and minus 1 kev. Assuming the $Al(p,\gamma)$ resonance to lie at 993.3 kev we then found the $F(p,\gamma)$ resonance at 873.1 kev and the $Li^7(p,n)Be^7$ threshold at 1880 kev agreeing with Herb, Snowden, and Sala's values to well within our voltage ripple. This scale then puts the $T^3(p,n)He^3$ threshold at $1019_{-1}^{+\frac{1}{2}}$ kev, giving a Q value of -763.7 kev, where the mass factor used was 0.749_6.

Assuming zero-neutrino mass and a beta-ray end point of 18.3 kev this threshold measurement gives the $n-H$ mass difference as 782 ± 1.5 kev. The 18.3-kev end point which gives the T^3-He^3 mass difference has been measured in this laboratory by Graves and Meyer[3] and by McKibben and Shurig.[4] This value is in agreement with other recent results.[5]

The neutron-proton mass difference determined here is roughly 18 kev lower than that given by Bell and Elliott[6] and that reported by Jenkins.[7] The systematic errors, aside from the absolute energy scale, which one would expect in our experiment are all such as to raise the threshold of $T^3(p,n)He^3$ and therefore the neutron-hydrogen mass difference.

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[2] Herb, Snowden, and Sala, Phys. Rev. **75**, 246 (1949).
[3] E. R. Graves and D. Meyer, private communication, to be submitted for publication in Phys. Rev.
[4] J. L. McKibben and A. Shurig, private communication.
[5] Curran, Angus, and Cockcroft, Nature **162**, 302 (1948).
[6] R. E. Bell and L. G. Elliott, Phys. Rev. **74**, 1552 (1948).
[7] Reported by Mr. Jenkins at the February, 1949, meeting of the American Physical Society in Berkeley, California, Supplementary Programme.

On the Liberation of Ions by Electron Bombardment

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TRUMP and Van de Graaff[1] have recently proposed that the electrical breakdown between metallic electrodes in a high vacuum is due to the emission of positive ions from the anode under the electron bombardment. From direct measurements they deduce the liberation coefficient A of ions by electrons, *viz.* the average number of ions emitted per incident electron, as a function of its energy. The curve they give for a steel anode has a marked maximum of 10^{-3} ions/electron at approximately one thousand volts and a flat minimum, *viz.* an almost constant value of $A=2.10^{-4}$, between 20 and 180 kv; a steep increase follows as far as 225 kv. The maximum is attributed to ionization of the residual gas.

We have studied with greater care the phenomenon in the low voltage range with the aim of reducing as much as possible the effect of the residual gas, in order to determine a possible threshold for the phenomenon of the secondary emission of ions. The arrangement of the electrodes is shown in Fig. 1.

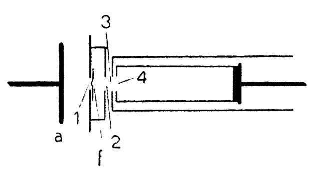

FIG. 1. Arrangement of the electrodes.

The diaphragms 1, 2, 3, 4 have, in this order, the following diameters: 2.7, 3.2, 3.7, 4 mm. The distance between the anode a and the cathode f (the emitting part of which is at the level of diaphragm 1), can be altered as desired. The vacuum, maintained by oil diffusion pumps and liquid nitrogen traps, is measured by means of a radiometer foil gauge and an ionization gauge.

The electrons hit a small area of the anode a around the axis of the system, and it is possible to observe directly the dark mark they produce if the apparatus contains a small amount of organic vapor.

The geometry of the experiment and the configuration of the field is such that if there is emission of ions from a due to incident electrons, the ions will follow the same path as the electrons but in the opposite direction. They will mainly fall in the Faraday cylinder c, which is functioning as a collector, as in the analogous arrangement of Trump and Van de Graaff, provided the initial speed of the ions is negligible as we think is true in our case.

The curves 1 and 2 of Fig. 2 have been obtained with a steel anode at distances of 6 and 12 mm from the cathode respectively, at the lowest pressure we could reach—about 10^{-6} mm Hg. In curve 1 the maximum is hardly noticeable; in the other it is well marked. It has been very difficult to obtain a curve without the maximum, working as we usually did with the apparatus under continuous pumping, for it is extremely sensitive to small variations in the conditions of vacuum and of the anode surface. The flat portion of the curve is, on the contrary, easily reproducible. The ordinates of this portion—and of the maximum too—are less than those found by Trump and Van de Graaff by a factor between 200 and 2000 in the interval examined by us (i.e. to 70 kv).

To reproduce approximately the values of these authors we took off the liquid nitrogen from the trap, so admitting the oil vapors (Apiezon oil B) into the apparatus. The pressure shown by the radiometer gauge (which in this range of pressure reads approximately absolute pressures) increases by a factor $\sim$5, while the ionization gauge shows an increase by a factor $\sim$10: the current in the Faraday cylinder increases by a factor $\sim$200 with the same electronic current, as represented by curve 3 of Fig. 2 (distance 12 mm) for which the ordinate axis on the R.H.S. of the diagram has been used.

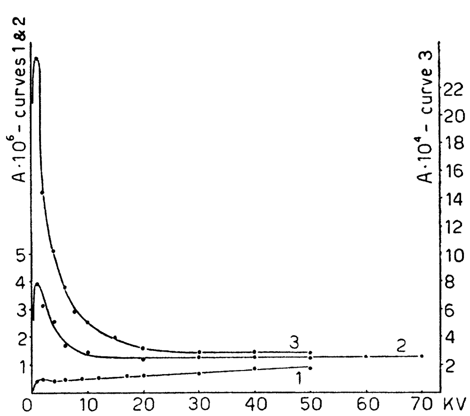

FIG. 2. Liberation coefficient A as a function of electron energy.

It seems to us that the values of these factors make it difficult to explain the current observed in these conditions as a mere effect of the ionization of the gas. We may note also in this connection that the maxima in the effective cross section of ionization by collision in the known gases generally occur at speeds of the order of a hundred and not of a thousand volts, as would be shown in the curves of Trump and Van de Graaff and in ours. We could perhaps postulate some surface effect on the anode, due to the presence of organic vapors. Only further experiments may settle this question.

The residual current observed by us in the best vacuum conditions—10^{-6} mm Hg—may be certainly explained as an effect of ionization by collision if an effective cross section of the order 1 $cm^2/cm^3 \cdot$mm Hg is attributed to the gas, in agreement with the results normally obtained.

Regarding the fundamental effect we were investigating, *viz.* the liberation of ions from the metal in the best conditions of surface purity and of vacuum, we infer from our measurements that, if it occurs, its threshold must be over 70 kv. This is also compatible with the shape of the curve of Trump and Van de Graaff. Our present arrangement did not permit measurements at higher voltages.

It is a pleasure for us to thank the Consiglio Nazionale delle Ricerche which has granted a scholarship to one of us (I.F.) and the S. A. Vetrocoke of Porto Marghera (Venice) which kindly put at our disposition the liquid nitrogen necessary for the experiments.

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The Generalized Schrödinger Equation in the Interaction Representation

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A GENERALIZED Schrödinger equation has been used by Schwinger[1] and Tomonaga and collaborators[2] in the interaction representation for a system whose state is given on a space-like surface. In both presentations a form for the Hamiltonian is assumed and then shown to satisfy necessary conditions. We will deduce the generalized Schrödinger equation by a method which, when applied to a particular case, gives the Hamiltonian directly, even when the interaction involves derivatives and the Hamiltonian consequently contains terms dependent on the direction of the surface.

Let $x^\mu=(ct, \mathbf{r})$. σ is a general space-like surface through x^μ. n_μ is the normal to the surface at $x^\mu(n_\mu n^\mu=-1)$. The Lagrangian of the field is $\mathfrak{L}(\varphi^\alpha, \varphi_\mu{}^\alpha)$ where $\varphi_\mu{}^\alpha=\partial\varphi^\alpha/\partial x^\mu$. The energy momentum tensor is $U_{\mu\nu}=(\partial\mathfrak{L}/\partial\varphi_\mu{}^\alpha)\varphi_\nu{}^\alpha-\mathfrak{L}g_{\mu\nu}$.

Define the canonical conjugate of φ^α to be

$$\pi_\alpha=(\partial\mathfrak{L}/\partial\varphi_\mu{}^\alpha)n_\mu. \tag{1}$$

The generalized commutation relations of Schwinger can then be written for a Bose field

$$[\varphi^\alpha(x), \pi_\beta(x')]_-=0 \quad x\neq x', \quad \int[\varphi^\alpha(x), \pi_\beta(x')]_- d\sigma'=i\hbar c\delta_\beta{}^\alpha, \tag{2}$$

and

$$\{\varphi^\alpha(x), \pi_\beta(x')\}_+=0 \quad x\neq x', \quad \int\{\varphi^\alpha(x), \pi_\beta(x')\}_+ d\sigma'=i\hbar c\delta_\beta{}^\alpha \tag{3}$$

for a Fermi field.

Define the Hamiltonian

$$\mathfrak{H}=U_{\mu\nu}n^\mu n^\nu. \tag{4}$$

This reduces to the usual expression when σ is flat. Choose a particular Lorenz frame such that at the point x^μ the coordinates $x^r=u^r$ lie in the tangent plane to σ and $x^0=\omega$ is in the normal direction. The equations of motion of the field, derivable from the Lagrangian are by (2), (3), and (4)

$$\begin{aligned}&\int[\pi_\alpha(x'), \mathfrak{H}(x)]d\sigma'=i\hbar c\partial\pi_\alpha/\partial\omega,\\ &\int[\varphi^\alpha(x'), \mathfrak{H}(x)]d\sigma'=i\hbar c\partial\varphi_\alpha/\partial\omega.\end{aligned} \tag{5}$$

Introducing the condition of Kanesawa and Tomonaga[2] that elementary regions are scale form flat, we have $\partial\varphi^\alpha/\partial\omega =(\delta/\delta\sigma(x))\{\int\varphi^\alpha(x')\cdot d\sigma'\}$. Thus the second of Eqs. (5) can be written in the form

$$i\hbar c\delta\varphi_H/\delta\sigma=[\varphi_H, \mathfrak{H}(\varphi_H, \pi_H)]. \tag{6}$$

If $\mathbf{\Phi}$ and $\mathbf{\Psi}$ are corresponding wave functions in the Heisenberg and Schrödinger representations, then $\mathbf{\Psi}=U(\sigma)\mathbf{\Phi}$. Denote the Heisenberg variable by φ_H and the Schrödinger variable by φ_S. Then

$$\varphi_H=U^{-1}\varphi_S U. \tag{7}$$

Also $\delta\varphi_S/\delta\sigma=0$ and by an argument similar to that used by Dirac[3]

$$i\hbar c\delta U/\delta\sigma=\mathfrak{H}(\varphi_S, \pi_S)U. \tag{8}$$

Therefore

$$i\hbar c\delta\mathbf{\Psi}/\delta\sigma=\mathfrak{H}(\varphi_S, \pi_S)\mathbf{\Psi}. \tag{9}$$

If two fields interact the Schrödinger equation in the Schrödinger representation is

$$i\hbar c\delta\mathbf{\Psi}/\delta\sigma=\{\mathfrak{H}_1(\varphi_S)+\mathfrak{H}_2(\psi_S)+\mathfrak{H}_{12}(\varphi_S, \psi_S)\}\mathbf{\Psi}. \tag{10}$$

To go over into the interaction representation we make the transformation $\mathbf{\Psi}\rightarrow U_1U_2\mathbf{\Psi}$ where U_1 and U_2 are determined by (8) for $\mathfrak{H}_1$ and $\mathfrak{H}_2$, respectively. Then

$$i\hbar c\delta\mathbf{\Psi}/\delta\sigma=\mathfrak{H}_{12}(\bar{\varphi}, \bar{\psi})\mathbf{\Psi}, \tag{11}$$

where $\bar{\varphi}=U_1{}^{-1}\varphi_S U_1$ and $\bar{\psi}=U_2{}^{-1}\psi_S U_2$. By (7) the motion of the field variables $\bar{\varphi}, \bar{\psi}$ and their respective canonical conjugates is determined by the Hamiltonians of the two separate fields without interaction. This is the distinguishing characteristic of the interaction representation which makes it possible to calculate the commutation relations between field variables at points with time-like separation.

The value of $\mathfrak{H}_{12}$ for the various meson, photon, electron or nucleon interactions can be calculated directly. The results are in agreement with those of Tomonaga and his collaborators.[2] Note that in calculating $\mathfrak{H}$ defined by (4) the normal component of φ_μ must be eliminated by (1). Thus if $\pi=(j_\mu-\varphi_\mu)n^\mu$ say, a term such as $\varphi_\mu\varphi^\mu$ occurring in $\mathfrak{H}$ must be written $\varphi_\mu\varphi^\mu+(\varphi_\mu n^\mu)(\varphi^\rho n_\rho)-(\pi-j_\mu n^\mu)(\pi-j^\rho n_\rho)$. $\mathfrak{H}_{12}$ can then be derived after a short calculation.

The above presentation shows that the error in Section II of a recent paper by Dyson,[4] corrected by him in a note added in the proof, arose from the neglect of the dependence of the Hamiltonian on the direction of the surface.

Equation (2) was deduced by Weiss[5] for the Poisson Bracket in classical field theory. Weiss' work has been developed independently by Roberts[6] to derive the classical analog of the Schwinger-Tomonaga theory.

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On the Gamma-Radiation From I^{131}

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THE beta-spectrum of I^{131} has recently been studied in this laboratory[1] with a view toward determining its possible complexity. In connection with this study a number of internal conversion lines were found and were assigned as K, L and M lines of four gamma-ray energies. Metzger and Deutsch[2] also have recently made a study of the I^{131} radiations. Their report did not include a gamma-ray of 163 kev energy