be produced by a large solenoid. We have taken a long step in this direction, and although our resolution does not yet permit observation of the true line width, we believe the results obtained so far are of some interest.

Apart from the increased resolution, our experiment provides a severe test of the theory, for the "strong" magnetic field, H_0, is here actually comparable in magnitude to the perturbing intermolecular magnetic fields.

The proton resonance in one liter of water at room temperature was observed at a frequency of 50 kilocycles per second, in a magnetic field of approximately 11.7 gauss. The magnetic field was produced by a solenoid four feet long, one foot in diameter, with a correcting coil 16 inches long wound over the central portion. The calculated magnetic field inhomogeneity over the volume of the sample was of the order of one milligauss. The signal was detected by means of a lock-in amplifier with a band-width of about one cycle per second. The observed signal voltage was roughly 15 times noise.

The observed line width, taken here as the distance between the points of maximum and minimum slope on the absorption curve, was 7 milligauss; on a frequency scale, this is comparable to the 30 cycles-per-second modulation frequency, a situation seldom occurring in previous experiments. In such cases there is reason to believe that the line width is determined by the modulation frequency rather than by field inhomogeneities. A similar effect has been observed and explained in Stark effect patterns in the microwave region.[2,3]

These experiments are being continued with the aim of increasing the resolution and of testing for possible shifts of the resonant frequency from γH_0, arising from second-order effects.

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On the Diffraction of a Plane Wave by a Semi-Infinite Conducting Sheet

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IN the usual discussion[1,2] of the Sommerfeld theory of diffraction by a semi-infinite conducting plane, approximate expressions are derived which are valid only in the regions outside two parabolas.[3] At optical frequencies this is not a serious restriction, but at radar frequencies the detecting unit is much smaller than the excluded region.

Dr. R. B. Watson and the writer are engaged in some diffraction measurements that involve the diffraction field inside the parabolas mentioned above. For this region, approximate analytic expressions have been developed which, in connection with the formulas already known, cover the entire plane. To save space, all of the symbols used are those defined by Baker and Copson.[2]

When the electric vector is parallel to the edge of the screen, the electric vector in regions S_2 and S_3, except for the parts inside the parabola $T'^2=1/\pi\epsilon^2$, is given by

$$\begin{aligned} d_z=&(1/2)\exp\{ik\rho\cos(\phi-\phi')+ikct\}\\ &+\{\csc\tfrac{1}{2}(\phi+\phi')/2(2\pi k\rho)^{\frac{1}{2}}\}\exp\{-ik\rho+ikct+i3\pi/4\}\\ &\pm\{(C^2+S^2)/2\}^{\frac{1}{2}}\exp\{ik\rho\cos(\phi-\phi')\\ &\qquad -i\tan^{-1}S/C+ikct+i\pi/4\}. \end{aligned} \quad (1)$$

The plus and minus signs apply in the illuminated and in the shadow regions, respectively. C and S denote the Fresnel integrals as defined by Jahnke and Emde.[4] The argument of these functions is $k\rho\{1+\cos(\phi-\phi')\}$. The error of Eq. (1) is not greater than $\pi\epsilon^3/2$. It is seen readily that the first two terms of (1) represent a plane and a cylindrical wave, respectively.

When the electric vector is parallel to the edge of the screen, the electric vector in regions S_1 and S_2, except for the parts inside the parabola $T^2=1/\pi\epsilon^2$, is given by

$$\begin{aligned} d_z=&\exp\{ik\rho\cos(\phi-\phi')+ikct\}\\ &-(1/2)\exp\{-ik\rho\cos(\phi+\phi')+ikct\}\\ &+\{\sec\tfrac{1}{2}(\phi-\phi')/2(2\pi k\rho)^{\frac{1}{2}}\}\exp\{-ik\rho+ikct+i3\pi/4\}\\ &\mp\{(C^2+S^2)/2\}^{\frac{1}{2}}\exp\{-ik\rho\cos(\phi+\phi')\\ &\qquad -i\tan^{-1}S/C+ikct+i\pi/4\}. \end{aligned} \quad (2)$$

The minus and plus signs apply inside and outside the region of geometric reflection, respectively. C and S again represent the Fresnel integrals, but the argument is $k\rho\{1-\cos(\phi+\phi')\}$. The error of (2) is not greater than $\pi\epsilon^3/2$. It is readily seen that the first term of (2) is the incident plane wave, the second term is a plane wave traveling in the direction of the reflected wave, while the third term is a cylindrical wave diverging from the edge.

When the incident plane wave is polarized so that the magnetic vector is parallel to the diffracting edge, the above discussion will apply to the magnetic vector, provided that the second term on the right side of (1) has its sign changed, and that the second and fourth terms on the right side of (2) have their signs changed. The corresponding electric field can be readily calculated.

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The Influence of the Length of a Hot Wire on the Measurements of Turbulence*

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LET us consider a perfectly compensated hot-wire anemometer with a wire of non-negligible length l. If I_b is a factor by which the measured intensity of longitudinal turbulence should be multiplied to give the correct intensity of turbulence then $(1/I_b^2)=(2/l^2)\int_0^l(l-s)R_y(s)ds$ where R_y is the transverse correlation coefficient. This relation was first found by H. K. Skramstad[1] by a rather complicated method and was obtained by the author[2] by a much simpler method. In the case of isotropic turbulence I_b can be expressed[2] in function of the longitudinal correlation coefficient R_x by $(1/I_b^2)=(1/l)\int_0^l R_x(s)ds$.

If the length l is large compared with the transverse scale of turbulence $L_y=\int_0^\infty R_y(s)ds$ then[3] $(1/I_b^2)=2[(L_y/l)-(L_y^{(1)}/L_y^2)(L_y/l)^2]$ where $L_y^{(1)}=\int_0^\infty sR_y(s)ds$. In homogeneous isotropic turbulence $L_y^{(1)}=0$ and $L_x=2L_y$. With wires for which $l\gg L_x$ we have then $(1/I_b^2)=L_x/l$. As

$$\lim_{(l/L_y)\to\infty}(1/I_b^2)=0,$$

it appears that the longitudinal turbulent energy measured with a hot wire of an indefinitely increasing length (compared to L_y) will approach zero even if the real energy is not negligible.

If now we consider the case when l/λ (λ being the microscale of turbulence) is small, then developing $R_y(y)$ in a Taylor's series and computing the value of the factor I_b we find[3] $(1/I_b^2)=1-l^2/6\lambda^2+Gl^4/120\lambda^4$, where $G=\lambda^4R_x^{\mathrm{IV}}(0)$. If l/λ is sufficiently small the simple relation $(1/I_b^2)=1-l^2/6\lambda^2$ can be used to correct the measured intensity of longitudinal turbulence.

When the length of the wire cannot be considered as very small or very large, then it is necessary to represent the correlation curve by a theoretical or empirical function in order to compute I_b. In the particular case when $R_x = \exp(-\pi x^2/4L_x^2)$ the factor $I_b = 1$ and no correction is necessary.

If $R_{b,y}$ represent the transverse correlation coefficient measured with two parallel hot wires of length l then it is found[1,2] that $R_{b,y} = (2I_b^2/l^2)\int_0^l (l-s)R_y(y^2+s^2)^{\frac{1}{2}}ds$. For the longitudinal correlation coefficient we find[2] in isotropic turbulence $R_{b,x} = (I_b^2/l)\int_0^l R_x(x^2+s^2)^{\frac{1}{2}}ds$.

Be $\lambda_{b,x}$ and $\lambda_{b,y}$ the microscales of turbulence found using the correlation curves $R_{b,x}$ and $R_{b,y}$ as measured with hot wires of non-negligible length. The square of the correlation factors by which these measured microscales should be multiplied to obtain λ are given[3] by the relations

$$\frac{\lambda^2}{\lambda_{b,x}^2} = \frac{18 - G(l/\lambda)^2}{18 - 3(l/\lambda)^2} \quad \frac{\lambda^2}{\lambda_{b,y}^2} = \frac{24 - G(l/\lambda)^2}{24 - 4(l/\lambda)^2}.$$

Using these relations it is also possible to express I_b and the correction factor for λ in function of $l/\lambda_{b,x}$ or $l/\lambda_{b,y}$, instead of l/λ (assuming a given value of G).

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On a New Equation of State

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A NEW equation of state describing many solids, compounds, and liquids has been obtained from very general principles of quantum mechanics and statistics, starting from a more precise analysis of the electron gas, and taking advantage of the fact that the zero-point pressures are *generally* very great compared to obtainable applied pressures.

The equation derived is:

$$P = \pi_r - \pi_a = \beta\left(\frac{e^{\beta V_0/\lambda RT} - e^{\beta V/\lambda RT}}{e^{\beta V/\lambda RT} - 1}\right)$$

in which π_r denotes the *repulsive* electron gas pressure, π_a the *attractive* compensating internal pressure, V_0 the initial volume, β, the zero-point pressure, λ, the effective number of electrons (per atom in the monatomic case, for example), and other quantities having their usual significance.

According to the underlying physical picture, this equation of state should apply to the conducting solids, certain valence crystals, and liquids, *but not* to ionic crystals or, for example, such molecular solids as sulfur and phosphorus. This has been found to be the case and agreement (within $\frac{1}{2}$ percent) is obtained with the data of P. W. Bridgman in all cases. Similar excellent agreement has been found for the change in relative resistivity under pressure—and the specific heat equation resulting from this theory is of the Einstein form *but* with a factor λR (instead of $3R$) which, for example, predicts the high temperature anomaly of Fe and Pb, with the values of β, λ obtained from (any) two points of the compressibility data.

As is to be expected, the zero-point pressures are enormous but usually somewhat smaller than for an ideal Fermi gas. To illustrate the natural sequence of β-values obtained (which should increase with the hardness of the substance), we list in round figures some illustrative values which are in kg/cm², or essentially atmospheres: Na (12,500); Pb (48,750); Ag (86,500); Cu (112,500); Fe (142,500); Si (280,000); C (diamond: 700,000).

A paper giving the detailed theory and comparison of calculated and observed compressibilities, resistivities, and specific heats, is now in preparation.

Vacuum Polarization

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IT has recently been shown by Schwinger[1] that the induced charge-current density δj_μ due to the polarization of the vacuum by an electromagnetic field A_μ is, in the first approximation, given by

$$\delta j_\mu(x) = 2e^2 \int S_{\mu\nu}(\xi) A_\nu(x') d^4x', \tag{1}$$

where $\xi_\mu = x'_\mu - x_\mu$, and

$$S_{\mu\nu} = \left\{\frac{\partial\Delta}{\partial\xi_\mu}\frac{\partial\Delta_1}{\partial\xi_\nu} + \frac{\partial\Delta}{\partial\xi_\nu}\frac{\partial\Delta_1}{\partial\xi_\mu} - \delta_{\mu\nu}\left(\frac{\partial\Delta}{\partial\xi_\lambda}\frac{\partial\Delta_1}{\partial\xi_\lambda} + m^2\Delta\Delta_1\right)\right\}(-\xi_0)/|\xi_0|. \tag{2}$$

Owing to the vacuum polarization there is an energy of interaction

$$H_{\text{v.p.}} = -\tfrac{1}{2}\delta j_\mu A_\mu. \tag{3}$$

Wentzel[2,3] has recently shown that the value of an integral involving the Δ-functions depends very much on the way in which the integration is performed, and that the photon self-energy obtained from Eq. (3) can either be infinite or finite and non-vanishing.

It seems to the present writer worth while to evaluate the integral in Eq. (1) along the lines of the customary calculations in radiation problems. It will be sufficient to consider the case of a plane wave

$$A_\nu(x) = a_\nu \exp(ik_\lambda x^\lambda). \tag{4}$$

The value of δj_μ depends then on the integral

$$R_{\mu\nu} = \int S_{\mu\nu}(\xi)\exp(ik_\lambda\xi^\lambda)d^4\xi. \tag{5}$$

Using the ordinary three-dimensional integral representation of the Δ-functions and integrating with respect to the ξ_μ, we obtain

$$R_{\mu\nu} = (2\pi)^{-3}\int\frac{d^3q}{p_0p_0'}\frac{1}{(p_0+p_0')^2 - k_0^2}\{p_\mu p_\nu'[(p_0+p_0')(1+a_{\mu\nu}) - k_0(1-a_{\mu\nu})] - \delta_{\mu\nu}(p_0+p_0')(p_\lambda p_\lambda' - m^2)\} \tag{6}$$

where

$p = -k/2 - q$, $p' = -k/2 + q$, $p_0 = (m^2+p^2)^{\frac{1}{2}}$, $p_0' = (m^2+p'^2)^{\frac{1}{2}}$, $a_{\mu\nu} = 1$ if μ, ν are both space indices or both 4, and $a_{\mu\nu} = -1$ otherwise. Following a method of Pauli and Rose[4] we obtain

$$\left.\begin{aligned} R_{i\nu} &= (m^2/8\pi^2)\{(k_ik_\nu - \delta_{i\nu}k_\lambda k_\lambda)J_1 + 4\delta_{i\nu}I\} \quad (i=1,2,3) \\ R_{4\nu} &= (m^2/8\pi^2)(k_4k_\nu - \delta_{4\nu}k_\lambda k_\lambda)J_1 \end{aligned}\right\} \tag{7}$$

where J_1 was given in reference 4 and

$$I = \int_0^1 dz \frac{z^2 - z^4/3}{(1-z^2)^2}. \tag{8}$$

The presence of the terms involving the integral I spoil the Lorentz covariance of the quantities $R_{\mu\nu}$, a curious situation which seems to be connected with the singular nature of the Δ-functions. It spoils also the gauge invariance and gives rise to an infinite self-energy for a photon.

The above result for $R_{\mu\nu}$ involves two diverging integrals I and J_1. We can separate $R_{\mu\nu}$ into a finite part

$$R_{\mu\nu}^0 = (m^2/8\pi^2)(k_\mu k_\nu - \delta_{\mu\nu}k_\lambda k_\lambda)(J_1 - J_2), \tag{9}$$

where J_2 is the value of J_1 for $k_\lambda = 0$, and a divergent part $R'_{\mu\nu}$ obtained from Eq. (7) by substituting J_2 for J_1. The interaction (3) corresponding to $R'_{\mu\nu}$ is

$$H'_{\text{v.p.}} = (m^2e^2/2\pi^2)(\pi J_2 j_\mu A_\mu - IA^2). \tag{10}$$

A finite, Lorentz- and gauge-covariant result will be obtained if we subtract the expression (10) from the Hamiltonian.