The residual current observed by us in the best vacuum conditions—10^{-6} mm Hg—may be certainly explained as an effect of ionization by collision if an effective cross section of the order 1 $cm^2/cm^3 \cdot$mm Hg is attributed to the gas, in agreement with the results normally obtained.

Regarding the fundamental effect we were investigating, *viz.* the liberation of ions from the metal in the best conditions of surface purity and of vacuum, we infer from our measurements that, if it occurs, its threshold must be over 70 kv. This is also compatible with the shape of the curve of Trump and Van de Graaff. Our present arrangement did not permit measurements at higher voltages.

It is a pleasure for us to thank the Consiglio Nazionale delle Ricerche which has granted a scholarship to one of us (I.F.) and the S. A. Vetrocoke of Porto Marghera (Venice) which kindly put at our disposition the liquid nitrogen necessary for the experiments.

[1] J. G. Trump and R. J. Van der Graaff, J. App. Phys. **18**, 327 (1947).

The Generalized Schrödinger Equation in the Interaction Representation

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March 8, 1949

A GENERALIZED Schrödinger equation has been used by Schwinger[1] and Tomonaga and collaborators[2] in the interaction representation for a system whose state is given on a space-like surface. In both presentations a form for the Hamiltonian is assumed and then shown to satisfy necessary conditions. We will deduce the generalized Schrödinger equation by a method which, when applied to a particular case, gives the Hamiltonian directly, even when the interaction involves derivatives and the Hamiltonian consequently contains terms dependent on the direction of the surface.

Let $x^\mu=(ct, \mathbf{r})$. σ is a general space-like surface through x^μ. n_μ is the normal to the surface at $x^\mu (n_\mu n^\mu=-1)$. The Lagrangian of the field is $\mathfrak{L}(\varphi^\alpha, \varphi_\mu{}^\alpha)$ where $\varphi_\mu{}^\alpha=\partial\varphi^\alpha/\partial x^\mu$. The energy momentum tensor is $U_{\mu\nu}=(\partial\mathfrak{L}/\partial\varphi_\mu{}^\alpha)\varphi_\nu{}^\alpha-\mathfrak{L}g_{\mu\nu}$.

Define the canonical conjugate of φ^α to be

$$\pi_\alpha=(\partial\mathfrak{L}/\partial\varphi_\mu{}^\alpha)n_\mu. \tag{1}$$

The generalized commutation relations of Schwinger can then be written for a Bose field

$$[\varphi^\alpha(x), \pi_\beta(x')]_-=0 \quad x\neq x', \quad \int[\varphi^\alpha(x), \pi_\beta(x')]_- d\sigma'=i\hbar c\delta_\beta{}^\alpha, \tag{2}$$

and

$$\{\varphi^\alpha(x), \pi_\beta(x')\}_+=0 \quad x\neq x', \quad \int\{\varphi^\alpha(x), \pi_\beta(x')\}_+ d\sigma'=i\hbar c\delta_\beta{}^\alpha \tag{3}$$

for a Fermi field.

Define the Hamiltonian

$$\mathfrak{H}=U_{\mu\nu}n^\mu n^\nu. \tag{4}$$

This reduces to the usual expression when σ is flat. Choose a particular Lorenz frame such that at the point x^μ the coordinates $x^r=u^r$ lie in the tangent plane to σ and $x^0=\omega$ is in the normal direction. The equations of motion of the field, derivable from the Lagrangian are by (2), (3), and (4)

$$\begin{aligned}&\int[\pi_\alpha(x'), \mathfrak{H}(x)]d\sigma'=i\hbar c\partial\pi_\alpha/\partial\omega,\\ &\int[\varphi^\alpha(x'), \mathfrak{H}(x)]d\sigma'=i\hbar c\partial\varphi_\alpha/\partial\omega.\end{aligned} \tag{5}$$

Introducing the condition of Kanesawa and Tomonaga[2] that elementary regions are scale form flat, we have $\partial\varphi^\alpha/\partial\omega =(\delta/\delta\sigma(x))\{\int\varphi^\alpha(x')\cdot d\sigma'\}$. Thus the second of Eqs. (5) can be written in the form

$$i\hbar c\delta\varphi_H/\delta\sigma=[\varphi_H, \mathfrak{H}(\varphi_H, \pi_H)]. \tag{6}$$

If $\mathbf{\Phi}$ and $\mathbf{\Psi}$ are corresponding wave functions in the Heisenberg and Schrödinger representations, then $\mathbf{\Psi}=U(\sigma)\mathbf{\Phi}$. Denote the Heisenberg variable by φ_H and the Schrödinger variable by φ_S. Then

$$\varphi_H=U^{-1}\varphi_S U. \tag{7}$$

Also $\delta\varphi_S/\delta\sigma=0$ and by an argument similar to that used by Dirac[3]

$$i\hbar c\delta U/\delta\sigma=\mathfrak{H}(\varphi_S, \pi_S)U. \tag{8}$$

Therefore

$$i\hbar c\delta\mathbf{\Psi}/\delta\sigma=\mathfrak{H}(\varphi_S, \pi_S)\mathbf{\Psi}. \tag{9}$$

If two fields interact the Schrödinger equation in the Schrödinger representation is

$$i\hbar c\delta\mathbf{\Psi}/\delta\sigma=\{\mathfrak{H}_1(\varphi_S)+\mathfrak{H}_2(\psi_S)+\mathfrak{H}_{12}(\varphi_S, \psi_S)\}\mathbf{\Psi}. \tag{10}$$

To go over into the interaction representation we make the transformation $\mathbf{\Psi}\rightarrow U_1U_2\mathbf{\Psi}$ where U_1 and U_2 are determined by (8) for $\mathfrak{H}_1$ and $\mathfrak{H}_2$, respectively. Then

$$i\hbar c\delta\mathbf{\Psi}/\delta\sigma=\mathfrak{H}_{12}(\bar{\varphi}, \bar{\psi})\mathbf{\Psi}, \tag{11}$$

where $\bar{\varphi}=U_1{}^{-1}\varphi_S U_1$ and $\bar{\psi}=U_2{}^{-1}\psi_S U_2$. By (7) the motion of the field variables $\bar{\varphi}, \bar{\psi}$ and their respective canonical conjugates is determined by the Hamiltonians of the two separate fields without interaction. This is the distinguishing characteristic of the interaction representation which makes it possible to calculate the commutation relations between field variables at points with time-like separation.

The value of $\mathfrak{H}_{12}$ for the various meson, photon, electron or nucleon interactions can be calculated directly. The results are in agreement with those of Tomonaga and his collaborators.[2] Note that in calculating $\mathfrak{H}$ defined by (4) the normal component of φ_μ must be eliminated by (1). Thus if $\pi=(j_\mu-\varphi_\mu)n^\mu$ say, a term such as $\varphi_\mu\varphi^\mu$ occurring in $\mathfrak{H}$ must be written $\varphi_\mu\varphi^\mu+(\varphi_\mu n^\mu)(\varphi^\rho n_\rho)-(\pi-j_\mu n^\mu)(\pi-j^\rho n_\rho)$. $\mathfrak{H}_{12}$ can then be derived after a short calculation.

The above presentation shows that the error in Section II of a recent paper by Dyson,[4] corrected by him in a note added in the proof, arose from the neglect of the dependence of the Hamiltonian on the direction of the surface.

Equation (2) was deduced by Weiss[5] for the Poisson Bracket in classical field theory. Weiss' work has been developed independently by Roberts[6] to derive the classical analog of the Schwinger-Tomonaga theory.

[1] Julian Schwinger, Phys. Rev. **74**, 1492 (1948).
[2] S. Tomonaga, Prog. Theor. Phys. **1**, 27 (1946); Koba, Tati, and Tomonaga, Prog. Theor. Phys. **2**, 101 (1947); S. Kanesawa and S. Tomonaga, Prog. Theor. Phys. **3**, 1 (1948); Y. Myamoto, Prog. Theor. Phys. **3**, 124 (1948).
[3] P. A. M. Dirac, *Quantum Mechanics* (Oxford University Press, New York, 1947), Chapter V.
[4] F. J. Dyson, Phys. Rev. **75**, 486 (1949).
[5] P. Weiss, Proc. Roy. Soc. **A169**, 102 (1938).
[6] K. V. Roberts, unpublished.

On the Gamma-Radiation From I^{131}

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February 25, 1949

THE beta-spectrum of I^{131} has recently been studied in this laboratory[1] with a view toward determining its possible complexity. In connection with this study a number of internal conversion lines were found and were assigned as K, L and M lines of four gamma-ray energies. Metzger and Deutsch[2] also have recently made a study of the I^{131} radiations. Their report did not include a gamma-ray of 163 kev energy