary to those reported by Kallmann.[2]

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[2] H. Kallmann, Research **2**, 62 (1949).

## Beta-Decay Spectrum of $Ag^{110}$

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The disintegration of $Ag^{110}$ has been studied previously by several authors.[1-3] Of the two modes of disintegration into stable isobars, $Pd^{110}$ and $Cd^{110}$, the disintegration by electron emission into $Cd^{110}$ is well substantiated.[2] The maximum energy of the beta-spectrum was found to be 0.59 Mev. It was reported that the beta-spectrum is complex with more than half of the disintegration electrons absorbed by 2 mg $cm^{-2}$ of aluminum.[1] The first mode of disintegration, into $Pd^{110}$, has not been studied as yet either with regard to positron emission or to orbital electron capture. Several gamma-rays, of energies 0.66 Mev −44 percent, 0.90 Mev −47 percent partially converted and 1.40 Mev −9 percent have been found. In the most recent publication a converted gamma-ray of 114 kev has been reported.[3]

In the present work the disintegration of $Ag^{110}$ into the isobars $Cd^{110}$ and $Pd^{110}$ was studied by means of a cloud chamber since disintegration into $Pd^{110}$ by emission of positrons *a priori* could not be excluded.

The silver 110 of high specific activity was obtained from Oak Ridge National Laboratory, AEC. It was purified and mounted in the center of a cloud chamber on zapon film. Pictures of tracks were taken with and without magnetic field applied. The first series of tracks were photographed in air with a field of 372.5 gauss. The distribution of electrons *vs.* energies showed: (a) the existence of converted electrons corresponding to the gamma-rays of $Ag^{110}$, and (b) electrons of a continuous spectrum.

The Kurie plot produced a straight line in the energy range from 150 kev to 550 kev with upper limit about 0.59 Mev. The presence of converted electron groups limited observation of a continuous spectrum in a cloud chamber.

Several single positrons and pairs were found. The upper ratio of number of positrons, for which pairs could not be established, to the number of the electrons of the continuous spectrum did not exceed 0.2 percent. The experimental data obtained allowed the conclusion that there is no positron disintegration in $Ag^{110}$ with branching ratio $\beta^+/\beta^-$ higher than 0.2 percent.

The second series of tracks were photographed in helium with a field of 149 gauss. The subtraction of the 0.59-Mev continuous spectrum and recalculations for a second Kurie plot produced evidence for the existence of a continuous spectrum of electrons with upper energy $90\pm 10$ kev. Independently it has been found that the electrons of the $90\pm 10$-kev spectrum are in coincidence with gamma-rays.[4] The electron groups in the energy interval 90–150 kev were not sufficiently resolved.

It has been shown recently that excess of low energy electrons in Kurie plots very often are due to imperfect mounting.[5]