Magnetostriction and Order-Disorder

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An experimental and theoretical study of the relationship between saturation magnetostriction and the order-disorder transformation in an iron-cobalt alloy has been carried out. For the experimental work Fe-Co alloys in the neighborhood of the 50 atomic percent composition containing 0.75 percent Cr were employed. Measurements of both electrical resistivity and neutron diffraction have been used to confirm the presence of the superlattice. Measurements of the magnetostriction were made with a resistance strain gauge technique. An increase in the saturation magnetostriction is observed on ordering. Theoretical analysis based on the influence of order on the fluctuation of local electron density predicts a variation in agreement with experiment and also accounts for the influence of composition.

INTRODUCTION

MANY magnetic phenomena depend very critically upon the fundamental magnetostrictive constants. Becker and Kersten[1] have shown that the coercive force and initial permeability should depend very markedly upon the saturation magnetostriction, λ_s. The high coercive force in such materials as cobalt magnet steel can be traced directly to this prediction.[2] It has been felt that many hitherto unexplainable anomalies in coercive force and permeability might become resolved if the magnitude of λ_s can be related to some fundamental structure characteristics of alloys. Since many of these anomalies are exhibited by binary alloys in which there is an order-disorder transforma-

FIG. 1. Circuit diagram of magnetostriction measuring apparatus.

tion, it was decided to investigate an alloy system of this type in order to ascertain whether there exists a relationship between order-disorder and magnetostriction. Theoretical analysis of the influence of order on local electron density and thus on magnetization and magnetostriction also indicated the possibility of an appreciable effect.

EXPERIMENTAL

Samples of the alloy were prepared by melting pure electrolytic iron and pure cobalt in a high-frequency vacuum furnace. 0.75 percent chromium was included in the melt in order to permit the material to be properly worked.[3] The ingot was then forged and cold-rolled into strip 0.038 cm in thickness and annealed for 10 hours at 850°C in wet hydrogen in order to remove any carbon that may be present and to further homogenize the material. In its final form, the amount of carbon present in the sample was less than 0.005 percent. The samples were prepared in the form of strips 15 cm×3 cm×0.038 cm. This is actually half of a conventional Epstein strip. All the strips were then heated to 900°C in a neutral atmosphere and quenched in cold water. In order to keep the samples flat, the annealing as well as the quenching was accomplished by placing the strip between two ¼-inch
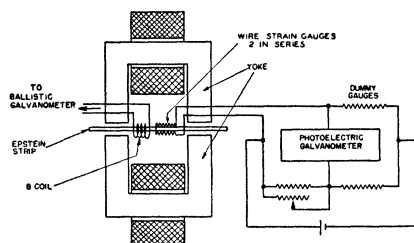
[1] See, for example, the recently published compilation by Kersten, *Foundations of the Theory of Ferromagnetic Hysteresis and Coercive Force* (Edwards Brothers, Inc., Ann Arbor, Michigan, 1946), (in German).
[2] J. E. Goldman, Trans. A.S.M. **37**, 212 (1946).
[3] Subsequent measurements on chromium-free alloys indicated the effect of this addition to be negligible.

copper bars. The ordered samples were annealed for several hours at a temperature slightly below the critical temperature for order which is 732°C.

Magnetostriction measurements were made with the resistance strain gauge technique which was first suggested and employed for this purpose by one of us.[4] These strain gauges operate on the principle that if a thin wire is stretched, its resistance will undergo a change proportional to the degree of strain. The constant of proportionality, the gauge factor,

$$G.F. = \frac{\Delta R/R}{\Delta l/l},$$

varies from 1 to as much as 12.1 depending upon the material, the magnitude of the stress, and other factors. The fine wire is bonded to a small piece of treated paper which is in turn bonded to the member whose strain is to be measured. These experiments have been carried out with Type A gauges (advance) of the SR-4 series manufactured by the Baldwin-Southwark Division of the Baldwin-Locomotive Works. The isoelastic gauges although possessing a higher G.F. are unsuitable for this application—because of the magnetoresistance of the gauge material.

In order to obtain the desired sensitivity of 2×10^{-8} in the apparatus, it was necessary to measure $\Delta R/R$ of the order of 4×10^{-8}. This was accomplished by means of a conventional type d.c. Wheatstone bridge circuit with a Rubicon photoelectric galvanometer as the detecting device operating as a deflection instrument rather than as a null indicator. The effect of temperature or other electromagnetic and atmospheric variations is minimized by employing identical gauges in all four arms of the bridge, all mounted on or close to the sample under test, and a single shielded and thermally insulated decade resistance box in parallel with one arm for balancing and calibration. The active arm of the bridge consists of two gauges in series, one mounted on each side of the strip under test. This minimizes the error due to bending of the sample. The magnetizing field is supplied by a symmetrical Hipernik yoke. The yoke, sample, and bridge are kept in a large thermally insulated box to reduce still further the effect of air currents and
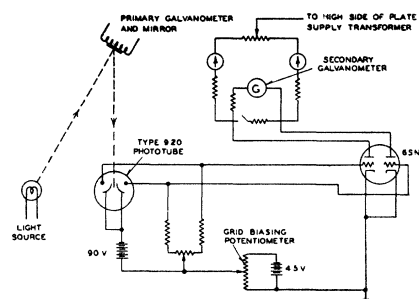
4 J. E. Goldman, Phys. Rev. 72, 529 (1947).

FIG. 2. Amplifying circuit for photoelectric galvanometer.

changes in ambient temperature. In order to minimize flux linkage by the gauge leads, all leads are twisted and brought out symmetrically. A schematic representation of the circuit is shown in Fig. 1. Details of the photo-galvanometer are shown in Fig. 2.

With this arrangement and with drift as well as pick-up reduced to a negligibly small value, a change in resistance corresponding to a strain of 10^{-6} gives a deflection on the secondary galvanometer of 46 mm. This corresponds, therefore, to a strain sensitivity, $\Delta l/l$ per mm $= 2.15 \times 10^{-8}$. Since we can estimate without difficulty to a fraction of a mm, it is felt that measurements of strain down to 10^{-8} can be made with good accuracy and reproducibility. The maximum uncertainty due to drift during the course of a measurement is of the order of 0.25 mm.

Measurements of magnetostriction were made by saturating the strip in the yoke and measuring the change in length when the flux density in the strip was reduced to zero. Magnetic measurements were made with a search coil and ballistic galvanometer.

There are several inherent advantages in this type of measurement which it may be of value to point out. (1) The measurement of the magnetostrictive strain utilizes only a very small portion of the sample, usually no greater than 1 inch, and measures the magnetostriction over this region. This has the dual advantage of permitting the use of very small samples and eliminating end effects due to shearing strains, non-uniformity of flux, etc. (2) The nature of the method is such that the effect of temperature and other variables is essentially eliminated. (3) A closed magnetic path may be used, thus

TABLE I. Saturation magnetostriction and electrical resistivity of quenched and annealed Fe-Co alloys of the 50 atomic percent composition. The degree of order as evidenced by the neutron diffraction patterns is included in the table.

	Quenched	Annealed
Saturation magnetostriction	6.5×10^{-5}	9.2×10^{-5}
Electrical resistivity (micro-ohm-cm)	24.2	18.82
Neutron diffraction	Slightly ordered	Well ordered

assuring a greater uniformity of flux distribution and eliminating possible errors due to form effect. (4) Finally, and perhaps most significant, by using square samples and type AX gauges, measurements can be made quite readily in two perpendicular directions, thus eliminating to a great extent the uncertainty in the knowledge of the demagnetized state, and errors due to the possible non-randomness of the initial distribution of domain directions.

Resistivity measurements were made by the standard method of using a type K potentiometer and measuring the voltage drop across two fixed points in the sample as well as the potential drop in a standard cell in series with the sample, the latter measurement determining the current. To detect the state of order by direct means, samples were given to Dr. S. Siegel who used neutron diffraction techniques[5] at the Clinton pile of the Oak Ridge National Labora-
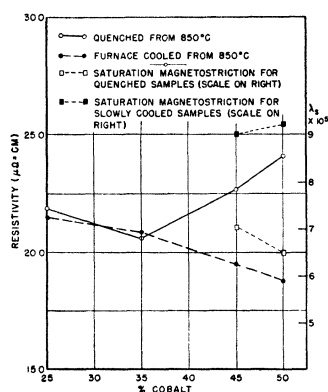

FIG. 3. Resistivity of quenched and annealed Fe-Co alloys containing various amounts of cobalt (0.75 percent Cr). Saturation magnetostriction for the 45 and 50 percent alloys is shown by the dashed curve. Scale on the right indicates magnetostriction.

[5] S. Siegel and C. G. Shull, Bull. Am. Phys. Soc. 23, No. 4, 5 (1948); also, S. Siegel, private communication to the authors.

tory to detect superlattice lines in the alloy. The x-ray atomic scattering factors of iron and cobalt being so similar, it is difficult to employ the standard x-ray techniques for this purpose.

RESULTS

The results of the investigation are shown in Table I. The values given are averages for several samples. The deviation for the magnetostriction values is in no case greater than ± 10 percent for the quenched samples and only a few percent for the annealed samples.

It is seen that the magnetostriction at saturation increases by about 40 percent upon ordering. It must be pointed out, however, that the neutron-diffraction patterns indicate that even the quenched samples are seen to be to some extent ordered. Photomicrographic evidence precludes the possibility of this change as well as the change in electrical resistivity being due to any metallurgical effect such as a dispersed phase or grain size effect.

It may be of interest to point out the variation in electrical resistivity as a function of cobalt content for samples quenched and air cooled from 850°C. This data, shown in Fig. 3, appears to be consistent with the hypothesis proposed by Ellis and Greiner[6] that a certain amount of ordering takes place at 35 percent cobalt and increases in degree with increasing cobalt content. The saturation magnetostriction for the 45 and 50 percent Co alloy measured on the same samples has been included in the figure. It has already been pointed out above that the quenched samples are to some extent ordered and the state of order would therefore be quite sensitive to the quenching rate.

THEORY AND DISCUSSION

There are, in general, two ways of approach to the problems of magnetic properties within the domains of a ferromagnetic metal. In the case of volume properties one treats the whole domain as a unit without specified crystallographic directions, and the approach is partly based on the band theory of metals. The magnetization appears as strongly dependent upon the relative size of unfilled $3d$-shells as compared

[6] W. C. Ellis and E. G. Greiner, Trans. Am. Soc. Metals 29, 415 (1941).

to the interatomic distances. This theory is particularly successful in explaining the saturation moments of the ferromagnetic metals, influence of alloying elements, volume magnetostriction, and other aspects[7] which depend upon the average electron density and do not involve explicit crystallographic directions. The notion of an average electron density in the $3d$-shells in a metal, and in particular in an alloy, can be interpreted either in terms of a $3d$-band or, perhaps preferably, in terms of an additive interaction between $3d$-shells of individual atoms. The dependence of the interaction forces (exchange forces) between the $3d$ electrons upon the ratio of the interatomic distance to the radius of the $3d$-shell has been pointed out by Slater[8] and later more quantitatively expressed by Néel[7] in his plot of the coefficient of molecular field against distance between the $3d$-shells (see Fig. 4). This way of approach, however, is not suitable for the treatment of linear magnetostriction and a literal interpretation of the above curve, as representing interaction between a pair of atoms in the crystal lattice, is not immediately justified. As shown below, the magnetostriction of iron is due to the angular dependence of the interaction between the magnetic moments of the atoms rather than to the dependence of that interaction upon distance as one might like to infer from the curve in Fig. 4. Magnetostriction of iron estimated on the latter basis appears to have a wrong sign and a wrong order of magnitude. Also an attempt to explain the influence of order on magnetostriction, by taking into account the difference of the radii of the $3d$-shells of iron and of cobalt and estimating the shift of the representative points on the curve in Fig. 4, fails.

The other type of approach which is much more successful in the treatment of the linear magnetostriction is that given by Becker.[9] Every atom is supposed to behave as a magnetic dipole and the interaction between parallel dipoles is given by the well-known formula

$$U = \frac{\mu^2}{r^3}(1 - 3\cos^2\alpha), \qquad (1)$$

[7] L. Néel, Ann. de physique 8, 237 (1937). R. Smoluchowski, Phys. Rev. 59, 309 (1941); 60, 249 (1941).
[8] J. C. Slater, Phys. Rev. 36, 57 (1930).
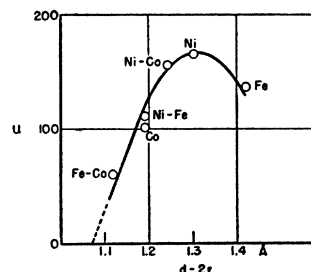[9] R. Becker, Zeits. f. Physik 62, 253 (1930).

Fig. 4. Curve giving the molecular field coefficient as a function of the distance between 3d shells (after Néel).

where r is the distance and α the angle between the line joining the atoms and the direction of the dipoles. This model is clearly over-simplified; but, apart from a numerical factor, it describes the linear magnetostriction of iron satisfactorily, although it is less successful in the case of nickel. It gives, however, no clue as to the probable dependence of magnetostriction on composition of alloys and upon the various volume properties which lead to the curve shown in Fig. 4.

No attempt will be made to estimate theoretically the possible dependence of the exchange forces between unfilled $3d$-shells upon the angle between the direction of magnetization and the line joining the two atoms. Instead, in our theory, which will be developed for a body-centered cubic lattice, we shall follow essentially the dipole approximation including, however, certain features of the other point of view. For simplicity we shall limit ourselves to the interactions between an atom and its nearest and second nearest neighbors, which in a body-centered lattice, are only about 15 percent further away. Also we shall limit ourselves to the magnetostriction in the direction of easy magnetization. Both these simplifications can be easily removed, but the treatment becomes then unnecessarily cumbersome. We shall consider the magnetostriction in the [100] direction, which is the direction of easy magnetization in iron and in some iron-cobalt alloys and is accounted for on the basis of Becker's theory. According to Shih,[10] there is indication that in a 50:50 iron-cobalt alloy the direction of easy magnetization is the [111] direction. This case has been examined in detail and it appears that all the results of our theory

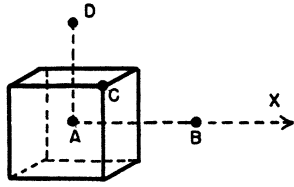[10] J. W. Shih, Phys. Rev. 46, 139 (1934).

FIG. 5. The unit cell of a body-centered cube showing the three kinds of neighbors of an atom in a domain.

including the quantitative influence of ordering apply here also except for the difference in sign of the magnetostriction as in Becker's theory. Figure 5 represents an atom and its neighbors in a body-centered cubic lattice in a domain which is magnetized along the [100] direction which we chose as the direction of the X coordinate. The atom A has three kinds of neighbors: nearest neighbors of the type C; second nearest neighbors in the direction of magnetization, type B; and second nearest neighbors in the directions perpendicular to the direction of magnetization, type D. In iron and in iron-cobalt alloys the magnetostriction is positive; thus, in the magnetic condition the interatomic distances parallel to the direction X are increased by a factor $(1+\lambda)$. In the other two perpendicular directions the distances are decreased by a factor $(1-\lambda/2)$. Thus, we use the condition that magnetostriction occurs without a change of volume, although a correction could be easily introduced for the known Poisson's ratio. Differentiation of the expression (1) gives easily the following contributions of the three types of neighbors to the change of the magnetic interaction as the function of the parameter λ:

$$\Delta U_C = -\frac{16}{9}\sqrt{3}\frac{\mu^2}{a^3}\lambda; \quad \Delta U_B = 6\frac{\mu^2}{a^3}\lambda; \quad \Delta U_D = \frac{3}{2}\frac{\mu^2}{a^3}\lambda$$

where a is the lattice constant. It is interesting to note that the only negative term is that due to the C type atoms. All other contributions to the magnetic energy are positive and oppose a positive magnetostriction. The distance AC remains unchanged within the approximation of terms linear in λ, and so the negative terms, and, thus, the positive magnetostriction is solely due to the angular displacement of the atom C.

Since we are going to consider ordering phenomena, we have to specify the contributions

from atoms in the various positions, and we will use the subscripts AB, AC, and AD. The energy of the dipole interaction of one atom with its neighbors is thus:

$$\frac{2}{a^3}\left(6\mu_{AB}^2 + 3\mu_{AD}^2 - \frac{64}{9}\sqrt{3}\mu_{AC}^2\right)\lambda, \qquad (2)$$

which, for the assumption of all dipoles being identical, simplifies to

$$-6.6\frac{\mu^2}{a^3}\lambda.$$

The density of energy is then expressed by

$$E_d = -\frac{1}{2}N\frac{\mu^2}{a^3}6.6\lambda,$$

where N is the number of dipoles per cubic centimeter which, for a body-centered lattice, is equal to $2/a^3$ thus:

$$E_d = -1.65N^2\mu^2\lambda. \qquad (3)$$

This decrease of magnetic energy is naturally opposed by an increase of the elastic energy. The latter, in our case, is given by

$$E_e = \tfrac{3}{2}G\lambda^2, \qquad (4)$$

where G is the shear modulus. Thus, at equilibrium, magnetostriction will be obtained from the condition

$$\frac{\partial}{\partial\lambda}(E_d + E_e) = 0,$$

which gives us

$$\lambda = \frac{N^2}{G}\frac{64\sqrt{3}\mu_1^2 - 81\mu_2^2}{54}, \qquad (5)$$

where subscripts 1 and 2 indicate contributions from nearest and from second nearest neighbors, respectively. The numerical factor differs slightly from that given by Becker because of the summation which here extends only over the first two layers of neighbors. In the case of a random alloy, the average magnetic moments of nearest and second nearest neighbors are the same and the formula simplifies to

$$\lambda = \frac{N^2}{G}0.56\mu^2. \qquad (6)$$

It is thus necessary to determine the influence of ordering on μ_1 and μ_2.

If we could consider the total magnetization as due to a sum of independent contributions of Fe-Fe, Co-Co, and Fe-Co pairs of neighbors, then we would obtain a quick qualitative estimate of the effect of ordering. Clearly, the pair Fe-Co would have to contribute more to the magnetization than the other two kinds. Inasmuch as in an ordered alloy all nearest neighbors are of the Fe-Co type, while in a random alloy only half of them are of that type, formula (5) would give a larger magnetostriction in an ordered state. However, such an assumption has many drawbacks and is difficult to reconcile with the present theory of magnetism which relates the magnetic moment to the electron density rather than to an interaction between pairs of atoms.

The dependence of magnetization per atom upon the electron concentration, as summarized in the well-known curve shown in Fig. 6, indicates a maximum around 8.3 valence electrons per atom, and is well explained on the basis of the band theory of metals.[11] The main fact underlying the theoretical interpretation of the curve in Fig. 6 is the fundamental importance of the electron density rather than the type of atoms which contribute electrons. We shall consider the total magnetization as being determined, not so much by an average electron density of the whole metal, but rather by a sum of contributions of local electron densities. Also we shall consider the curve in Fig. 6 as applying to local electron concentrations, although the experimental points on this curve, except for pure metals, were obtained for random alloys, and thus in that sense are average values. The local electron concentration naturally depends upon the neighboring atoms and these are influenced by the degree of order.

In order to make our theory quantitative, we have to take into account the fluctuations of the charge density in a random and in an ordered alloy and estimate the contributions of local electron densities to the final magnetic moment. The probability that an atom in a random 50–50 alloy will have r identical neighbors among its

n neighbors is

$$\frac{1}{2^n}\binom{n}{r}. \tag{7}$$

From this we can easily compute the probability of finding a given average electron concentration among the eight neighbors of an atom. The magnetic moment characteristic of the given atom and its neighborhood is then determined by the arithmetical mean of the number of valence electrons of the atom in question and of the average valence electron density of its neighborhood. Thus, in the case of an iron atom with its eight valence electrons, the effective density will vary anywhere from 8, in case it is surrounded by 8 iron atoms, to 8.5 if it is surrounded by 8 cobalt atoms. Similarly, the effective electron density for a cobalt atom will vary from 8.5 to 9. The corresponding probability of the occurrence of these density fluctuations is obtained from the expression (7). The magnetic moment associated with each of the thus computed electronic densities is read off the curve in Fig. 6, in which we use a linear relationship, expected by theory, and somewhat more perfect than that given by experimental data. Thus the magnetization increases linearly from the value 2.22 for Fe to 2.53 at 35 percent Co, and then decreases linearly to the value for Co. Pure cobalt has a close-packed lattice so that the direct use of its moment, 1.7 in our case, may not be correct, since both iron and iron-cobalt alloys are body-centered cubic up to 75 percent Co. By extrapolating the magnetization values of body-centered iron-cobalt
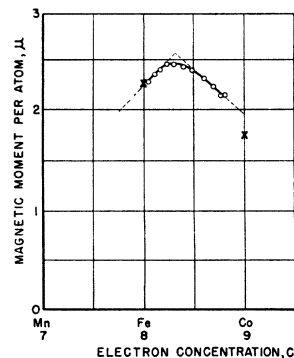

Fig. 6. Magnetic moment per atom in Bohr magnetons vs. electron concentration for iron, cobalt and their body centered alloys.

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[11] See, for example, F. Seitz, *Physics of Metals* (McGraw-Hill Book Company, 1943), p. 308.

	50 Co	45 Co
Observed	40	30
Theoretical	85	70

alloys, we obtain the value 1.88 for pure body-centered cobalt. This correction is not important and it introduces only small numerical differences in the result. If we now add all the contributions of the various local electron concentrations with the proper probabilities, we obtain a magnetic moment of 2.27 which is about 4 percent smaller than the value 2.37 which we considered as characteristic of the average density of 8.5 electrons. In a perfectly ordered alloy, each iron atom is surrounded by 8 cobalt atoms and each cobalt atom by 8 iron atoms so that the local electron density will be everywhere 8.5 and, thus, the corresponding magnetic moment 2.37.

If we now try to compute the change of magnetostriction on ordering, we have to substitute the proper μ values in formulae 5 and 6. For a random alloy $\mu_1 = \mu_2 = \mu = 2.27$; for an ordered alloy the magnetic moment for the nearest neighbors μ_1, according to the previous paragraph, will be 2.37; on the other hand, the magnetic moment for the second nearest neighbors, which are always of the same kind as the central atom, will be given by the average of the magnetic moments of pure iron and of pure cobalt, thus $\mu_2 = 2.05$. The ratio of the two magnetostrictions turns out to be 1.85, thus an increase of 85 percent. This is to be compared with the observed value of about 40 percent. It is important that the small change of 4 percent of the magnetic moment, μ, between the ordered and disordered state does not play a significant role in the change of magnetostriction. It is rather the change in the second nearest neighbors which produces the big difference.

It is perfectly understandable why the theoretical increase of magnetostriction is higher than the experimental result. One would not expect our experimental treatments to correspond so closely to a comparison of a perfectly random with a perfectly ordered state. As men-

tioned before, the results of neutron diffraction experiments confirm this explanation. Another important aspect is the fact that "random" alloy may have a certain degree of short-range order. On the basis of this theory, one might expect magnetostriction to be quite sensitive to short-range order.

The theory here outlined can be easily extended to the 55 percent Fe 45 percent Co alloy for which experimental data are available. First, for a random alloy, instead of formula (7) we have

$$\binom{n}{r} p^{n-r} q^r \tag{8}$$

where p and q are the concentrations of cobalt and of iron, respectively. This gives us the statistical neighborhood of any atom in a random alloy. Among these atoms 45 percent are cobalt atoms and thus by averaging the electron densities we obtain the contributions of local neighborhoods around cobalt atoms to the total magnetization. A similar procedure gives the contributions of neighborhoods of iron atoms. The result is $\mu = 2.26$ for a random alloy.

A perfectly ordered alloy containing 45 percent cobalt can be represented as a perfectly ordered 50:50 alloy in which 10 percent of the cobalt atoms are substituted by iron atoms. This substitution is random and all cobalt atoms have iron atoms as nearest neighbors. Among the iron atoms some have all nearest neighbors cobalt atoms, some have one iron atom among them, some two, three, and so on. The relative percentages are 21.5; 19; 7.4; etc. Finally the substitutionally dissolved iron atoms have all neighbors iron atoms. Computation of the local densities and the corresponding probabilities gives $\mu_1 = 2.33$. A similar calculation for the second neighbors gives $\mu_2 = 2.10$. The increase of magnetostriction as obtained from formula (5) is 70 percent which is to be compared with the experimental value of about 30 percent. Presumably the low observed value is, as in the previous case, due to incomplete order and imperfect randomness. The decrease of the influence of order on magnetostriction at lower cobalt content is accounted for satisfactorily by the theory. Table II summarizes the results.

Recent work of Fine and Ellis[12] indicates an interesting volume increase of ordered Fe-Co alloys. As indicated by one of us,[13] the increase of the average interatomic distance on ordering can be interpreted as an increase of repulsion between the partly filled $3d$-shells. According to the theoretical explanation of the saturation moment in ferromagnetic alloys (Fig. 6), the $3d$-shells consist of two parts, one of which is just filled up at the electron concentration corresponding to the maximum value of the magnetization. As we know from band theory of metals, full electronic shells produce strong repulsive forces. The apparent increase of repulsion between the iron and cobalt atoms can be interpreted as due to a more perfect filling of the lower part of the $3d$-shells in both kinds of atoms

in the state of perfect ordering. In the random state, as indicated before, only part of the atoms will have the proper neighborhood to assure the optimum local electron concentration, and the repulsion forces will be weaker.

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[12] M. E. Fine and W. C. Ellis, to be published in Trans. A.I.M.E.
[13] R. Smoluchowski, discussion to paper in reference 12.

Note added in proof: Preliminary calculations indicate that a similar mechanism might explain the rapid decrease of magnetostriction in Fe-Si alloys at the composition of about 12 at. percent Si, where ordering is known to take place. These results were reported by the authors at the National Research Council Conference on Phase Changes held at Cornell University in August, 1948.

Irreversible Magnetic Effects of Stress

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When Rayleigh's law for small hysteresis loops is interpreted in terms of the elementary process of domain wall displacement, its generalization to include effects of stress becomes possible. In this way theoretical formulas have been derived for the behavior of a soft iron or steel specimen which is first put into a state of normal magnetization at small magnetizing force H and is then subjected to a small tension cycle. According to the theory the magnetization should increase upon application of tension T by an amount $g(4.00 \times 10^{-9}HT + 9.86 \times 10^{-18}T^2)$ in c.g.s. e.m.u., where $4\pi g$ is the slope of the normal permeability *vs.* H curve, and should remain at the new value upon removal of the tension. An experimental test of the theory verified these predictions within the precision claimed for them. On the basis of the theoretical model, certain effects of diminishing alternating fields and stresses are analyzed quantitatively; the results are used to explain qualitatively the observed long-time magnetic behavior of ships.

INTRODUCTION

THE changes of magnetization produced by stress are cause for concern whenever magnetic devices are subject to uncontrollable mechanical disturbances. Such changes are characterized by hysteresis and are, therefore, difficult to treat analytically. However, the stresses usually encountered are magnetically equivalent to rather small fields (as compared with those

necessary to erase the previous history completely), and this suggests that it would be useful to extend the analytical description of small hysteresis loops, known as Rayleigh's law,[1] to cases in which the independent variable is stress rather than magnetizing force. The legitimacy of such an extension and the form that it must take both follow at once if one merely assumes that Rayleigh's law, with appropriate changes in the

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[1] R. Becker and W. Döring, *Ferromagnetismus* (Verlag Julius Springer, Berlin, 1939), p. 218 ff.