Note added in proof: Since this paper went to press, Lonsdale and Owston have pointed out to us by correspondence that the parameters given by Bernal and Fowler, namely, $y=0.105$ and $Z=0.037$, are incorrect and should be changed to $y=0.115$ and $Z=0.0435$. This change in the value of the parameters would affect somewhat the intensity calculations given in column 3 of Table I but this change could almost certainly not produce good agreement between this model and our observed results since the coherent scattering by the hydrogen atoms in a model in which these atoms are fixed at definite sites will always be twice as great as that produced by the Pauling model. The calculated intensities given in Table I for the Bernal-Fowler model are all higher than the observed intensities given in column 7 of the table and it would not be expected that a revision of the parameters would alter this situation.

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The Scattering of High Energy Neutrons by Nuclei

S. FERNBACH, R. SERBER, AND T. B. TAYLOR
Radiation Laboratory, Department of Physics, University of California, Berkeley, California
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The experiments of Cook, McMillan, Peterson, and Sewell on the cross sections of nuclei for neutrons of about 90 Mev indicate that the nuclei are partially transparent to high energy neutrons. It is shown that the results can be explained quite satisfactorily using a nuclear radius $R=1.37A^{\frac{1}{3}}\times 10^{-13}$ cm, a potential energy for the neutron in the nucleus of 31 Mev, and a mean free path for the neutron in nuclear matter of 4.5×10^{-13} cm. This mean free path agrees with that estimated from the high energy *n-p* cross section, but the results are not sensitive to the choice of mean free path.

In a previous paper by one of the writers[1] it has been pointed out that to a high energy bombarding particle a nucleus appears partially transparent, since at energies of the order of 100 Mev the scattering mean free path for a neutron or proton traversing nuclear matter becomes comparable to the nuclear radius. This transparency effect is strikingly apparent in the experiments of Cook, McMillan, Peterson, and Sewell[2] on the scattering by nuclei of neutrons of about 90 Mev. In the present paper it will be shown that the observed scattering cross sections can be quite satisfactorily accounted for, using a mean free path of the expected magnitude.

The problem is that of the scattering of the neutron wave by a sphere of material characterized by an absorption coefficient and an index of refraction. The index of refraction is determined by the mean potential energy, V, of the neutron in the nucleus. If $k=(2ME)^{\frac{1}{2}}/\hbar$ is the propagation vector of the wave outside the nucleus, its propagation vector inside is $k+k_1$, with

$$k_1=k[(1+V/E)^{\frac{1}{2}}-1].$$

For $E=90$ Mev, $k=2.08\times 10^{13}$ cm^{-1}. The potential V is generally taken to be about 8 Mev larger than the energy of the Fermi sphere. The latter depends on the assumed nuclear density. If we use for the nuclear radius the value $R=1.37A^{\frac{1}{3}}\times 10^{-13}$ cm, deduced by Cook, McMillan, Peterson, and Sewell from the 14–25 Mev scattering results of Amaldi, Bocciarelli, Cacciapuoti, and Trabacchi,[3] and Sherr,[4] we find a Fermi energy of 22 Mev, and $V=30$ Mev. This gives $k_1=3.22\times 10^{12}$ cm^{-1}. The absorption coefficient in nuclear matter is equal to the particle density times the cross section for scattering of the neutron by a particle in the nucleus,

$$K=3A\sigma/4\pi R^3.$$

In terms of the *n-p* and *n-n* cross sections,

$$\sigma=[Z\sigma_{np}+(A-Z)\sigma_{nn}]/A.$$

Cook *et al.*,[2] give for the scattering of a 90 Mev neutron by a free proton $\sigma_{np(\text{free})}=8.3\times 10^{-26}$ cm^2. This cross section must be reduced to allow for the effect of the exclusion principle on the scattering by a proton bound in the nucleus; according to Goldberger,[5] the factor is $\sigma_{np}=\frac{2}{3}\sigma_{np(\text{free})}$. Assuming a $1/E$ dependence of the cross sections we find, for $E=90+30=120$ Mev, $\sigma_{np}=4.15\times 10^{-26}$ cm^2. If, following Goldberger, we take $\sigma_{nn}=\frac{1}{4}\sigma_{np}$, and use the previously quoted radius formula, we obtain $K=2.4\times 10^{12}$ cm^{-1} for $Z/A=\frac{1}{2}$, $K=2.1\times 10^{12}$ cm^{-1} for $Z/A=0.39$ (U). It will be seen from these numbers that in the ensuing calculations it will be a reasonable approximation to suppose that $kR\gg 1$, but k_1/k and $K/k\ll 1$, so that k_1R and KR are of order one.

[1] R. Serber, Phys. Rev. **72**, 1114 (1947).
[2] L. J. Cook, E. M. McMillan, J. M. Peterson, and D. C. Sewell, Phys. Rev. **75**, 7 (1949).
[3] E. Amaldi, D. Bocciarelli, B. N. Cacciapuoti, and G. C. Trabacchi, Nuovo Cimento **3**, 203 (1946).
[4] R. Sherr, Phys. Rev. **68**, 240 (1945).
[5] M. L. Goldberger, Phys. Rev. **74**, 1268 (1948).

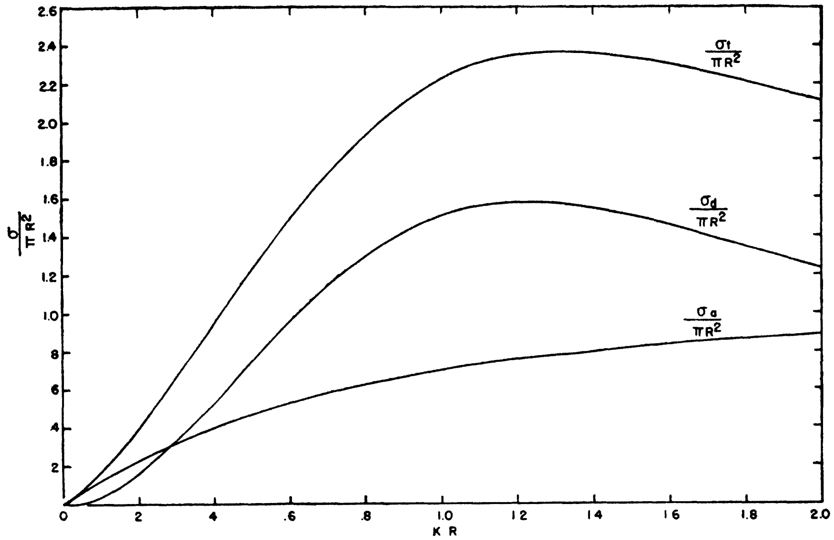

FIG. 1. Absorption, diffraction and total cross section as a function of the nuclear radius measured in mean free paths. These curves are for $k_1/K=1.5$.

The scattering cross section consists of two parts. The first, the "absorption cross section," is just πR^2 times the probability that the neutron collides with a particle in the nucleus. This is not true absorption: inelastic scattering and scattering with exchange are included. The second part, the "diffraction scattering," is elastic scattering arising from the disturbance of the incident plane wave by the nucleus. To illustrate the calculation, we first consider the scattering from a disk of radius R and thickness T. We suppose there is a boundary layer at the surface of the disk in which k_1 and K rise to their interior values in a distance larger than $1/k$.[6] There will then be no scattering at the surfaces, and, for unit amplitude of incident wave, the wave transmitted through the disk will have an amplitude and relative phase $a=\exp(-\frac{1}{2}K+ik_1)T$. The absorption cross section is

$$\sigma_a=\pi R^2(1-|a|^2)=\pi R^2(1-e^{-KT}). \qquad (1)$$

The diffraction cross section can be found from the consideration that on a plane behind the disk the wave is no longer plane, but differs from a plane wave by an amplitude $1-a$ in the shadow of the disk. This amplitude represents a scattered wave, and the corresponding cross section is

$$\sigma_d=\pi R^2|1-a|^2 =\pi R^2(1-2e^{-\frac{1}{2}KT}\cos k_1T+e^{-KT}). \qquad (2)$$

It can easily be shown that the angular dependence of the scattered amplitude is

$$f(\theta)=k\int_0^R(1-a)J_0(k\rho\sin\theta)\rho d\rho =(1-a)RJ_1(kR\sin\theta)/\sin\theta, \qquad (3)$$

which gives the differential scattering cross section

$$d\sigma_d(\theta)=|f(\theta)|^2d\Omega =(\sigma_d/\pi)[J_1(kR\sin\theta)/\sin\theta]^2d\Omega. \qquad (4)$$

The absorption cross section is, of course, always less than πR^2, but the diffraction cross section may be either larger or smaller, depending on the magnitude of the phase shift. For large KT, $\sigma_a=\sigma_d=\pi R^2$. In the opposite limit of small KT and k_1T, we have

$$\sigma_a=\pi R^2KT=A\sigma,$$
$$\sigma_d=\pi R^2(\tfrac{1}{4}K^2+k_1^2)T^2=\tfrac{1}{4}A\sigma[1+4(k_1^2/K^2)]KT.$$

Thus for low density or small thickness, σ_a approaches the sum of the scattering cross sections of the separate nucleons. The diffraction cross section, however, vanishes in the limit, being proportional to the probability of double scattering.

The corresponding calculations for a sphere are only slightly more complicated. The portion of the wave which strikes the sphere at a distance ρ from a line through the center of the sphere emerges after traveling a distance $2s$, with $s^2=R^2-\rho^2$. Its amplitude on emerging is $a=\exp(-K+2ik_1)s$, so that, in place of (1) we have

$$\sigma_a=2\pi\int_0^R(1-e^{-2Ks})\rho d\rho=2\pi\int_0^R(1-e^{-2Ks})sds =\pi R^2\{1-[1-(1+2KR)e^{-2KR}]/2K^2R^2\}. \qquad (5)$$

This formula for the absorption cross section has previously been given by Bethe.[7] Similarly, in

[6] In terms of the model being employed, the finite intercept of the R vs. $A^{1/3}$ line obtained from the data on the lower energy scattering could be interpreted by the more careful examination of the boundary conditions which in this case would be necessary.

[7] H. A. Bethe, Phys. Rev. 57, 1125 (1940).

place of (2), we have

$$\sigma_d=2\pi\int_0^R |1-e^{(-K+2ik_1)s}|^2\rho d\rho$$
$$=\pi R^2[1+(1/2K^2R^2)\{1-(1+2KR)e^{-2KR}\}$$
$$-(1/(\tfrac{1}{4}K^2+k_1^2)^2R^2)\{(\tfrac{1}{4}K^2-k_1^2)$$
$$+e^{-KR}[2k_1R(\tfrac{1}{4}K^2+k_1^2)+k_1K]\sin 2k_1R$$
$$-e^{-KR}[(\tfrac{1}{4}K^2-k_1^2)+KR(\tfrac{1}{4}K^2+k_1^2)]$$
$$\times\cos 2k_1R\}]. \quad (6)$$

In deriving (5) and (6) we have neglected refraction at the surface of the sphere. It can easily be seen that this is legitimate, since it gives an effect of order $(k_1/k)k_1R$.

For the angular distribution we find, in analogy to (3),

$$f(\theta)=k\int_0^R[1-e^{(-K+2ik_1)s}]J_0(k\rho\sin\theta)\rho d\rho. \quad (7)$$

For $KR\to\infty$, we again obtain (4), but we have not found a convenient expression in the general case. The amplitude for forward scattering is easily evaluated, and is found to be

$$f(0)=\frac{kR^2}{2}\left\{1+\frac{(k_1-\frac{1}{2}iK)^2[1-(1+KR-2ik_1R)e^{(-K+2ik_1)R}]}{2(\frac{1}{4}K^2+k_1^2)^2R^2}\right\}. \quad (8)$$

For purposes of calculation, the integral can be converted to a sum; letting $l+\frac{1}{2}=k\rho$ and using the relation $J_0((l+\frac{1}{2})\sin\theta)=P_l(\cos\theta)$, valid for large l and small θ, we find

$$f(\theta)=\tfrac{1}{2}k\sum_{l=0}^{l+\frac{1}{2}<kR}(2l+1)(1-e^{(-K+2ik_1)s_l})P_l(\cos\theta), \quad (9)$$

where

$$s_l=[k^2R^2-(l+\tfrac{1}{2})^2]^{\frac{1}{2}}/k.$$

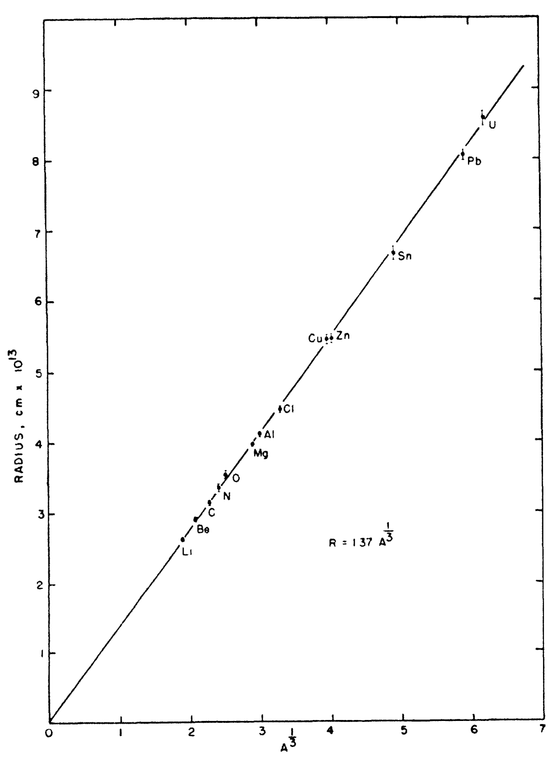

FIG. 2. Nuclear radii deduced from the total cross section measurements of Cook, McMillan, Peterson, and Sewell, plotted against the cube roots of the mass numbers. The straight line is $R=1.37A^{\frac{1}{3}}\times10^{-13}$ cm.

This expression can also be obtained by a partial wave analysis, using the WKB method to evaluate the phase shifts. This gives

$$\delta_l=(k_1+\tfrac{1}{2}iK)s_l,$$

whence we immediately obtain (9), and for σ_a and σ_d,

$$\sigma_a=(\pi/k^2)\textstyle\sum_l(2l+1)(1-e^{-2Ks_l}), \quad (10)$$

$$\sigma_d=(\pi/k^2)\textstyle\sum_l(2l+1)|1-e^{(-K+2ik_1)s_l}|^2. \quad (11)$$

Converting the sums in (10) and (11) to integrals we again obtain (5) and (6).

In Fig. 1 we have plotted $\sigma_a/\pi R^2$, $\sigma_d/\pi R^2$, and the total cross section $\sigma_t/\pi R^2=(\sigma_a+\sigma_d)/\pi R^2$ as functions of KR. The ratio $\sigma_a/\pi R^2$ is a function only of KR; the other two depend on k_1/K as well. The curves in Fig. 1 have been plotted for $k_1/K=1.5$, about the ratio indicated by our earlier consideration of the expected magnitude of the constants. Using this plot it is possible to determine, once a value of K is chosen, the radius required for each nucleus to give the measured total cross section. The radii calculated in this way from the observed cross sections, using the value[8] $K=2.2\times10^{12}$ cm^{-1}, are shown in Fig. 2. It will be seen that they lie quite closely on the line $R=1.37A^{\frac{1}{3}}\times10^{-13}$ cm; the self-consistency of our description of the scattering process is thus established. The value $K=2.2\times10^{12}$ cm^{-1} corresponds to a mean free path in nuclear matter of 4.5×10^{-13} cm. The associated value, $k_1=3.3\times10^{12}$ cm^{-1}, corresponds to $V=30.8$ Mev.

The question now arises as to the accuracy with which the constants K and k_1 are determined by the scattering data. If k_1 is decreased, keeping K constant, it is found that the radius curve, Fig. 2, is pulled up in the middle; the resultant curve can

[8] The small dependence of K on Z/A is unimportant, as we shall see later.

be approximated by two straight lines, the light elements lying on a steeper line through the origin, while the heavy elements lie on a less steep line with a positive intercept. Increasing k_1 has the opposite effect. A variation in k_1 of $\pm 0.2\times 10^{12}$ cm^{-1}, or in V of ± 2 Mev, begins to produce appreciable bending. A reduction in K, with fixed k_1, introduces a curvature in the radius line, the center being pulled down and the two ends raised. The curvature becomes noticeable if K is reduced to less than $K=1.9\times 10^{12}$ cm^{-1}, however K can be almost doubled before the opposite curvature becomes very pronounced. For example, $K=3.0\times 10^{12}$ cm^{-1} gives an about equally good straight line, $R=1.39A^{\frac{1}{3}}\times 10^{-13}$ cm. The total cross-section measurements thus determine the potential fairly well, but are quite insensitive to the absorption coefficient. Measurements of σ_a and of the differential diffraction scattering are required for a better evaluation of K. It should be noted that while k_1 and K are determined directly from the cross sections, the evaluation of V depends also on the energy of the incident neutrons. Cook *et al.* state that the energy of the neutrons detected in their experiment may be a little lower than 90 Mev, lying somewhere between 80 and 90 Mev. If we took $E=80$ Mev, we would find $V=28.8$ Mev.

For $K=2.2\times 10^{12}$ cm^{-1}, the values of KR range from 0.58 for Li to 1.87 for U. It will be seen from Fig. 1 that the nuclear opacity, $\sigma_a/\pi R^2$, would vary from 0.52 for Li to 0.88 for U. It will also be seen that over this range of values of KR it would be expected that σ_d will be nearly twice as large as σ_a.

If one plots the angular distribution of the diffraction scattering given by (9) (i.e., $d\sigma_d(\theta)/d\sigma_d(0)$ *versus* $kR\sin\theta$) one finds curves for the heaviest nuclei which are indistinguishable from that for an opaque nucleus (Eq. (4)), at least as far as the first minimum of the diffraction pattern. For the lighter nuclei, the form of the curve is closely the same, but with an altered scale of abscissa, corresponding to using an effective radius somewhat smaller than the true radius. The increase in the half width of the diffraction peak is zero for $KR=1.78$ (Pb), 3.7 percent for $KR=1.20$ (Cu), 6.2 percent for $KR=0.90$ (Al) and 9.6 percent for $KR=0.63$ (Be). Measurements of the diffraction scattering and of the absorption are now in progress in this laboratory.

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He^3 Isotopic Abundance*

J. H. Coon
Los Alamos Scientific Laboratory, Los Alamos, New Mexico
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The isotopic abundance of He^3 in one sample each of "well" helium and "atmospheric" helium has been measured by detecting the $He^3(n,p)H^3$ disintegrations induced by thermal neutrons. The helium gas was put into a proportional counter, the disintegration rate compared to that with nitrogen in the counter, and the He^3 content deduced from the known ratio of the He^3 and N disintegration cross sections.

INTRODUCTION

PREVIOUS measurements[1,2] have indicated that He^3 is present in natural helium in amounts of the order of one part in 10^6 to 10^7, and that the abundance varies by more than a factor of 10 depending on the source of the helium. He^3 concentration determination is of special current interest in connection with nuclear investigations of interactions between elementary nuclei, and in connection with investigations of the thermodynamic behavior of He^3 and He^4 at temperatures of liquid helium.

In the present work measurements were made of the isotopic abundance of He^3 in two samples of natural helium: one from wells near Amarillo, Texas, and one from air reduction processing. The presence of He^3 was detected by counting ionization pulses arising from the disintegration products of the reaction $He^3(n,p)H^3$ induced by thermal neutrons. The cross section for this reaction is about 5000 barns,[3] which is sufficiently large to make it possible to detect the He^3 in natural helium samples. Data were also taken with nitrogen in the counter, in which case one detects the $N^{14}(n,p)C^{14}$ disintegrations. Since the protons from the He^3 and N reactions have very closely the same range, the "wall effect" corrections will be similar for the two cases. If counting is done on nitrogen and on

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[1] L. T. Aldrich and A. O. Nier, Phys. Rev. **70**, 983 (1946); **74**, 1225 (1948); **74**, 1590 (1948).
[2] L. W. Alvarez and R. Cornog, Phys. Rev. **56**, 613 (1939); **56**, 379 (1939).
[3] J. H. Coon and R. A. Nobles, Phys. Rev. **75**, 1358 (1949).