Measurements of the Absolute Intensity of Cosmic Radiation at Sea Level

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A cosmic-ray telescope is described which measures the absolute intensity of the cosmic radiation through the use of trays of overlapping Geiger counters to eliminate insensitive areas. The results obtained, carefully corrected for accidentals, counter efficiency, side showers, absorption, and barometric pressure, are compared with those of previous authors.

I. INTRODUCTION

PREVIOUS sea level determinations of the total cosmic ray intensity show surprising discrepancies in the values obtained by different authors, over a range of thirty to forty percent.[1–5] These discrepancies are much greater than the normal daily and seasonal variations. The greatest single sources of error are in the corrections for side showers and absorption in the counter walls and material overhead.

The Geiger-counter telescope described below utilizes larger and symmetrical sensitive areas and yet retains good angular resolution. The larger areas give greater real counting rates so that good statistical accuracy can be achieved in short periods of time. This advantage, together with that of using thin-walled counters of relatively low absorbing power, and a novel method of minimizing errors due to variations in the effective length of the counter, indicated that more reliable results could be obtained than those previously reported.

This work was done as a supplement to that previously described[6,7] using equipment largely adapted from the previous work.

II. COUNTER TELESCOPE FOR ABSOLUTE DETERMINATION OF COSMIC-RAY INTENSITY

The telescope is composed of three trays of Geiger counters rigidly supported, one below the other, by a boxlike aluminum framework. Each tray is made up of eleven counters, nine of which have their axes parallel and which overlap so that there is no insensitive area between the counters. The other two counters are placed transversely, one across each end of the main group. Each tray of the telescope has 25.69-cm effective length and 24.52-cm effective width. The spacing between the two outer trays of the telescope is 98.5 cm between corresponding planes through the trays.

The Geiger counters used,[6] were made from copper-plated steel cylinders of 0.025-cm wall thickness. The diameter is 3.3 cm and the length approximately 25 cm. The counters were carefully chosen to have nearly equal thresholds and were operated at constant battery voltage approximately 100 volts above threshold. The individual counting rates of the counters agreed very closely when they were tested under identical conditions, using a thorium source to stimulate rapid counting rates.

From purely geometrical considerations it is possible to calculate the ratio of the counting rate of the telescope, n, in counts per minute, to the absolute intensity, j_0, of the cosmic rays, in ionizing particles per unit solid angle per cm^2 per minute, considering only cases where one particle strikes all three trays. If the telescope is pointing vertically, if the trays are of dimensions "$2a$" and "$2b$," if the separation between the outer trays is "d," and if the intensity of the radiation is assumed to vary as the square of the cosine of the zenith angle, then we have

$$n/j_0 = 16a^2b^2(1-2a^2/d^2)(1-2b^2/d^2)/d^2, \qquad (1)$$

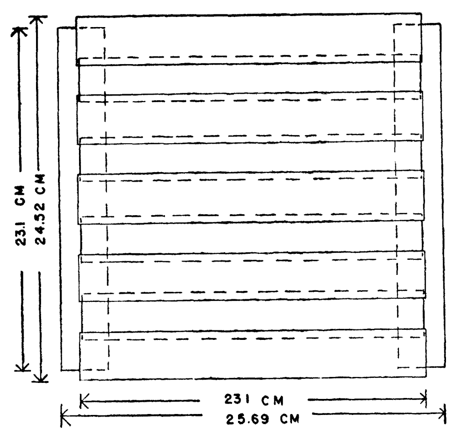

FIG. 1. Plan view of a single tray of Geiger counters. The effective dimensions of the individual counters are shown. Note that the counters overlap so that only about $\frac{4}{5}$ of the effective area of each is utilized.

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[1] T. H. Johnson, Phys. Rev. **43**, 307 (1933).
[2] J. C. Street and R. H. Woodward, Phys. Rev. **46**, 1029 (1934).
[3] D. K. Froman and J. C. Stearns, Can. J. Research **A**, 29 (1938).
[4] K. Greisen, Phys. Rev. **61**, 212 (1942).
[5] K. Greisen and N. Nereson, Phys. Rev. **62**, 316 (1942).
[6] A. T. Biehl, R. A. Montgomery, H. V. Neher, W. H. Pickering, and W. C. Roesch, Rev. Mod. Phys. **20**, 360 (1948).
[7] See reference 6, p. 353.

TABLE I. Counting rate of apparatus.

	Counting rate (coincidences per minute)	Date	Barometer (inches Hg)	Total counts	j_0 (uncorrected)
Transverse counters connected	28.05	Nov. 5–7, 1947	29.42	72,000	0.726
Transverse counters unconnected	23.40	Nov. 13–14, 1947	29.21	22,500	0.724

where if a/d is less than $\frac{1}{8}$, then any higher order terms may be neglected with an error of less than $\frac{1}{4}$ percent.

For the particular telescope used, we must make a correction for the insensitive areas at the corners of the tray and also a correction for the finite thickness of the trays. Figure 1 shows a plan view of one of the trays showing the sensitive areas of the counters making up the tray. The effective width of each counter is assumed to equal the inside diameter of the cylinder.[4] The effective counter length was determined by the method of Street and Woodward[2] to be 23.1 cm as an average for five counters. Figure 2 shows the experimental curve for this determination.

The insensitive area constitutes 0.66 percent of the total area so that the correction due to this factor is -1.3 percent because of the dependence of the counting rate on the square of the tray area. The cylindrical shape of the counters may be shown to add to the effective tray area in amount of 0.5 percent when the transverse counters are effective in the circuit.

Applying these two geometrical corrections and substituting for "a," "b," and "d," we have $n/j_0 = 38.60$.

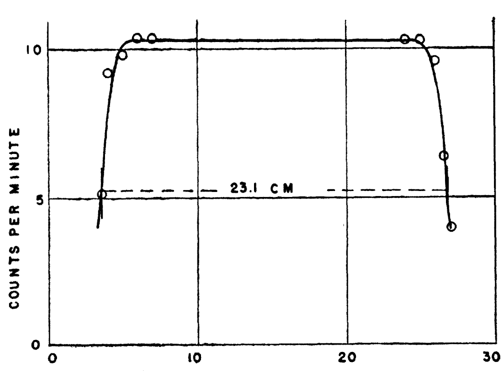

FIG. 2. Effective length of counter as determined for five counters simultaneously using the method of Street and Woodward. The shape of the curve depends on the diameter of the counters, the effective length being given by the distance between half-maximum points.

If the transverse counters are disconnected, then we have an entirely different telescope admittance. The circular ends of the counters now contribute to the effective area and solid angle of the telescope. An integration is performed to determine the number of rays that can pass through one set of circular ends on one tray, and which are so directed as to pass through the other extreme tray. This result, multiplied by four for the other similar sets of counter ends, amounts to a total of 3.2 percent for the geometry used. The cylindrical shape of the counters affects only the effective width in this case and hence the correction is but half of that when the traverse counters are connected, i.e., 0.25 percent. If we apply these two corrections and now use the effective length of a single counter as that of the tray, we now have, using Eq. (1), $n/j_0 = 32.30$.

The ratio of the geometrical factors for the telescope with and without the transverse counters connected is $38.60/32.30 = 1.195$. It is possible to get an internal check on the consistency of these results by comparing the counting rates of the telescope with and without the transverse counters connected. This was done using both double and triple coincidences. The results were: for triples, 1.19 ± 0.01; for doubles, 1.215 ± 0.01.

III. DETERMINATION OF ABSOLUTE INTENSITY OF COSMIC RAYS

The counting rate of the above apparatus was determined at Pasadena on the roof of Bridge Laboratory. The residual absorbing material, in addition to the counter walls, was $\frac{1}{4}$-inch plywood. (See Table I.)

IV. CORRECTIONS FOR ABSOLUTE DETERMINATION

These results must be corrected for the following factors:

(1) Accidental coincidences and loss of efficiency;
(2) changes in barometric pressure;
(3) absorption in the counter walls and wood overhead;
(4) additional coincidences due to side showers.

Measurements were taken over 24-hour periods to minimize the daily variations.

V. CORRECTIONS FOR ACCIDENTALS AND EFFICIENCY

The resolving time of the electronic circuits is such that the number of sea-level accidental coincidences is completely negligible for a three-tray arrangement.

As discussed by Johnson,[8] the efficiency of a train of "n" coincidence counters is E^n where $E = E_\sigma E_p$. E_σ is the efficiency as limited by the "dead time" of the counters, and E_p is the efficiency

[8] T. H. Johnson, Rev. Mod. Phys. 10, 206 (1938).

as limited by the pressure in the counters and the kind of gas used to fill them.

The measurements of the efficiency due to "dead time" have been described.[6] The efficiency in this respect is 99.9 percent since the sea-level counting rate per tray is only about 20 counts pèr second.

The probability of detection of a single particle of path length "L" through a counter, filled to pressure "p" with a gas of specific ionization "N," is $(1-\exp(-NLp))$. The over-all efficiency is obtained by integrating the efficiency for a given path length over the distribution of possible path lengths. For a cylindrical counter of diameter "D," the over-all efficiency for particles whose paths are normal to the axis of the cylinder is

$$E_p = 1 - \int_0^1 \exp(-NpD(1-y^2)^{\frac{1}{2}})dy,$$

where "y" is the distance of closest approach of the ray to the counter axis, measured in units of the cylinder radius. Taking the value of $N=30$ ions per cm per atmos. for argon,[8, 9] $p=0.075$ atmos., and $D=3.25$ cm, then NpD equals 7.4.

The integration performed graphically in Fig. 3 shows that the over-all counter efficiency is 97.9 percent if the counter is used singly. In the trays used for the absolute determination, the counters overlapped in such a manner that only $\frac{4}{5}$ of the effective diameter of each counter is utilized. The efficiency under these conditions is given by the same expression as above, with the upper limit now 0.8 in the integral. This works out to be 99.8 percent. The tray efficiency is now 99.6 percent if we consider the tray as made up of eight counters, each 99.8 percent efficient and one counter 97.9 percent efficient. The over-all efficiency for three trays in coincidence is 98.8 percent for rays traveling parallel to the axis of the telescope.

The correction for inefficiency of the mechanical recorder is negligible since the resolving time is approximately 0.01 second, and the counting rate is only about 0.5 count per second.

VI. PRESSURE COEFFICIENT OF COSMIC-RAY INTENSITY

Widely different values have been reported in the literature for the pressure-intensity coefficient of the vertically incident cosmic rays at sea level. Clay and Bruins[10] give a value of -6.4 percent per cm Hg change in pressure, from ionization measurements with the chamber shielded with from 12- to 120-cm iron, for rays incident at any angle. Barnothy and Forro[11] report values of -3.74 ± 0.59 percent per cm Hg at the vertical with an angular opening in their Geiger-counter telescope of $10°\times40°$. Their results indicated that the coefficient was slightly smaller away from the vertical. These results differ by nearly a factor of two from those of Clay and Bruins. Unpublished measurements by Pickering show -3.65 percent per cm Hg change in pressure from sea-level observations.

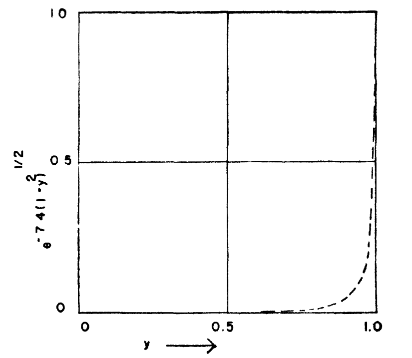

FIG. 3. Graphical integration of $\exp-7.4(1-y^2)^{\frac{1}{2}}dy$. The area under the dotted curve gives the fractional loss in efficiency due to the probability that no ions will be formed by some of the ionizing rays in passing through the counter.

In order to redetermine this coefficient, two of the cosmic-ray balloon telescopes[6] were mounted in a truck and operated at three different altitudes ranging from zero to 1300 feet above sea level. The pressures were recorded with an accurate Paulin barometer which was calibrated against a meteorological standard. The counting rate at each altitude was determined to within a statistical probable error of less than 0.5 percent. Figure 4 shows a curve of the natural logarithm of the counting rate *vs.* the pressure in inches of Hg for several instruments. The average slope corresponds to a coefficient of -9.1 ± 0.6 percent per inch of Hg or -3.6 ± 0.3 percent per cm Hg. These results agree very well with those of Barnothy and Forro, and those of Pickering.

The assumption is made in these measurements that the pressure change due to a change in altitude is equivalent, in its effect on the cosmic-ray intensity, to an equal pressure change at the same altitude. This assumption is justified if an atmosphere in equilibrium is postulated.[12]

VII. ABSORPTION IN THE COUNTER WALLS AND WOOD OVERHEAD

The change in the counting rate of a two-tray telescope was measured as absorbers, corresponding to equivalent numbers of counter wall thicknesses, were placed between the two trays. In Fig. 5 the logarithm of the counting rate is plotted against the number of equivalent wall thicknesses of ab-

[9] J. Cosyns, Bull. Tech. Assn. Eng. Brussels, 173 (1936).
[10] J. Clay and E. M. Bruins, Rev. Mod. Phys. 11, 158 (1939).
[11] J. Barnothy and M. Forro, Zeits. f. Physik. 100, 742 (1936); and 104, 535 (1936).
[12] B. Rossi, Rev. Mod. Phys. 11, 297 (1939).

TABLE II. Principal determinations of the absolute intensity, j_0 of cosmic rays.

Author	Date	Altitude	Pressure inches/Hg	j_0	Corrected	Magnetic lat.
Johnson	1933	189 m	—	0.70		57°
Street and Woodward	1934	0 m	—	0.80 ±0.028		55°
Froman and Stearns	1938	37 m	—	0.973 ±0.032		58°
Greisen	1941	259 m	29.2	0.735	0.709*	55°

* Behind ¼-inch of wood.

sorber. Sheets of copper-plated steel, identical to those from which the cylinders were rolled, are used as absorber. Six thicknesses total 1.4-g/cm² of absorber.

If we consider that as many rays are produced as absorbed in the upper wall of the top tray,[13] then the correction for the absorption in the other five counter walls of a triple coincidence telescope would be 4.0±0.5 percent by extrapolation of the curve. The absorption in the ¼-inch of wood overhead is approximately 1.5 percent, as determined by noting the change in counting rate when an additional equivalent layer of wood is placed above the telescope.

VIII. CORRECTION FOR ADDITIONAL COUNTS DUE TO SIDE SHOWERS

A first estimate of the effect of side showers can be obtained by displacing the center tray of the triple-coincidence telescope so that no one particle can traverse all three trays if it travels in a straight line. If the telescope is vertical, and the center tray is displaced in a horizontal plane, a curve may be obtained showing the variation of counting rate with the distance of displacement.

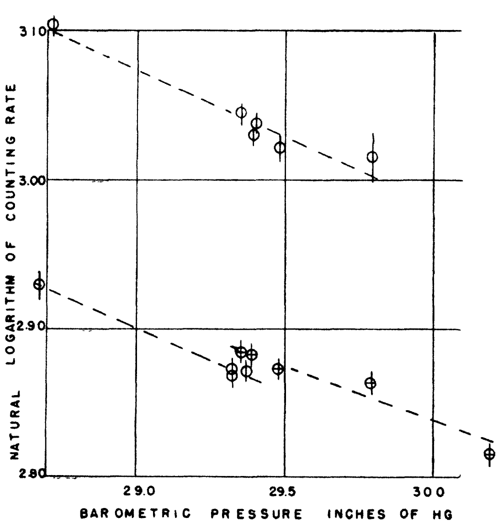

FIG. 4. Variation of intensity of vertically incident cosmic radiation with pressure at sea level. The average slope corresponds to a coefficient $k=0.091$ for the variation of the natural logarithm of counting rate with barometric pressure, measured in inches of Hg.

[13] H. Schwindler, Zeits. f. Physik, **72**, 625 (1931).

We consider the totality of coincidences due to side showers in two groups, *viz.*, (1) those coincidences due to the passage of only two associated rays, and (2) those coincidences due to the passage of more than two associated rays. The number of the latter will be nearly independent of small displacements, relative to the shower extent, of the center tray.

The number of counts due to only two particles, $n_2(x)$, will be proportional to the number of shower particles which pass at discrete intervals of time through a sub-telescope, made up of the middle tray of the main telescope and one of the outer trays, multiplied by the probability that each particle will be accompanied by another shower particle which excites the third tray. If this probability be considered constant, and if the angular distribution of the discrete shower particles is assumed to vary as the square of the cosine of the zenith angle, then $n_2(x)=KA(\theta)w(\theta)\cos^2\theta$, where $A(\theta)$ is the tray area normal to the sub-telescope axis at angle θ, and $w(\theta)$ is the sub-telescope solid angle. If "d" and "x" are respectively the separation between the outer trays of the telescope and the distance of horizontal displacement of the intermediate tray from the telescope axis, then $\cos\theta=d/(d^2+x^2)^{\frac{1}{2}}$.

Now $A(\theta)=A(0)\cos\theta$ and $w(\theta)=A(0)\cos^3\theta/(d/2)^2$, hence, $n_2(x)=C_1\cos^6\theta$ where C_1 is a constant of the geometry. The total number of counts due to side showers, $n_s(x)$ is therefore of the form $n_s(x)=C_1\cos^6\theta+C_2$, where C_2 represents the coincidences due to the passage of more than two rays. The number of counts due to side showers when the three trays are in the normal telescope arrangement, may be estimated from the extrapolation of

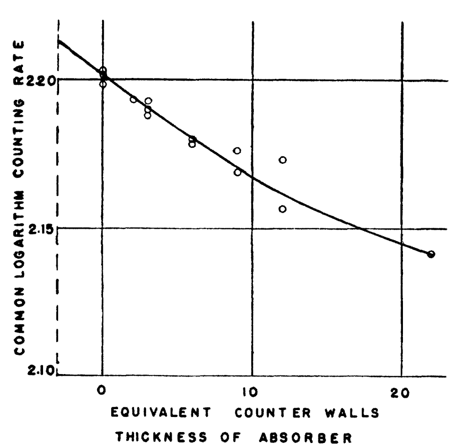

FIG. 5. Variation of coincidence counting rate with equivalent counter walls thickness of absorber using double coincidences. The curve is extrapolated to give the absorption in three additional counter walls.

the curve of $n_s(x)$ *versus* x or it may be independently determined in the following manner.

The standard telescope is arranged with four evenly spaced trays and counting rates are determined with double, triple, and quadruple coincidences. When the results are corrected for pressure changes and for accidentals, the differences represent the contributions of the side showers since the absorbing material is held constant. The results corrected to a common pressure of 29.07 inches Hg are, if s_2, s_3, and s_4 are the number of counts due to side showers for double, triple, and quadruple coincidences, respectively,

Counting rate (counts/min.)			s_2-s_3	s_3-s_4
Doubles	Triples	Quadruples		
31.70	29.15	28.20	2.55	0.95

If we assume that $s_4/s_3=s_3/s_2$, then solving we have $s_3=1.5$ counts per minute.

Figure 6 shows the experimental points obtained for the coincidence counting rate as a function of the horizontal displacement distance superimposed on the theoretical curve which fits s_3 equal to 1.5 counts per minute for $x=0$ and a background of 0.30 count per minute. The theoretical curve fits the experimental points quite well, indicating that the results of the two methods of estimating the shower effect are consistent.

In considering the number of counts observed with the one tray displaced, we must allow for the fact that we can observe coincidences between particles, one of which arrives in the permitted cone of the telescope, and the second, traveling in a parallel path, strikes the displaced tray. That these occurrences are relatively rare is shown by the following experiment.

Two counter telescopes are arranged side by side, both pointing in the vertical direction. It is arranged to count only simultaneous coincidences in both telescopes. The observed counting rate in this experiment is only 0.11 count/min., a number small compared with the contribution due to side showers.

The correction to be made to the triple-coincidence measurements is therefore $1.5/29.15=0.05$ or a 5 percent correction with a probable error of about $\frac{1}{2}$ percent.

IX. SUMMARY OF CORRECTIONS

The corrections other than geometrical are then:

(1) efficiency, 1.3 ± 0.5 percent;
(2) absorption, 4.0 ± 0.5 percent in counter walls plus 1.5 ± 0.5 percent in the wood overhead;
(3) side showers, -5.0 ± 0.5 percent;
(4) barometric pressure -3.6 ± 0.3 percent per cm Hg.

No corrections are made for temperature variations. The results, corrected to standard atmosphere pressure of 76-cm Hg are

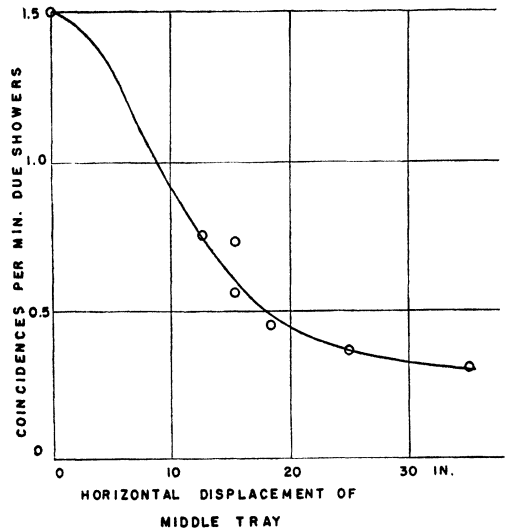

FIG. 6. Variation of number of triple coincidences observed due to side showers with the horizontal displacement of the middle tray of the telescope. The curve shown is the theoretical one adjusted to give correct values on the axis and at a distance large compared to the tray size.

	$\frac{1}{4}''$ wood overhead	no absorber
Transverse counters connected	0.695	0.705
Transverse counters unconnected	0.680	0.690

The results are expressed in ionizing particles per unit solid angle per cm² per minute.

X. COMPARISON WITH PREVIOUS RESULTS

The results given above serve generally to confirm those of Greisen,[4] as corrected in the later paper by Greisen and Nereson,[5] when their results have also been corrected for the residual absorbing material which consists of 2.3 g/cm² of brass plus $\frac{1}{4}$-inch wood.

The results calculated from Greisen's data for absorption are consistent with the results given in Section VII, each being about 3 percent per g/cm² of absorber. Greisen's results must also be corrected for the shower factor which he estimates at 3 percent from his data on sixfold coincidences.

Table II summarizes the principal determinations to date, which may be compared with those in Section IX.

The present results are therefore approximately three percent lower than those of Greisen. It is interesting to attempt to correlate the experimental difference in terms of the world-wide changes in cosmic-ray intensity as measured by Forbush.[14] These records show that the mean daily sea-level cosmic-ray intensity as measured at Cheltenham, Maryland, was higher in the midsummer of 1941 by 5 percent than the daily average in the period of these measurements in the fall of 1947. Since

[14] Records made available to Dr. H. V. Neher through the kindness of Dr. S. E. Forbush.

Pasadena is at magnetic latitude of 40°N, there should be negligible variation with latitude of the sea-level cosmic-ray intensity between this location and more northerly locations.[15]

[15] R. A. Millikan and H. V. Neher, Phys. Rev. 50, 15 (1936).

In conclusion the author wishes to record his deep appreciation for the invaluable advice and counsel of Dr. H. V. Neher and Dr. W. H. Pickering. He sincerely thanks the Carnegie Institution of Washington for their financial assistance.

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Paramagnetic Absorption in Single Crystals of Copper Sulfate Pentahydrate*

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We have studied paramagnetic resonance in $CuSO_4 \cdot 5H_2O$, at 9375 mc/sec. Although two absorption peaks were expected, only one was found; Van Vleck has explained this in terms of exchange coupling between Cu^{++} ions. We have found electronic gyromagnetic ratios for a large number of orientations of the crystal (with respect to the external field). Theory agrees with experiment if the gyromagnetic ratios that correspond to the three principal susceptibilities are 2.39, 2.39, and 2.07. We also find a variation in absorption line width with orientation. The shapes of the absorption lines agree with a theory of exchange coupling advanced by Gorter and Van Vleck.

I. INTRODUCTION

THE paramagnetic anisotropy of $CuSO_4 \cdot 5H_2O$ has been studied, both experimentally by Krishnan and Mookherji[1] and theoretically by Polder.[2] Since the discovery of paramagnetic resonance absorption, further studies of this salt in the form of single crystals have been made by Kip,[3] by us,[4] and by Bagguley and Griffiths.[5] In microwave resonance experiments, the measured quantity is usually the electronic gyromagnetic ratio (g factor).

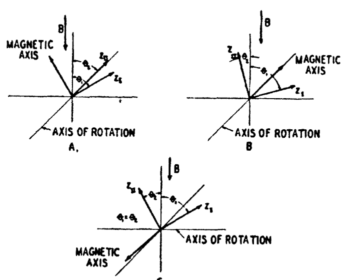

FIG. 1. Geometry of the measurements.

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[1] K. S. Krishnan and A. Mookherji, Phys. Rev. 50, 860 (1936); 54, 533 (1938); 54, 841 (1938). KM will be used hereinafter when referring to these papers.

[2] D. Polder, Physica 9, 709 (1942).

[3] R. D. Arnold and A. F. Kip, Phys. Rev. 73, 1247(A) (1948).

[4] J. Wheatley, D. Halliday, and J. H. Van Vleck, Phys. Rev. 74, 1211(A) (1948).

[5] D. M. S. Bagguley and H. E. Griffiths, Nature 162, 538 (1948).

II. CRYSTAL STRUCTURE

Beevers and Lipsom[6] have worked out the structure of copper sulfate. The unit cell contains two Cu^{++} ions. Each ion is surrounded by four negatively charged oxygens (parts of water molecules) in an approximate square, 2.8A on a side; there are also two other oxygens (parts of sulfate groups), each of which is 3.1A from any one of the oxygens in the square. These six oxygens form an octahedron about the cupric ion; the body diagonal perpendicular to the square is the longest. Polder[2] has calculated that this arrangement produces an electric field of nearly tetragonal symmetry at each Cu^{++} ion. The x-ray measurements show that the configuration about each ion is nearly the same and that the angle between the two tetragonal axes is 98°. Magnetic measurements show that this angle is close to a right angle.

III. PREVIOUS MAGNETIC MEASUREMENTS

Krishnan and Mookherji[1] found that two of the principal susceptibilities of the crystal—those along the bisectors of the angles between the tetragonal axes—are nearly equal. They are greater than the third—which is taken normal to the plane formed by the two tetragonal axes. We shall call these tetragonal axes Z_I and Z_{II} following KM.[1] The direction of lowest susceptibility is an axis of magnetic symmetry; we will call it the magnetic axis. From their measurements of the principal susceptibilities of the crystal, KM were able to calculate susceptibilities for directions both parallel

[6] C. A. Beevers and H. Lipsom, Proc. Roy. Soc. London A146, 570 (1934).