are much narrower than one expects on the basis of known internal magnetic fields. Also, we find that the absorption in $CuSO_4 \cdot 5H_2O$ reaches negligible values only after the applied field is about 1500 gauss greater than that at the absorption maximum. This is shown in Fig. 4. A line shape theory has been worked out by Gorter and Van Vleck,[11] and in detail by Van Vleck,[12] on the assumption of exchange interaction between the cupric ions. Their results show that when exchange is important, the usual assumption of a Gaussian curve for the absorption line is a poor one. Instead, they find that the absorption line should be peaked in the center and should have a long tail. To test this, we have drawn a Gaussian curve with the same area and second moment as the absorption line (Fig. 4). The Gaussian curve is a very poor approximation to the experimental curve. In Fig. 5 is shown the strong field side of the upper absorption line of $CrK(SO_4)_2 \cdot 12H_2O$ (steady field oriented along the 111 crystallographic direction). Exchange effects here are probably small. We draw here, too, a Gaussian curve with the same area and second moment as the absorption line. In this case the Gaussian curve is a reasonable approximation to the experimental curve. The difference between the absorption line shapes of the two salts can thus be explained on the basis that there is exchange interaction between the cupric ions.

We wish to acknowledge the continued aid and encouragement of Dr. A. J. Allen.

[11] C. J. Gorter and J. H. Van Vleck, Phys. Rev. **72**, 1128 (1947).
[12] J. H. Van Vleck, Phys. Rev. **74**, 1168 (1948).

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The Second Viscosity of Liquids*

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The second coefficient of viscosity is concerned with the viscous forces generated by compression (or dilatation). In the absence of knowledge of its magnitude in liquids it has been customary in hydrodynamics to assume that the coefficient of dilatational viscosity, n', could be approximated by the ideal gas value $n' = -2n/3$, where n is the coefficient of shear viscosity. A method has been developed for obtaining values for the dilatational viscosity which is based on Eckart's theory of acoustical streaming; the non-periodic motion of the fluid in the vicinity of a sound source is dependent on the two coefficients of viscosity. Values for the coefficient of dilatational viscosity for a variety of organic liquids and for water are given in the table. The coefficient of dilatational viscosity was found to be positive in sign and greater in magnitude than the shear viscosity. For example, the dilatational viscosity of water was found to be 2.4 centipoise and that for carbon disulfide greater than 200 centipoise. There is no correlation between the magnitude of the shear and dilatational viscosities for the liquids studied. Temperature variation measurements on water show that the temperature dependence of dilatational and shear viscosity in this substance is identical. Introduction of values for the dilatational viscosity into acoustical calculations eliminates the well-known discrepancy between theory and observation of sound absorption in liquids at very high frequencies.

I. INTRODUCTION

THE classical theory of viscosity is closely analogous to the theory of elasticity. In the classical elasticity theory two coefficients of elasticity appear, the Lamé coefficients, λ and μ. In an analogous manner there are two coefficients of viscosity, n and n'. The coefficient, n is familiar; it is simply the coefficient of shear viscosity. Values for this coefficient are determined with the use of a rotating cylinder or a transpirational viscosimeter and are to be found for a variety of substances in handbooks.

The second coefficient of viscosity, n', is less familiar; it might be called the coefficient of dilatational viscosity. It is a measure of the viscous forces which arise when a volume of fluid is compressed or dilated without change in shape. The magnitude of the viscous forces depends on the *rate* of compression or dilatation. A conventional viscosimeter does not measure the dilatational viscosity inasmuch as the dilatation or compression in these instruments is obviously negligible. No dilatational viscosimeter exists nor may values for the dilatational viscosity be found in the literature.

In hydrodynamics it has been customary in dealing with actual liquids to assign for the value of the dilatational viscosity, that calculated for an ideal gas; the dilatational viscosity of an ideal gas may be shown to be equal to $-2n/3$.[1] Most textbooks[2] do not attempt to justify this assumption

* This work represents one of the results of research carried out under contract with the Bureau of Ships, Navy Department.

[1] Horace Lamb, *Hydrodynamics*, 6th edition (Cambridge University Press, London, 1932), p. 574.
[2] For example: Lord Rayleigh, *Theory of Sound* (Dover Publications, New York, 1945), p. 314.

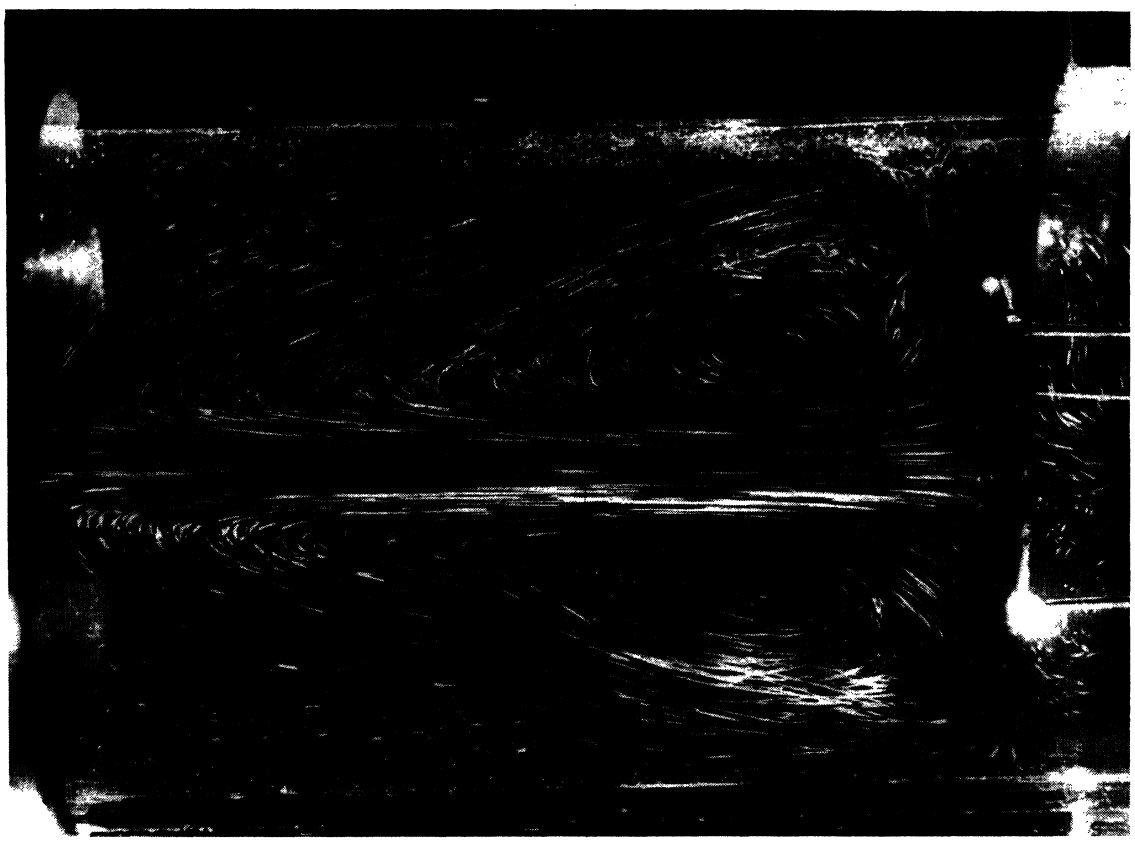

FIG. 1a. Photograph of acoustic streaming from a sound source; the motion of the liquid is made visible by a suspension of tiny particles of aluminum.

but attribute the proof of the argument to Stokes. However, Stokes was well aware of the deficiencies of his argument: ". . . I have always felt that the correctness of the value $n/3$ for the last term of this equation (the term which includes the dilatational viscosity) does not rest on as firm a basis as the correctness of the equation of motion of an incompressible fluid, for which the last term does not come in at all."[3]

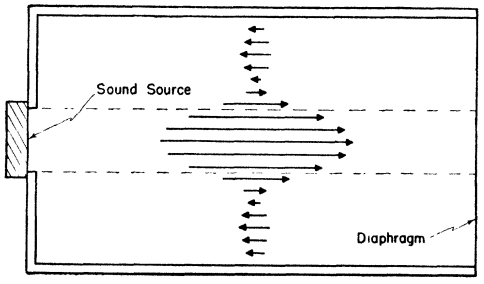

FIG. 1b. A schematic of the experimental arrangement used in a. The liquid is contained in a glass cylinder; the sound beam is circular in cross section and is transmitted through the diaphragm without reflection. The arrows indicate the theoretical distribution of velocity in the mid portion of the cylinder.

[3] G. G. Stokes, Math. Phys. Papers, **3**, 136 (1851).

It is not surprising that this assumption should have persisted so long in hydrodynamical theory. In most problems liquids may be treated as incompressible and the value of the dilatational viscosity does not arise. Even when the compressibility may not be neglected the motion is often divergenceless; the stress in this type of motion may be shown to be independent of n'. However the generation of acoustical waves in liquids and in gases does produce divergent motion; in acoustics the effect of the dilatational viscosity can be expected to be an extremely important factor. It is in this subject particularly that one may expect to find defects in the theory which result from the neglect of dilatational viscosity.

The succeeding sections will discuss in turn, the effect of dilatational viscosity on the absorption of sound; an unusual second-order acoustical effect which has its origin in part in the dilatational viscosity; and finally, actual values of the dilatational viscosity obtained experimentally for a variety of liquids.

II. VISCOSITY AND SOUND ABSORPTION

As previously mentioned, divergent motion is associated with acoustic waves; hence the dilata-

tional viscosity as well as the shear viscosity can be expected to exert an important influence on the propagation of these waves. The effect of the two viscosities is conveniently studied by beginning with the usual second-order equation of hydrodynamics,[4]

$$\rho(Du/Dt)+\nabla p=n\nabla^2 u+(n+n')\nabla\nabla\cdot u, \qquad (1)$$

where u is the particle velocity, ρ is the density, and p the pressure. This equation may be considerably simplified for the study of acoustic waves: for plane waves in the x direction Eq. (1) becomes

$$\rho(\partial u/\partial t)+(\partial p/\partial x)=(2n+n')(\partial^2 u/\partial x^2). \qquad (2)$$

Equation (2) together with the equation of continuity gives the usual plane wave solution:

$$u=u_0\exp[-i(\omega t+kx)] \qquad (3)$$

in which k is complex if ω is real. The imaginary part of k is the negative of the absorption coefficient. To a sufficiently good approximation the absorption coefficient is given by:

$$\alpha=\omega^2(2n+n')/2\rho c^3, \qquad (4)$$

where ω is 2π times the frequency, and c is the velocity of sound. In the ideal gas approximation $n'=-2n/3$ and the quantity $(2n+n')$ in Eq. (4) becomes $4n/3$. The latter will be recognized as the form given in most textbooks on sound for the calculation of absorption.

It is well known that the observed sound absorption, with few exceptions, is always in excess of the calculated values. Equation (4) suggests a reason for this disagreement: errors in the calculated absorption may be attributable to the use of the ideal gas approximation rather than the actual dilatational viscosity. This proposal has been advanced by several writers in recent years,[5] but unfortunately, sound absorption data, alone, cannot be used to establish unambiguous values for the dilatational viscosity. Absorption data do not necessarily represent the sound energy dissipated by viscous forces alone; for example, heat conduction and scattering processes may also dissipate sound energy. However it will be shown in a later section that sound absorption can, in fact, be correctly attributed to the combined effects of the dilatational and shear viscosities in the case of all liquids studied here.

III. ACOUSTICAL STREAMING AND VISCOSITY

The streaming of liquids or gases away from a vibrating sound source is a phenomenon which is

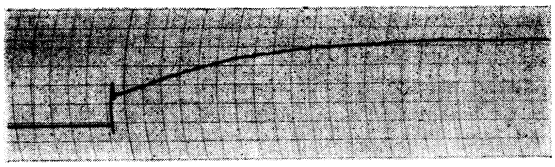

FIG. 2. A sample recording of forces on the suspended disk. The source was turned on: the abrupt rise is proportional to the radiation pressure on the disk; the slow increase represents the force of the streaming liquid as it approaches equilibrium velocity. The suspension is underdamped.

familiar to some experimenters. Figure 1a is a photograph illustrating this phenomenon. The streaming of a liquid is made visible by a suspension of fine particles of aluminum; an oscillating quartz crystal which is the sound source is on the left of the photograph.

Eckart[6] has recently developed a set of second-order equations of acoustics which include the streaming phenomenon as one of their solutions. The force which drives the motion is dependent on both the dilatational and shear viscosities; the retarding force is dependent only on the viscous shear forces. Eckart suggested therefore that the ratio n'/n might be determined from a study of the streaming phenomenon.

The solutions to Eckart's equations show that acoustical streaming consists of vortex motion, generated at the boundaries of the sound beam. As is generally true of vortex motion, these vortices diffuse from their origin, following closely the laws of heat diffusion. Quantitative considerations of these vortices are considerably simplified if the fluid is confined to a cylinder whose length is long compared to the radius; the sound beam is directed

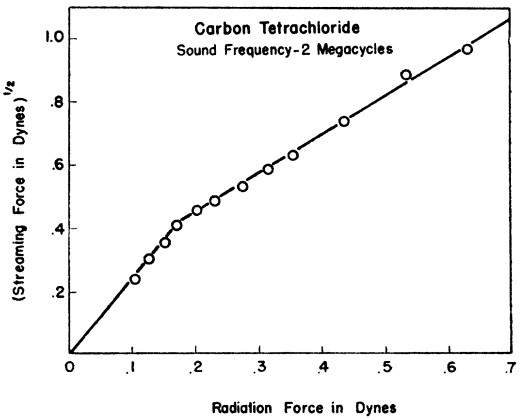

FIG. 3. Sample data for one liquid. The ordinate is proportional to the velocity of streaming; the abscissa is proportional to the sound intensity. Measurements were made over a range of sound intensities in order to define the two slopes. The slope which passes through the origin is used for the determination of the viscosity; the decrease in slope at higher streaming velocities represents turbulent motion.

[4] Compare with reference 2, pp. 574, 577.

[5] For example see: Laszto Tisza, Phys. Rev. **71**, 531 (1942); William Pitt Mason, *Electromechanical Transducers and Wave Filters* (D. Van Nostrand Company, Inc., New York, 1942), p. 306.

[6] Carl Eckart, Phys. Rev. **73**, 68 (1948).

TABLE I. Summary of viscosity ratio for a number of liquids.

Liquid	Frequency (megacycles)	Viscosity ratio (n'/n)	Coefficient of shear viscosity[a] (n in centipoise)	Coefficient of dilatational viscosity (n' in centipoise)	Viscosity ratio from sound absorption[b] (n'/n)
Water	5	2.4	1	2.4	2.1
Methyl alcohol	5	1.3	0.60	2.2	1.1
Ethyl alcohol	5	3.8	1.2	3.2	3.6, 2.6, 2.3
Acetone	5	3.1	0.30	10	3.8, 3.1, 2.9
Propyl alcohol (*iso*)	5	5.1	2.2	2.3	
Amyl acetate	5	9.9	0.89	11	10.2, 12.3
	2	9.6		11	
Xylol (*m*)	5	11	0.62	18	7.7, 6.5
	2	11		18	
Ethyl formate	5	15*	0.40	37*	21, 10
	4	22*		55*	
	3	31*		77*	
	2	85*		210*	
Chloroform	2	24	0.57	48	27, 34, 35
Carbon tetrachloride	2	28	2.0	14	27, 21, 22
Benzene	2	107	0.65	160	140, 130, 150
Carbon disulphide	2	>200	0.37	>500	

* These values are for the unmodified Eq. (5). Using the modified form (See Section IV, Part C) $n'/n=700$ for all frequencies.

[a] Taken from *International Critical Tables* (McGraw Hill Book Company, Inc., New York).

[b] Absorption data for water were taken from J. Pinkerton, Nature **160**, 128 (1948); the others are from Ludwig Bergmann, *Der Ultraschall* (Edwards Brothers Inc., Ann Arbor, 1944).

along the axis and passes without reflection through the region in which the liquid is confined. A schematic diagram of such an arrangement is shown in Fig. 1b. For the above conditions Eckart's equations lead to a velocity of streaming on the axis of the tube given by**

$$v=(\omega^2r^2GI/\rho c^4)(2+n'/n). \tag{5}$$

The sound beam has a radius r and an intensity I. The constant G in this equation is a geometrical factor which takes into account the relative size of the sound beam and the tube. If the sound beam fills the tube, $G=0$ and the streaming stalls; an increased axial velocity results as the tube diameter increases. For the special case outlined above

$$G=(r^2/r_0^2-1)/2-\log r/r_0, \tag{6}$$

where r_0 is the radius of the tube.

The predicted distribution of the velocity in other regions of the tube as well as on the axis is also of interest. The streaming flows along the axis away from the sound source and returns along the periphery. The result of a distribution calculation carried out for one set of dimensions used in these experiments is illustrated in Fig. 1b; the relative magnitude and direction of the velocity is illustrated by the vectors in the diagram; end effects of the tube have, or course, been neglected. This velocity distribution may be directly compared with the distribution of velocity in the photograph inasmuch as the geometry was identical.

IV. THE MEASUREMENT OF THE RATIO OF THE VISCOSITIES

A. Basis of the Method

It is seen from the preceding section that the ratio of the viscosities n'/n may be obtained from Eq. (5) provided all the other quantities in this equation are measurable or obtainable. Of these quantities, it is the measurement of the axial velocity of streaming and the absolute intensity of the sound beam which presents the major experimental problem.

The measurement of the axial streaming velocity can be made in either of two ways: particles can be suspended in a liquid in order to make the streaming visible and the velocity measured with the aid of intermittent photography; or the velocity can be obtained by calculation from the measurement of the forces on a disk set in the stream. The latter method makes use of the well-known relation between fluid velocity and the force on a disk,

$$F_S=K\rho v^2 \tag{7}$$

which obtains solely from momentum considerations. Viscous drag effects are eliminated inasmuch as the disk blocks the flow around itself.

Actually both methods were used in the determination of the streaming velocity. The constant of proportionality K, in the above expression, was determined experimentally by direct comparison with the velocity obtained by the photographic method for two liquids, water and carbon tetrachloride. As was expected, the value of the constant reproduced over a wide range of velocities. The more convenient disk method could then be used henceforth for all other liquids for which the geometry of the experiment was identical.

The use of the force on a disk for measuring streaming velocity conveniently lends itself to the simultaneous determination of the intensity of the sound beam. A disk suspended in a sound beam will, in general, be acted upon by the force of radiation pressure as well as by the force of streaming. The force of radiation pressure is given by,

$$F_R=2\pi IRr^2/c, \tag{8}$$

where I is the incident intensity of the sound beam and R is the reflection coefficient of the disk.***

** A typographical error occurs in Eq. (33) of Eckart's paper which if uncorrected leads to the first power of the frequency rather than the square of the frequency in the above equation.

*** The discussion of streaming in the previous section neglects the possibility of reflection. In the presence of sound reflection of relatively small intensity, the effective intensity is given by the difference between the incident and reflected intensities.

The use of radiation pressure to determine the intensity in a sound beam is a well-known method, but the fact that both radiation and streaming forces act simultaneously on the disc has long plagued investigators who were interested only in measuring the sound intensity.

The separation of the two forces can be effected by making use of the inertia of streaming. While radiation forces respond instantly to abrupt changes in sound intensity, the streaming velocity has a relatively long time constant. Hence a force-measuring device, capable of rapid response, will differentiate between the effects of radiation and of streaming.

A sample recording of the forces on the disk is shown in Fig. 2. Initially, the source was turned on; the abrupt rise is proportional to the force produced by radiation pressure; the slow increase represents the force on the disc as the streaming velocity increases and eventually approaches a maximum exponentially. On interruption of the source, the decay is analogous to the growth. Hence, the two effects of radiation and of streaming are separated but still may be measured practically simultaneously.

The actual determination of the ratio of the two viscosities is best illustrated by examining some typical data. Figure 3 gives a number of observations obtained for carbon tetrachloride at different sound intensities during one series. The abscissa of each point is the force on the disk (in dynes) due to radiation pressure. The ordinate of each point is the square root of the force of streaming on the disk. The ordinate is proportional to the streaming velocity; the abscissa is proportional to the sound intensity.

The slope of the curve shown in Fig. 3 like all others studied, surprisingly possesses two values, one at low sound intensities and one at high. The reason lies in the difference between laminar and turbulent streaming which is discussed at greater length in a succeeding section. The theory of streaming clearly applies only to laminar flow; hence the need for obtaining data in the form shown in the sample which clearly defines the two flow regions.

B. Apparatus for Studying the Viscosity Ratio

The apparatus used for these measurements is shown schematically in Fig. 4. The disk is mounted on a vertical magnesium deflection arm which is suspended by a stiff horizontal suspension ribbon. The deflection of the disk which is proportional to the force is electrically recorded by a variable reluctance device, consisting of an iron slug which changes the inductance of one of two iron core inductances forming part of an alternating current inductance bridge. The unbalance voltage is amplified and applied to a recording galvanometer.

The system was calibrated before and after each measurement by deflecting the suspension with a known force. A fine glass fiber is arranged in such a manner that its known bending force (of the order of 0.2 dyne) can be applied to the suspension.

The suspension is damped by an unconventional procedure. A tiny permanent magnet is mounted on the suspension; an electro-magnet mounted on the suspension support attracts or repells the magnet depending on the polarity of the polarizing voltage. The polarizing voltage is obtained after suitable amplification, from the derivative of the current supplied to the recording galvanometer. By varying the amount of the polarizing voltage the exact amount of the damping can be controlled. Usually the equipment was operated slightly underdamped rather than at critical damping.

Interchangeable disks, semitransparent to sound were used. These consisted of thin aluminum foils mounted on supporting rings. The accuracy of the final measurement is dependent on the accuracy with which the reflection coefficient of the various foils is known. These reflection coefficients were calculated using the usual expression for the reflection at normal incidence from a thin plate:

$$R=\frac{\left[\rho_1 c_1/\rho_2 c_2-\rho_2 c_2/\rho_1 c_1\right]^2}{4\cot^2\omega\tau/c+\left[\rho_1 c_1/\rho_2 c_2+\rho_2 c_2/\rho_1 c_1\right]^2}, \qquad (9)$$

in which τ is the foil thickness; the subscripts 1 and 2 refer to the liquid and to the aluminum. The practical application of this equation to thin foils was tested by measuring the reflectivity as a function of thickness for a wide variety of aluminum foils ranging in thickness from 2.5×10^{-4} to 10^{-2} cm.

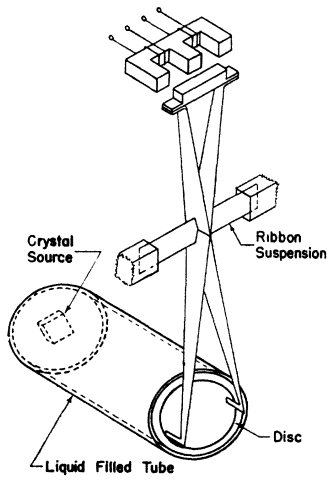

FIG. 4. A schematic diagram of the construction of the apparatus. The deflection of the disc produces an unbalance in the variable reluctance bridge located at the top.

Deviations from the above equation were found only for the foils which were thinner than 10^{-3} cm. The thinner foils were found to reflect more than is predicted, presumably owing to the slight contribution of the supporting ring. The foils used for the actual measurement were thicker than 10^{-3} cm.

V. THE RATIO OF THE VISCOSITIES

A. The Measured Viscosity Ratios

Table I summarizes the results of the determination of the viscosity ratio for a number of liquids. Several important conclusions are immediately evident from an inspection of the values in Table I: first, the viscosity ratios for all of the liquids studied have *positive values* rather than the value $-\frac{2}{3}$ which is predicted for an ideal gas. Secondly, the value of the ratio varies widely; there is *no direct correlation* between shear viscosity and dilatational viscosity. Thirdly, the value of the coefficient of dilatational viscosity for most liquids is *considerably greater* than the coefficient of shear viscosity; in benzene and carbon disulfide the dilatational viscosity is more than 100 times the shear viscosity. Clearly the ideal gas theory applied to the dilatational viscosity in liquids is a poor approximation.

B. Viscosity Ratios from Sound Absorption Data

Section II discussed the possibility of obtaining values for the dilatational viscosity with the use of absorption data. Absorption data for a number of liquids are given in Bergmann's book;[7] data on absorption in water has also been obtained from Pinkerton's work.[8] The consistency of the absorption measurements of liquids with the exception of water are rather poor. Water has been investigated with considerable care by several workers and Pinkerton's measurements are undoubtedly an excellent example as well as being the latest published work; his value is also in approximate agreement with previous work.

In accordance with the discussion in Section II, calculations were made assuming the observed sound absorption resulted exclusively from viscous dissipation. The values obtained in this manner with the aid of Eq. (4) are listed in the last column of Table I. It is seen that these values, in general, agree with those obtained from the present experiment within the consistency of the absorption data. In the case of water for which there exists reliable absorption data the agreement is excellent. This general agreement presents strong evidence that *the major part, if not all, of the sound absorption in most liquids is a result of viscous dissipation;***** the Stokes theory is found to agree with observation of liquids provided the dilatational as well as the shear viscosity is included in the calculation.

C. Temperature Dependence of the Viscosity Ratio

The data given in Table I were all taken with the temperature at approximately 17.4°C. The room temperature of the underground room in which these measurements were performed conveniently remained constant to within a few tenths of a degree and no attempt was made to control the temperature more precisely.

A set of observations were made on water to ascertain the temperature dependence of the viscosity ratio of this substance. The temperature was varied from 4°C to 25°C and the viscosity ratio measured at 2° intervals. Although the shear viscosity of water varies by the factor 2 in this temperature range, *the viscosity ratio showed no significant variation.* Absorption data for water as a function of temperature[8] also lead to the same result.

D. Frequency Dependence of the Viscosity Ratio

It will be noted that most of the liquids were studied at the five megacycle frequency; some were studied at two megacycles and a few were studied at both frequencies. The choice of frequencies was dictated by the magnitude of the viscosity ratio. Increasing the frequency increases the streaming forces. Hence, liquids having small viscosity ratios were studied at five megacycles; liquids having very large ratios were more conveniently studied at two megacycles; intermediate ratios could be studied at both frequencies.

Of the three liquids studied at the two frequencies, one liquid, ethyl formate, showed a remarkable dependence on frequency; the ratio increases with decreasing frequency. In order to explore this phenomenon further, additional measurements were made. The viscosity ratio of ethyl formate was found to be 15, 22, 31, and 85, at the respective frequencies 5, 4, 3, and 2 megacycles.

This behavior of the viscosity ratio may be explained if the assumption is made that the dilatational viscosity mechanism possesses a relaxation time which is considerably greater than that of the shear viscosity. (The relaxation time of the shear viscosity is usually assumed to be the mean free travel time of the molecules, of the order of 10^{-12} second.) This assumption of the relaxation time influences the frequency dependence of the streaming velocity. From Eq. (5) the streaming

[7] Ludwig Bergmann, *Der Ultraschall* (Edwards Brothers, Inc., Ann Arbor, 1944).

[8] J. Pinkerton, Nature **160**, 128 (1948).

**** One exceptional liquid is sea water. Recent work has shown that in addition to sound absorption resulting from the two viscosities, absorption at frequencies below one megacycle results from a chemical reaction involving one of the dissolved salts. Another well known exception is mercury, whose sound absorption results largely from its high heat conductivity.

velocity is proportional to

$$(2+n'/n)\omega^2 \equiv (4/3+\kappa/n)\omega^2, \tag{10}$$

where κ is defined as $n'+2n/3$. If the dilatational viscosity mechanism exhibits relaxation, the term $\kappa\omega^2/n$ in the above expression should be replaced by

$$\kappa\omega^2/n(1+\omega^2\theta^2) \tag{11}$$

in which θ is the relaxation time of the dilatational viscosity. In case Eq. (5) is applied without modification to streaming data where relaxation exists the viscosity ratio will be found to apparently decrease in the region of relaxation and to approach the limiting value $-\frac{2}{3}$ at extremely high frequencies. The application of the modified Eq. (5) to the ethyl formate data is found to give the approximately constant value $n'/n=700$ at the four frequencies. The relaxation time, θ, which was chosen to give the best fit, is 2×10^{-7} second.

VI. DISCUSSION

A. Dilatational Viscosity and Molecular Theory

It might be satisfying, if at this point a molecular model of dilatational viscosity could be presented. Considering the complexity of the structure of liquids it is not surprising that none can be given which has a real foundation in fact. There have, however, been numerous attempts in the past to formulate a molecular theory of shear viscosity in liquids. A brief review of a current theory is of interest in that it is indicative of the form which future theories of dilatational viscosity will undoubtedly assume.

The usual theory of shear viscosity assumes a liquid to be constructed of molecules which are bonded to one another but which may change equilibrium positions under the action of a shearing motion. The introduction of a relaxation time or transition probability for the transition process, following the Poisson-Maxwell procedure, gives the dependence of viscosity on the rate of shear. The number of molecules possessing sufficient energy to effect bond rupture, and hence transition, depends on the Boltzmann expression, $\exp(-B/RT)$, which leads to a reasonably correct temperature dependence of shear viscosity for some liquids,

$$n_T = n\exp(B/RT).$$

In a similar manner it might be proposed that dilatational viscosity is a result of molecular rearrangement induced by compression or dilatation. Presumably there would exist in a liquid two or more stable molecular configurations possessing different energies. Transitions between these states would be actuated by compression (or dilatation). A dependence of viscous dissipation on the rate of compression results as above from a finite relaxation time for the transition process. The temperature dependence of this process would be expected to be given by the Boltzmann expression just as in the shear viscosity model. Inasmuch as the activitation energy, B, of the process would be expected to be nearly the same as that for the shear process, the temperature dependence of the two viscosities might be expected to be nearly identical; this would agree with the observed equivalence of the temperature dependence of the shear and dilatational viscosities in water (see Section V, Part C). The similarity of the activiation energies of the two processes would not preclude large variations in the viscosity ratio; the magnitude of the viscosity depends on the relaxation time as well as on the activation energy of the process.

B. Flow Phenomena Associated with Acoustical Streaming

The hydrodynamics of the observed acoustical streaming is of interest quite apart from the study of dilatational viscosity. Several unusual phenomena are apparent from a study of the motion of the fluid: As can be seen from Fig. 3, the relationship between sound intensity and fluid velocity is not the same for all values of the sound intensity; instead there appear to be two clearly defined flow regions. In both regions the flow velocity is a linear

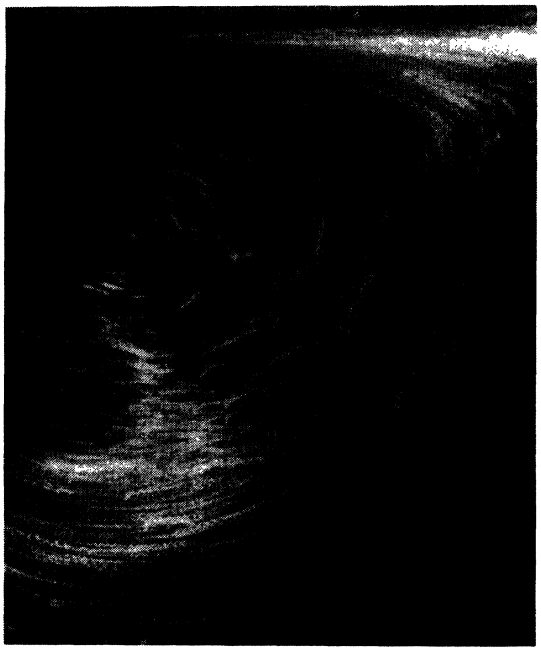

FIG. 5. A photograph of acoustical streaming similar to that shown in Fig. 1. A portion of the streaming has been enlarged in order to show that, surprisingly, the particle tracks are sinuous. It is believed that the particles rotate, which results in an epicycloidal motion.

function of the sound intensity but with different constants.

This behavior is strongly reminiscent of laminar and turbulent flow. For example, fluid flow in pipes is proportional to the pressure head up to a critical velocity; beyond this velocity the flow becomes turbulent and the constant of proportionality changes, or the relation may even be no longer linear. Such an explanation of the two flow regions in acoustical streaming has one disturbing feature: The Reynolds' Number at the critical streaming velocity is of the order of 100 which is a factor 20 less than the generally accepted value. Possibly the Reynolds' criterion for turbulence must be modified when applied to vortex motion of this kind. Visual inspection with the aid of dye or suspended particles exhibits no striking change during the transition between the two flow regions.

Photographs of the suspended particles in the stream show two phenomena which have not been mentioned earlier. It is apparent in Fig. 1a that the streaming lacks symmetry about a vertical line through the center of the photograph. On the other hand, Eckart's theory of streaming would predict perfect symmetry about the median line. Eckart has suggested[9] that this phenomenon may be related to 4th-order terms which were neglected in the theory. The significant 4th-order terms would take into account inertial effects in the fluid flow. In addition to disturbing the symmetry the magnitude of these terms at high fluid velocities might also be sufficient to disturb the linear relationship between fluid velocity and sound intensity. If this were the case, the linear relationship would be approached asymptotically with decreasing sound intensity. These 4th-order terms should become significant at a Reynolds' Number of about unity.

Another phenomenon associated with acoustical streaming may be seen in Fig. 5. This photograph is similar to Fig. 1 except that a portion of the acoustical streaming has been enlarged in order to make visible the individual tracks of the suspended particles. Close inspection shows that, surprisingly, each track is sinuous. It is believed that each particle rotates about an axis which is displaced from its center; when combined with translation this motion would be epicycloidal. The explanation for the rotation of the suspended particles is not obvious and this phenomenon must be given further study.

The author is indebted to Messrs. Stanley Lai and Victor Anderson for assistance in experimental work. He also wishes to express his appreciation to Professor Carl Eckart for valuable discussions.

[9] Carl Eckart, private communication.

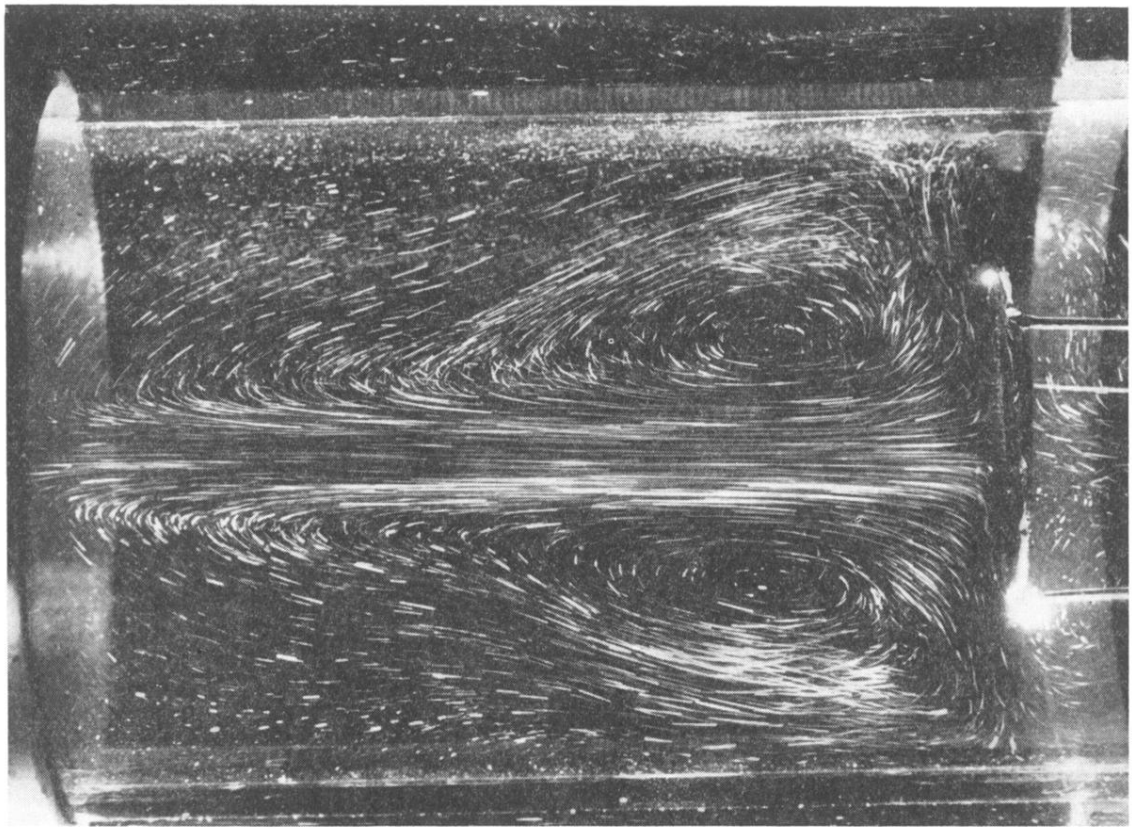

FIG. 1a. Photograph of acoustic streaming from a sound source; the motion of the liquid is made visible by a suspension of tiny particles of aluminum.

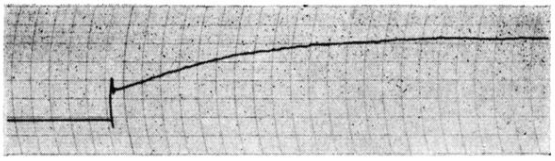

FIG. 2. A sample recording of forces on the suspended disk. The source was turned on: the abrupt rise is proportional to the radiation pressure on the disk; the slow increase represents the force of the streaming liquid as it approaches equilibrium velocity. The suspension is underdamped.

FIG. 5. A photograph of acoustical streaming similar to that shown in Fig. 1. A portion of the streaming has been enlarged in order to show that, surprisingly, the particle tracks are sinuous. It is believed that the particles rotate, which results in an epicycloidal motion.