Precision Measurement of the Wave-Length and Spectral Profile of the Annihilation Radiation from Cu^{64} with the Two-Meter Focusing Curved Crystal Spectrometer*

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The two-meter focusing curved crystal spectrometer has been used to make a direct precision wave-length measurement of the annihilation radiation from Cu^{64} by first-order reflection from both sides of the (310) planes of the curved quartz lamina. The instrument constant has been calibrated (by measurements of the K spectrum from a tungsten target x-ray tube) so as to give gamma-ray wave-lengths correct to ±0.01 x.u. Using a source of 2.5 curies of neutron-activated Cu^{64}, a sharp annihilation radiation line substantially symmetrical to within the precision of our observations was obtained. The measured wave-length at the peak of this line after conversion (by the multiplying factor $\lambda_s/\lambda_g=1.00203$) from the conventional Siegbahn scale of x.u. to angstroms was 0.024271±0.000010A which is in satisfactory agreement with the value 0.0242650±0.0000025A obtained by DuMond and Cohen for the Compton wave-length in their 1947 least squares evaluation of the atomic constants F, N_0, m_0, and h from a wide variety of sources of information. The present measurement therefore adds another important confirming datum to the above-mentioned 1947 set of least squares adjusted values. It also adds further independent evidence favorable to the higher iodine value of the Faraday and unfavorable to the silver value.

The profile of the observed annihilation "line" is slightly broader than a careful analysis of all instrumental causes seems capable of accounting for. The residual "natural" profile, after abstraction of the instrumental broadening, is roughly estimated to have a half-width of 0.096 x.u. If this were ascribed to a Doppler effect of the motions of recombining pairs of moving negative and essentially stationary positive electrons the velocities would correspond to electrons of only 16 ev, an order of energy which can hardly be associated with anything but the conduction electrons in copper. The 0.66-Mev positrons emitted by the Cu^{64} nuclei are so rapidly retarded by large parameter inelastic "collisions" with electrons as Heitler has shown, that less than two percent of them undergo annihilation before reaching thermal velocities. They then become virtually stationary targets for annihilation chiefly by conduction electrons and possibly M electrons. The repulsion of the nuclear fields probably explains why annihilation by K and L electrons is improbable.

INTRODUCTION

The purpose of the present investigation was twofold. Firstly, it was hoped that a precision determination of the Compton wave-length $\lambda_c = h/mc$ could be made experimentally by a direct crystal diffraction measurement of the wave-length of the 0.5-Mev annihilation radiation coming from the recombination of positrons (emitted by Cu^{64}) with negative electrons in a solid book of neutron-activated copper. Secondly, a study of the spectral distribution of the annihilation radiation from the copper was planned with the hope of obtaining more detailed information about the annihilation process.

The annihilation radiation "line" is frequently used as a reference in nuclear studies with magnetic β-ray spectrometers. Because such instruments have rather low resolving power and are difficult to calibrate with high absolute accuracy, considerable uncertainty exists as to (a) the true spectral distribution of this "line" and (b) the exact value of the wave-length or energy to be associated with it or with some of its more well defined features. There seems little doubt, in the ideal case of a pair recombination in free space where the two members have negligible initial kinetic energy, that a process should occur in which the pair disappears and two identical photons are emitted in opposite directions, each having exactly the quantum energy m_0c^2 and hence the wave-length $\lambda_c = h/(m_0c)$ since, if electrons and positrons have identical masses, any other result would violate the principles of equivalence of mass and energy and of conservation of energy and momentum.

Such an ideal case, however, never occurs experimentally. The possible departures from the ideal case may be described as follows: (i) In a reference system fixed to the center of mass, recombination of the pair may occur with a small residue of kinetic energy associated with the particles. For such cases the quantum energy of the radiation will be a little greater than m_0c^2 in the center of mass system. Since the positions of the two members of a recombining pair must coincide, their potential energies are of opposite sign and equal magnitude. Therefore, the potential energies of the particles, do not affect the total energy of the two annihilation radiation quanta.

(ii) According to calculations by Heitler[1] the probability of annihilation of a positron while in rapid motion in the copper block is very small indeed. The initial kinetic energy of the positrons from Cu^{64} is 0.66 Mev and according to Heitler's calculations less than two percent of all positrons will be annihilated before they have lost their initial kinetic energy (by imparting it chiefly in very small quanta through large parameter "inelastic collisions" with the electrons of the copper).

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[1] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, London, 1944), p. 230 ff.

The remainder, 98 percent, come safely to the ends of their ranges where they are stopped entirely save for the random motion of thermal agitation which they share with the copper lattice structure. They are then eventually annihilated, their mean lifetimes[2] in copper being about 0.55×10^{-10} second. If this picture is correct, we see that the positrons, most of which eventually have very low velocities and hence become virtually stationary targets, might perhaps be annihilated by recombination with electrons belonging to any of the different energy levels of the copper (K, L, M, or conduction electrons). The center of mass of such a recombining pair may thus have a considerable velocity in random directions relative to the laboratory coordinates. One would thus be led to expect that the structure of the annihilation line would exhibit, in the laboratory coordinates, a series of marked Doppler broadenings of quite different orders of magnitude for each class of electrons in copper. Thus one might expect to find a sharp core in the line with a breadth appropriate to the speeds of the conduction electrons superposed on several very much broader and more diffuse distributions ascribable to recombination with M, L, and K electrons. These broader distributions might exhibit some shift of their centers toward longer wave-lengths away from the value $h/(m_0c)$ because of the various kinetic energies of the negative members of the recombining pairs. A rather strong argument, however, against annihilation of positrons by electrons such as those of the K and L shells is that positrons of thermal energies would have very little chance of approaching sufficiently close to the nuclei because of the repulsion of the nuclear charge. The experimental investigation of the spectral structure of the annihilation radiation is, therefore, of considerable interest.

There is a good reason to expect that the shift in the wave-length of the "line" from recombination at finite relative velocity in the center-of-mass system will be a second-order effect as compared to the Doppler broadening just referred to. In fact, if in the laboratory coordinates we assume the positrons to be stationary and denote the speed of a class of electrons with which they recombine by $v=\beta c$ and if the half-breadth of the line from Doppler broadening is $\Delta\lambda$, the shift $\delta\lambda$ of the center of the line will be given by

$$-\delta\lambda/\lambda\cong(\Delta\lambda/\lambda)^2=\tfrac{1}{4}\beta^2 \qquad (1)$$

to a non-relativistic approximation. Thus, the square of the relative breadth of the observed "line" may be taken as a rough measure of its relative reliability for the determination of $\lambda_c=h/(m_0c)$.

To summarize then, we cannot in the experimental case expect the annihilation "line" to be (a) perfectly simple, sharp, and narrow and (b) exactly equal in wave-length to the ideal value, $h/(m_0c)$. Though it seems likely that the line should have an intense core in which these departures from the ideal case are small, the annihilation mechanism is so complex in the real case that an experimental investigation is desirable.

[2] See reference 1, pp. 204–208.

THE FOCUSING CURVED QUARTZ CRYSTAL SPECTROMETER

A description of the 2-meter focusing curved quartz crystal spectrometer in its earlier stages of development has been given in a previous article[3] and we shall give here only those features essential to an understanding of its somewhat novel[4] application to the present experiment. The diffracting crystal consists of an optically flat lamina of quartz 1 mm thick and 70 mm high by 50 mm wide cut from a very perfect crystal specimen in such a way that the (310) planes of the crystal are normal to the flat face of the lamina. This quartz plate is clamped between two hardened stainless steel blocks so that it is in optical contact with the accurately profiled convex cylindrical surface of one of them, a rubber gasket being placed between the quartz plate and the concave cylindrical steel surface of the other block to facilitate this. The blocks are provided with windows (traversed by ribs) through which the radiation can pass. By special shop methods previously described[5] it has

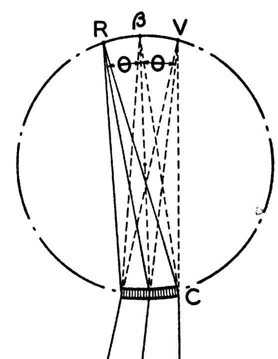

FIG. 1. Geometry of focusing curved crystal spectrometer of the transmission type. R and V are the real and virtual foci, θ is the Bragg angle, and β is the point at which the reflecting atomic planes of the crystal lamina would converge if produced. The crystal, at C, with initially optically flat faces is curved in a precision profiled clamp to a radius equal to the diameter of the focal circle shown here.

[3] J. W. M. DuMond, "A high resolving power, curved crystal focusing spectrometer for short wave-length x-ray and gamma-rays," Rev. Sci. Inst. **18**, 626 (1947).

[4] In the study of the 0.41-Mev line from Au^{198} [DuMond, Lind, and Watson, Phys. Rev. **73**, 1392 (1948)] this instrument was used in the same way but no description of the method was given in that report.

[5] DuMond, Lind, and Cohen, "A precision method of generating circular cylindrical surfaces of large radius of curvature for use in the curved crystal spectrometer," Rev. Sci. Inst. **18**, 617 (1947).

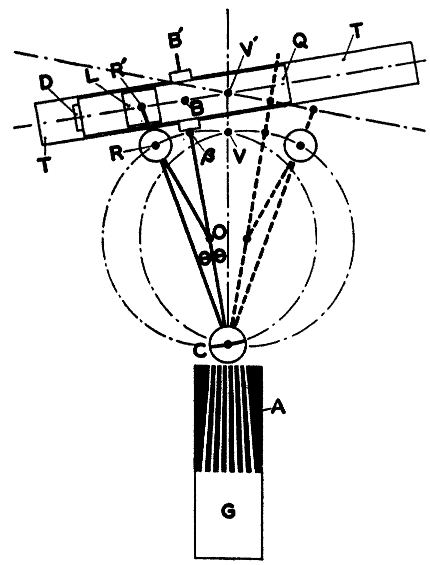

FIG. 2. Schematic plan view of focusing curved crystal gamma-ray spectrometer with the source at the focus and arranged so as to maintain the selectively reflected beam stationary as the wave-length scale is explored. The Bragg angle, θ, is here greatly exaggerated. A is the lead collimator which stops the transmitted beam and transmits only the reflected beam. G is the multicellular Geiger counter. C is the curved crystal, mounted on a support which is rigidly attached to the beam CB'. R is the shielded source at the focus of the curved crystal. L and Q are screw driven carriages and T is a track swivelling at V'. These last impart the appropriate motions to arms CR' and CB' so that wave-lengths can be read directly on a drum at D. The full and dotted lines show, respectively, the positions for measuring reflections to left and to right of the (310) planes of the curved crystal.

been found possible to profile the convex cylindrical steel surface to a radius of 2 meters so accurately that the crystal will focus x-rays of a specified wave-length to one and the same point to within about 0.06 mm anywhere on the focal circle from all parts of the crystal window. This window is 45 mm wide and 33 mm high and has a net free opening of about 13 cm^2 (after deduction of the ribs). The simple geometry of the transmission type focusing curved crystal spectrometer with its focal circle and its foci, one real and one virtual, is shown in Fig. 1 and explained in the caption.

The present instrument was purposely designed with the idea in view of establishing a precision linkage between the scales of x-ray and nuclear gamma-ray wave-lengths. To this end it was designed so that the wave-length scale is explored by a carriage driven by a precision screw, the travel being *linearly related to the wave-length* to a high degree of accuracy. In the present instrument the scale covers a range of wave-lengths from 500 x.u. to about 8 x.u., and one turn of the screw corresponds closely to 1 x.u. The instrument is so designed that the wave-length scale can be explored right through the point, $\lambda=0$, so as to permit measurements of reflections in the first order from *both sides* of the (310) planes.

It is clear from Fig. 1 that the transmission type curved crystal spectrometer, which is the type appropriate to the study of short wave-lengths, *has only one real focus*. Now it can be shown[6] that there is a great gain in intensity at a specified wave-length setting if the source is placed at this real focus. Nevertheless, in all other applications of this type of instrument up to the present, a more-or-less extended source has always been placed *on the convex side* of the crystal (away from the focus). The spectrum has then been studied either by placing a photographic film or plate on the focal circle or by moving a slit along the focal circle behind which some intensity measuring device such as an ion chamber or a counter was located. In the present application, the instrument is used in a radically different way with the source in concentrated form placed at the focus of the crystal. The wave-length scale is explored by moving this source along the focal circle while simultaneously measuring the intensity of the beam reflected by the crystal. Figure 2 shows schematically the arrangement for doing this while Fig. 3 is a perspective sketch of the instrument to clarify somewhat the actual construction. Corresponding features have the same reference letters in both figures. The divergent γ-ray beam coming from the source at R, after reflection at the crystal planes (at C), enters the lead collimator A and passes through it to the multicellular counter G where its intensity is measured. The purpose of the collimator is solely *to stop the γ-ray beam directly transmitted through the crystal* and to permit only the beam reflected by the crystal planes to reach the counter. The grating constant of the (310) planes of quartz is 1177.6 x.u. and the Bragg angle for the 0.510-Mev annihilation radiation (of wave-length about 24 x.u.) is therefore only about 34 minutes of arc. This means that the transmitted and reflected beams differ in direction by only slightly more than one degree. The present collimator is easily capable of discriminating between these and, in fact, its geometry is such that the direct beam only starts leaking through to the counter (by direct transmission) at settings as low as 7 or 8 x.u. The direct beam, which is far more intense than the reflected beam, does, however, give an increasingly intense background by scattering from the partititions of the collimator at the smaller Bragg angles. It should be emphasized that the collimator has nothing whatever to do with the spectral resolving power of the spectrometer. The

[6] In a paper now in preparation on the reflecting power of the (310) planes of quartz over a wide range of wave-lengths, it is shown that a gain of 730-fold is obtained in the present instrument when the source is placed at the focus as compared to the conventional arrangement.

latter is determined solely by the excellence of focus of the curved crystal and the narrowness of the source on the focal circle.

Since the collimator and the heavily shielded counter are by far the least mobile parts of the apparatus, the instrument is designed so that it is these which remain stationary. In the exploration of the spectrum both the crystal at *C* and the source at *R* on the focal circle move in such a way as to maintain the direction of the outgoing reflected beam constant and rigorously aligned with the collimator channels. The focal circle (dotted in Fig. 2) also rotates, as it should, in unison with the rotation of the crystal. The focal circle is swept out by a radius bar *OR* pivoted at *O* and linked to the source *R*. This latter is contained in a heavy spherical lead shield which is free to travel a short distance on ball bearing ways longitudinally along the beam CR'. The two major instrument beams, CR' and CB', each pivot independently about *C*. The lower beam CB' which carries the defining pivot *O* of the focal circle, has at *C* a vertical axle rigidly coupled to it which passes up through the pivot of the upper beam CR' and carries on its upper end the table on which the focusing crystal is mounted. Thus the focal circle, the focusing crystal, and the beam CB' form one rigid (though mobile) system of reference and the beam CR' which carries the source at *R* (constrained always to lie on the focal circle by the radius bar *OR*) forms an independent system. As the spectrum is explored, the upper beam CR' moves at twice the angular velocity of the lower beam CB' so as to maintain the direction of the reflected radiation invariable. This is accomplished by the system of a pivoted track *T* and two carriages *Q* and *L* situated one on top of the other. The long carriage *Q* contains two precision right-hand screws, one directly above the other, geared to turn at equal rates in opposite senses. The lower screw drives the carriage *Q* along the track *T* by means of a nut situated above the pivot V' of track *T*. The upper screw drives the smaller carriage *L* relative to *Q* by means of a nut situated below the pivot R'. The radial distances CR' and CV' are at all times equal and constant and the center of the lower beam CB' therefore forms the variable altitude of an isosceles triangle $CR'V'$. A cylindrical steel shaft terminating the beam CB' passes through two ball bearing slides on either side of the carriage *Q* so as to permit the change in the length *CB*. Thus, the point *B* (on the center line of beam CB') remains always exactly midway between R' and V'. The center point of travel of the carriages where the points, R', *B*, and V' are in the same vertical line is the point of zero wave-length setting. This geometry is clearly such that the travel of the upper screw, which determines the distance *BL*, is proportional to the sine of the Bragg angle θ so that rotation of this screw as read on the drum *D* gives a strictly linear scale of wave-

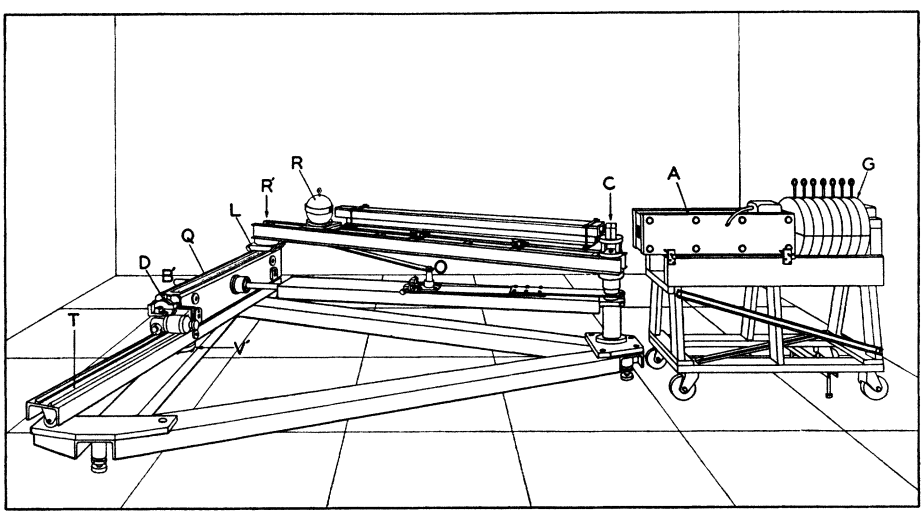

FIG. 3. Perspective line drawing of curved crystal gamma-ray spectrometer. A rough idea of the scale can be formed by recalling that the source (in its shielded holder) at *R* is two meters distant from the curved crystal at *C*. *A* is the lead collimator in its steel supporting sheath. *G* is the heavy lead shielding surrounding the multicellular Geiger counter. *R* is the shielded source mounted on the sliding carriage which is maintained on the focal circle by the radius bar pivoted at 0 on the lower beam. The crystal table at *C* is rigidly connected to the lower beam by an axle while the upper beam turns freely and independently of these on a cone bearing on this axle. Its weight is supported by ball thrust bearings. Where corresponding parts or points are designated the same letters are used in Figs. 2 and 3. The instrument is here shown at one extreme of its travel corresponding to the position for a setting of about 500 x.u. The wave-length drum *D*, the revolution counter, and the motor driving the two precision screws can be seen at the end of carriage *Q*.

lengths.[7] It is also clear how precision wave-length settings on the same screw can be made for reflections *from either side* of the atomic planes in the quartz crystal. The positions in which two such settings might be made are shown schematically by the dotted and full lines, respectively, in Fig. 2. In this figure the Bragg angle is, of course, greatly exaggerated.

The all-important multicellular Geiger counter in a somewhat earlier form has been described[8] previously. For the present application, where one of the chief difficulties was the low available intensity of the annihilation radiation, the number of cells of the counter was increased to nine (ten lead septa with nine intervening four-pronged tungsten wire spiders as anodes) and with this arrangement an efficiency of 25 percent was obtained for the radiation from a standard radium source. By this we mean that 25 percent of all photons entering the front window of the counter actually produce recorded counts. In spite of the large number of cells it was found possible to obtain as over-all characteristics of the device a counter threshold of 1050 volts and a flat plateau 100 volts wide. The counter occupies a brass cylinder 3.25 inches in inside diameter by 7 inches long. One of the advantages of this type of gamma-ray detector for the present application is that it is able to integrate a large fraction of the total intensity associated with a gamma-ray beam *of rather large cross sections and considerable penetrating power*, a property which would be difficult to obtain with scintillation counters or crystal counters in their present state of development. The counter is protected (a) from local stray radiation with 3-inch thick lead shields and (b) from penetrating cosmic-ray particles by means of six conventional cylindrical G-M counters which completely cover the top semicylinder of the multicellular counter and are connected in anticoincidence with it. These anticoincidence counters reduce the natural background to about half the value it otherwise would have.

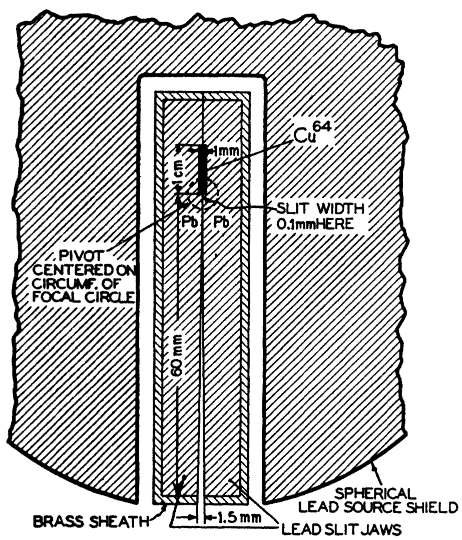

FIG. 4. To illustrate the geometry of the Cu^{64} source. This is a horizontal section through the spherical lead source holder R of Fig. 3 which consists of three parts, an upper and a lower lead hemisphere and a lead "waistline" 1.5 inches thick provided with a slot in which the source holder proper is located. This source holder consists of two carefully machined lead half jaws contained in brass stiffening sheaths. The strip of activated copper 1.0 mm by 10 mm by 30 mm is held in a cavity provided in these jaws and the annihilation radiation issues through a slit opening 0.1 mm wide between the tapering surfaces of the two lead jaws. The narrowest point of these lead jaws is centered exactly on the focal circle of the instrument by means of a pivot and socket underneath the source holder. The source holder stands on a small rotating table which permits exact centering of the jaws on the focal circle.

CALIBRATION OF THE SPECTROMETER WAVE-LENGTH SCALE

In a paper[9] now in press a more detailed description is given of the methods used for the precision calibration of the wave-length scale of the 2-meter instrument. The wave-lengths of the $K\alpha_1$ line of tungsten and of the $K\alpha_1$ line of molybdenum were first measured with great care using a precision two-crystal spectrometer to obtain their exact ratio with all possible precision. The $K\alpha_1$ line of molybdenum has received such careful attention in many precision x-ray measurements that it may well be taken as the standard for the conventional Siegbahn scale of x-ray wave-lengths and, since the ratio of this scale to the angstrom unit is now well established by the many well-known and beautiful precision measurements of x-rays with gratings by J. A. Bearden and others, this entire procedure essentially establishes the wave-length of tungsten $K\alpha_1$ in angstroms or centimeters to a precision of about $\pm 0.01 \times 10^{-11}$ cm. By measurements of the tungsten K spectrum with the 2-meter focusing spectrometer it is thus possible to establish the multiplying factor which must be applied to the nominal wave-lengths in x units given by the drum and precision screw of the instrument to convert them to true absolute units. This multiplying factor turned out to be 1.00023. The wave-lengths of all of the K spectrum lines, seven in all, and of the K absorption edge were measured in this way with the 2-meter instrument. These results when compared with the very careful work[10] of E. Ingelstam by a photographic method agree very satisfactorily,

[7] It can be shown that no correction for the refractive index of x-rays in quartz is needed in this transmission case.

[8] See Jesse W. M. DuMond, Rev. Sci. Inst. **18**, 637 (1947). A considerable number of improvements have been made since this appeared and these are described in a forthcoming paper.

[9] Watson, West, Lind, and DuMond, "A precision study of the tungsten K-spectrum using the 2-meter focusing curved crystal spectrometer," Phys. Rev. **75**, 505 (1949).

[10] E. Ingelstam, Arkiv. f. Mat. Astro. O. Fys. **27B**, No. 4 (1939).

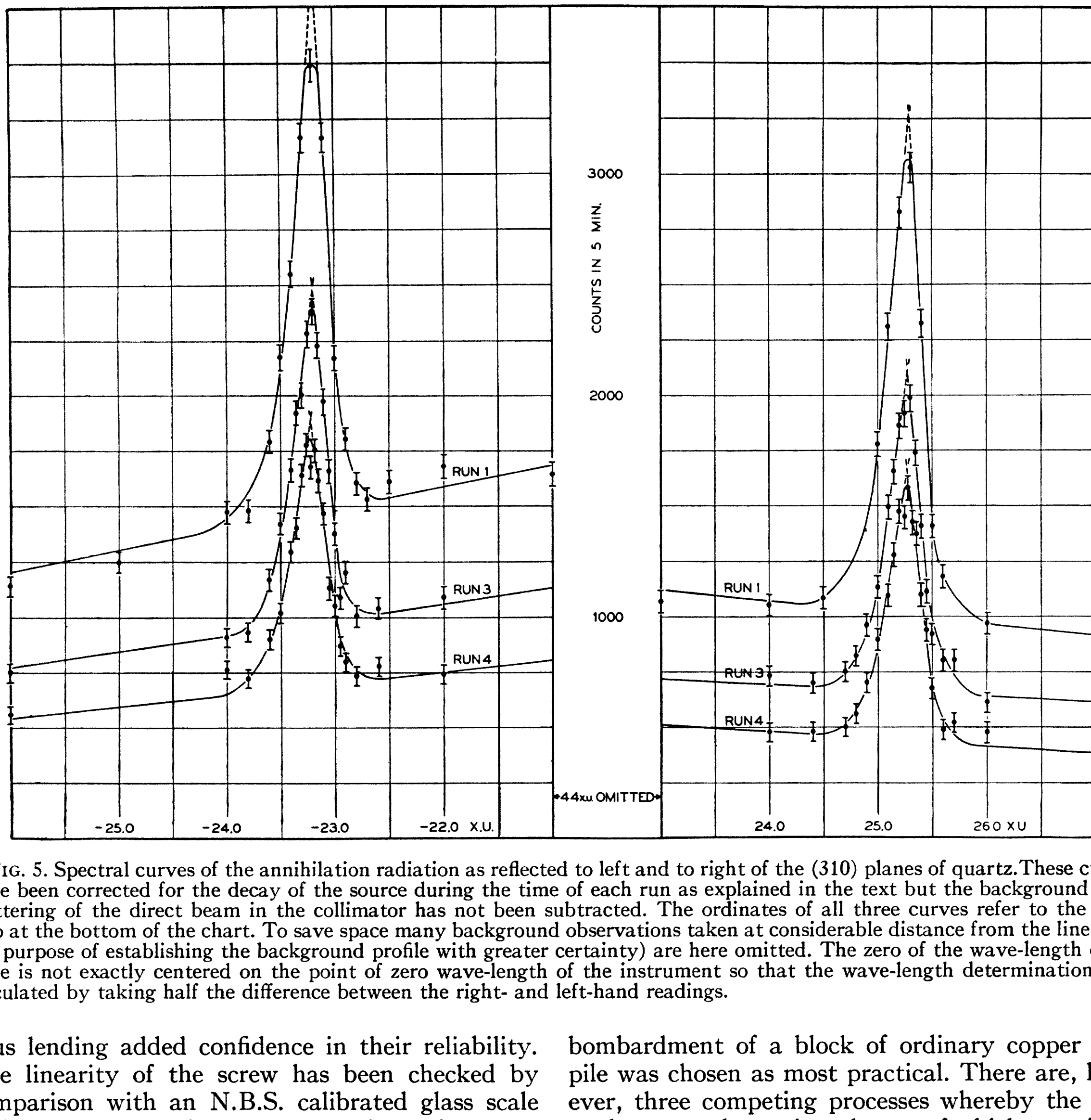

FIG. 5. Spectral curves of the annihilation radiation as reflected to left and to right of the (310) planes of quartz. These curves have been corrected for the decay of the source during the time of each run as explained in the text but the background from scattering of the direct beam in the collimator has not been subtracted. The ordinates of all three curves refer to the same zero at the bottom of the chart. To save space many background observations taken at considerable distance from the lines (for the purpose of establishing the background profile with greater certainty) are here omitted. The zero of the wave-length drum scale is not exactly centered on the point of zero wave-length of the instrument so that the wave-length determinations are calculated by taking half the difference between the right- and left-hand readings.

thus lending added confidence in their reliability. The linearity of the screw has been checked by comparison with an N.B.S. calibrated glass scale and a 100-power microscope over the entire range of travel between the first-order tungsten K spectra as reflected from both sides of the crystal planes, thus covering the entire range used in the study of nuclear gamma-radiation.

THE SOURCE OF ANNIHILATION RADIATION

The problem of obtaining a source of annihilation radiation of sufficient intensity for this spectral study was difficult for several reasons. Our studies[6] have shown that the reflecting power of the (310) planes of quartz falls off much more rapidly with decreasing wave-length than was at first anticipated. Our previous measurements indicated that roughly only about 0.2 percent of the annihilation radiation of wave-length 24 x.u. incident on the (310) planes of the quartz lamina would be reflected. As a source material 12.8-hr. Cu^{64} activated by neutron bombardment of a block of ordinary copper in a pile was chosen as most practical. There are, however, three competing processes whereby the Cu^{64} nucleus may decay, in only one of which a positron is emitted. Roughly, 40 percent of all disintegrations go to Zn^{64} with emission of β^-. Of the remainder about two-thirds go to Ni^{64} by K capture and one-third by β^+ emission. Thus, only about 20 percent of all disintegrations result in positrons. The 0.66-Mev positrons have a range of roughly 0.3 mm in copper and, as we have indicated in our introduction, the great bulk of them are probably not annihilated until after they have gone to the ends of their ranges. Because of the serious problem of getting enough intensity, it was therefore decided to work with a rather massive source of copper of dimensions 1 mm thick (in the direction of dispersion of the instrument) by 30 mm high by 10 mm deep and to delimit the effective size of the source on the focal circle by placing it behind a narrow lead slit with very thick jaws as shown in Fig. 4. At

TABLE I. Wave-length determination of annihilation radiation from Cu^{64}. Grand average value of observed annihilation radiation wave-length before correction 24.227 ± 0.0025; annihilation radiation wave-length after correction for instrument constant and for $\lambda_g/\lambda_s = 1.002030$, $\lambda_c = 0.024271 \pm 0.0000025$A.

Line centers by "median point" method					Line centers by "tangent intersection" method			
Run no.	Center left	Center right	Diff.	Average for run	Inter-sec. left	Inter-sec. right	Tangent diff.	Average for run
1	76.775	25.228	48.453	48.453	76.815	25.269	48.454	48.454
2	76.778	25.214	48.436	48.436	76.806	25.254	48.448	48.456
	76.775	25.212	48.437	48.436	76.795	25.260	48.465	48.456
3	76.770	25.225	48.455	48.455	76.797	25.256	48.459	48.459
4	76.762	25.223	48.461	48.464	76.775	25.254	48.479	48.474
	76.755	25.221	48.466	48.464	76.772	25.240	48.468	48.474
5	77.200	25.652	48.452	48.452	77.203	25.652	48.449	48.454
	77.199	25.646	48.447	48.452	77.199	25.662	48.463	48.454
	77.199	25.656	48.457	48.452	77.201	25.650	48.449	48.454
6	77.199	25.622	48.423	48.423*	77.197	25.620	48.423	48.423*
7	77.200	25.664	48.464	48.465	77.201	25.656	48.455	48.458
	77.199	25.665	48.466	48.465	77.201	25.661	48.460	48.458
Average by "median point" method 48.452 ± 0.005					Average by "tangent intersection" method 48.456 ± 0.005			

* This starred observation received only half weight.

the narrowest point where these jaws nearly met (on the focal circle) their separation was 0.1 mm and the long slim wedge-shaped opening between them defined the angular width of the gamma-ray beam which fell on the curved crystal two meters away. By this arrangement, the central portion of the copper slab directly behind the slit is the part in which annihilation is occurring most intensely, since it receives positrons from activated copper nuclei on both sides out to a distance equal to the maximum positron range. It is no more expensive to activate a thick block of copper than a thin one since the price is based on time in the pile rather than final activity or source strength. We calculated that approximately a threefold gain in radiant intensity was obtained with the present geometry by this device of a thick source behind a thin slit as compared with a thin source of the same thickness as the present slit. The disadvantage of this method is that it introduces some slight uncertainty as regards source width because of penetration of the radiation through the jaws of the slit. This is a disadvantage, however, only in analyzing the *breadth* of the annihilation line. It introduces no appreciable uncertainty in the wave-length measurement.

The source at the beginning of the experiment had a strength of 2.5 curies equivalent. We are much indebted to the Atomic Energy Commission for special permission to have this source irradiated at very high neutron flux level without which the present experiment would have been difficult to perform.

EXPERIMENTAL OBSERVATIONS AND RESULTS

Seven independent spectral curves were made, each giving the annihilation line as reflected both to left and to right of the (310) planes of the quartz crystal. At the end of these seven runs the source had decayed below a level where further reliable work could profitably be continued. Figure 5 shows a typical sample of these spectral curves. The abscissa scale is in nominal x.u. as read on the drum and vernier of the precision wave-length screw and a large gap in the wave-length scale is omitted between the lines to permit plotting them in reasonable compass. They would be about 8 times as far apart as here shown if the scale were plotted in its entirety. The true zero of the wave-length scale is slightly displaced from the nominal zero which accounts for the slight inequality in the numerical values at which the annihilation lines occur to left and to right. In each case the wave-length is computed by taking half the difference between the right- and left-hand positions.

The ordinate scale gives the actual number of counts observed for each plotted point multiplied by a correction factor for the decay of the source during the course of the run. This factor starts with the value unity at the beginning of each run (left-hand side of each of the curves) and increases from this value exponentially with the time at which each observation was made. In order to make this decay correction properly it must, of course, only be applied to that portion of each counter reading which is due to the presence of the Cu^{64} source (after subtraction of the constant natural background from cosmic rays and local radioactivity) and this was the procedure followed. The vertical stroke at each observed point in Fig. 5 is indicative of the statistical uncertainty of each observation and this was computed as the square root of the *actual total number of counts observed for the point before subtraction of the above-mentioned natural background or correction for source decay*. This uncertainty was then multiplied by the decay factor so as to plot it to appropriate scale.

Table I gives the results of all seven independent spectral curves. Two methods of locating the positions of the centers of the annihilation lines were used in Table I labeled "median point" and "tangent intersection," respectively. In the former method the center was taken at the midpoint of a horizontal line drawn across the width of the annihilation profile. In the latter, tangents to the profile at its steepest points on either side were projected upward until they intersected. This point of intersection was taken as defining the midpoint. In applying the median point method the height of the horizontal line to be bisected was always taken at half the height of the intersection point of the two steepest tangents above referred to. The lines, as can be seen in Fig. 5, are superposed upon a sloping background. As explained under our general description of the instrument, we believe this background comes chiefly from the primary undeviated beam transmitted without reflection in the crystal and then scattered on the partitions of the collimator into the counter. The rapid increase in this scattered intensity with diminishing scattering angle accounts for the rising slope of this background in the direction of diminishing wave-length. We have tried locating the centers of our annihilation lines (by the two methods already described) *both with and without* subtraction of this sloping background. In Table I the first row of figures in each numbered run gives the results *before* subtracting the sloping background and, when other rows of figures for a run appear below the first, these represent repeated independent determinations *after* subtracting the sloping collimator background. Ideally the method in which the background is subtracted is the correct one but, in practice, since the results by the two methods hardly differ significantly at all and since the more elaborate procedure perhaps introduces as much error through the additional manipulations as it corrects, we have attached equal weights to the two types of result and have averaged both. Only one-half the weight of the other runs has been given to run 6 because in this instance a considerable interruption occured between the observation of the left-hand and right-hand line profiles, thus

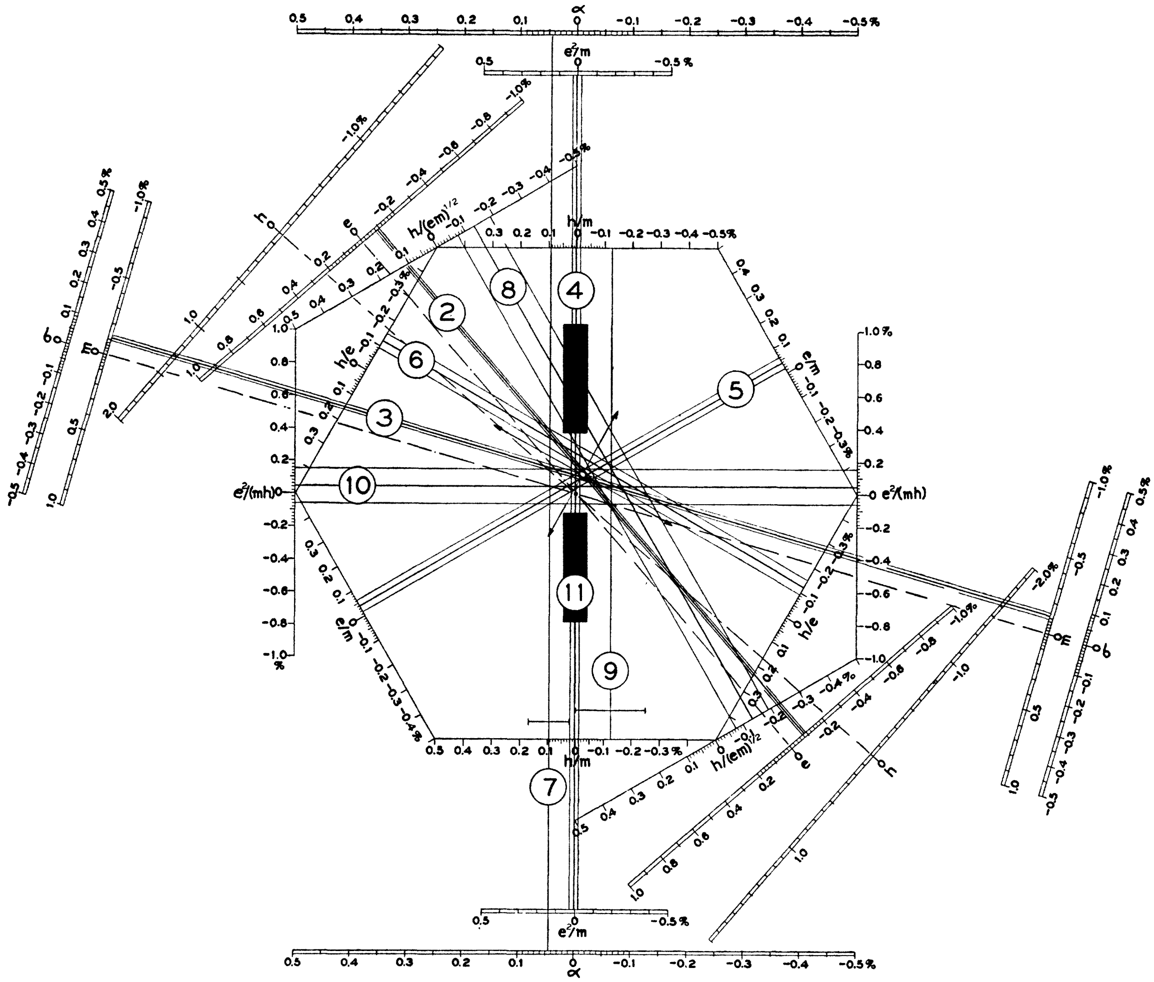

FIG. 6. Isometric consistency chart showing the consistency of the present annihilation radiation wave-length determination of the function h/m_0 compared with nine other independent types of determination of functions of the natural constants e, m, and h. The functions compared are numbered on the chart as follows: 2. Faraday constant (average of Ag and I voltameters). 3. Atomic mass of electron by spectroscopy (H_α and D_α; R_H and R_{He}). 4. e^2/m from Bearden's measurement of the x-ray refractive index of diamond. 5. e/m by direct deflection methods. 6. h/e Panofsky, Green, and DuMond, continuous x-ray limit. 7. Christy and Keller's evaluation of α. 8. Von Friesen's $h/(em)^{\frac{1}{2}}$. 9. Gnan's h/m. 10. Robinson and co-workers $e^2/(mh)$. 11. Present annihilation radiation measurement of Compton wave-length. The center of the small white ellipse of probable error gives our recommended least squares adjusted best value. The axes of this ellipse are indicated with arrows. This cut, save for the addition of measurement 11, is in all respects identical to Fig. 9, page 103, of the article "Our knowledge of the atomic constants F, N, m, and h in 1947 and of other constants derivable therefrom," (Rev. Mod. Phys. **20**, 82, 1948) and the reader is referred there for further explanatory details.

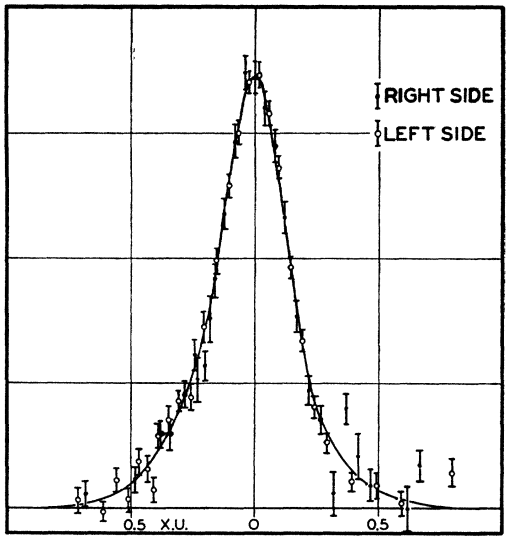

FIG. 7. Composite mean profile of the annihilation line obtained by the superposition of all available curves. Where points fall on the same ordinate an average has been taken.

introducing a considerably greater decay correction than in the other cases.

The final value after making correction for the instrumental screw constant and for the conversion from Siegbahn wave-lengths to absolute c.g.s. units is

$$\lambda_c = h/(m_0c) = 0.024271 \pm 0.000010\text{A}.$$

In giving the probable error of this final value we have arbitrarily multiplied the root-mean-square deviation from the mean from Table I by a factor 4 to take care of any possible unsuspected systematic instrumental errors. The assigned uncertainty ± 0.000010A corresponds to a motion of the upper carriage L in Figs. 2 and 3 of 0.01 mm.

This value of λ_c is in satisfactory agreement with the 1947 least squares adjusted best value from a wide variety of sources of information recently obtained by DuMond and Cohen,[11] namely,

$$\lambda_c = 0.0242650 \pm 0.0000025\text{A}.$$

The present measurement thus adds another important confirming datum to the above-mentioned least squares fitting with all of its consequent results regarding the numerical values of the atomic constants.[12]

[11] Jesse W. M. DuMond and E. Richard Cohen, "our knowledge of the atomic constants F, N, m and h in 1947, and of other constants derivable therefrom," Rev. Mod. Phys. **20**, 106 (1948).

[12] We purposely refrain from giving in the present article numerical values such as the rest energy, m_0c^2, in ergs or in electron volts (m_0c^2/e) *as consequences of the present measurement,* for in order to do this we would be obliged to combine our present observation, ***which can strictly be regarded only as a measurement*** of $\lambda_c = h/(m_0c)$, ***with one or more other independent observations*** of the atomic constants or functions thereof. Suppose we consider how the rest energy in electron volts might be computed utilizing the present result. There are some six types of combination suitable for this purpose, one of which, for example, would be to combine a measurement of h/e with the present measurement of h/m_0 (c being regarded as a fixed constant because of the superior accuracy with which it is known) so as to eliminate h and give e/m_0. Then, m_0c^2/e would yield the desired rest-mass energy in electron volts. Another one of these ways would be to combine a value of $e^2/(m_0h)$ (from Robinson's work on the $H\rho$ of photo-electrons with magnetic fields) with the present measurement of h/m_0 to give $[e^2/(m_0h)]\cdot h/m_0 = e^2/m_0^2$ from which m_0c^2/e can be directly computed. Any one of the types of determination not parallel to (4) in Fig. 6 can, in fact, be so combined with the present measurement. The point is that no two of these methods would give exactly concordant results (although the discords would not be great) and the choice of one method to the exclusion of the others would be completely arbitrary. We believe, therefore, the only acceptable procedure would be *to include the present measurement in a complete new least-squares solution* like the one referred to in reference 11. Now the present measurement is in such good accord with the latter, as is evident from Fig. 6 ,that in view of its probable error we feel safe in predicting that such a procedure would not modify the 1947 least squares adjusted values in any significant degree. We still recommend, therefore, 0.51079 ± 0.00006 Mev as the best present value of the electron rest energy.

To give a clearer idea of the satisfactory nature of this agreement we show in Fig. 6 the same isometric consistency chart which appeared as Fig. 9 of the above-mentioned article,[11] to which has been added (as a heavy black line) the present newly obtained datum. The width of this line indicates the assigned probable error of the present experimental observation.

The present measurement of λ_c is in remarkable agreement with the results of J. A. Bearden's determination of the x-ray refractive index of diamond (when the latter is combined with the experimental

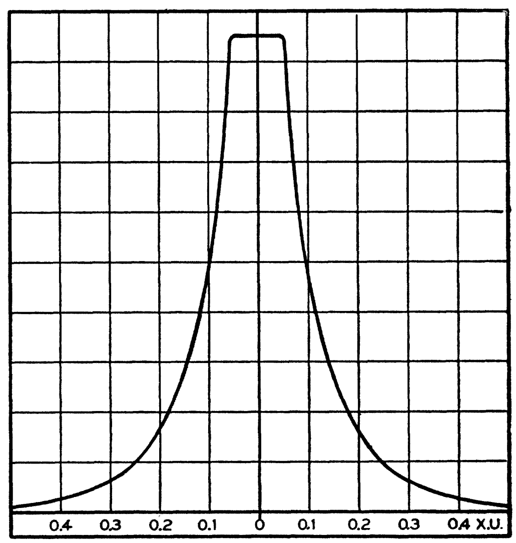

FIG. 8. The calculated source function curve coming from the combined effects of the finite opening of the lead slit jaws and the penetration of the annihilation radiation through their edges. The curving approaches are due to the penetration of the radiation through the edges of jaws. The differing obliquities of the rays in the different collimator slots are taken into account in the calculation.

value of the Rydberg), as can be clearly seen in Fig. 6.

The present result also *furnishes new and independent evidence favorable to a higher value of the Faraday.*[13] In Fig. 6 it is clear that three excellent determinations of independent functions of the atomic constants, namely, 3, the atomic mass of the electron by spectroscopy ($H\alpha$ and $D\alpha$; R_H and R_{He}), 4, e^2/m from Bearden's measurement of the x-ray refractive index of diamond and, 5, e/m by direct deflection methods all tend to agree on a diagram point which calls for a higher value of "e," by from one to three parts in 10^4 than that plotted. Now the value of "e," plotted in Fig. 6 as determination 2, is the direct result of averaging the silver and iodine values of the Faraday. If the silver value were abandoned and the iodine value adopted as the correct one, this would go a long way toward establishing a very perfect consistency between the four best determinations on the chart. Therefore, it is very interesting that we now have a fifth entirely independent piece of evidence, the present annihilation radiation determination, which also supports this shift toward the higher Faraday (see note at end of article).

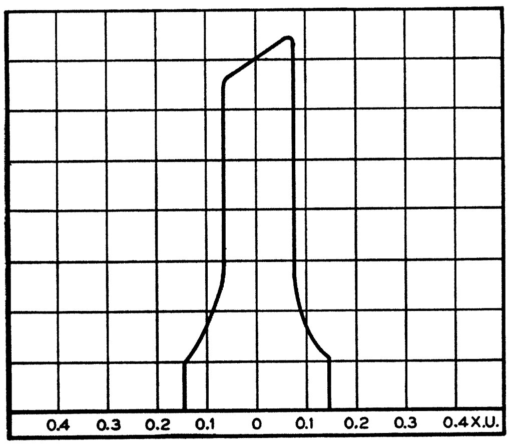

FIG. 9. The curve descriptive of the geometrical aberrations of focus of the curved quartz crystal. This curve has been determined from the observed line profiles of the 0.41-Mev line from Au^{198} in the way explained in the text.

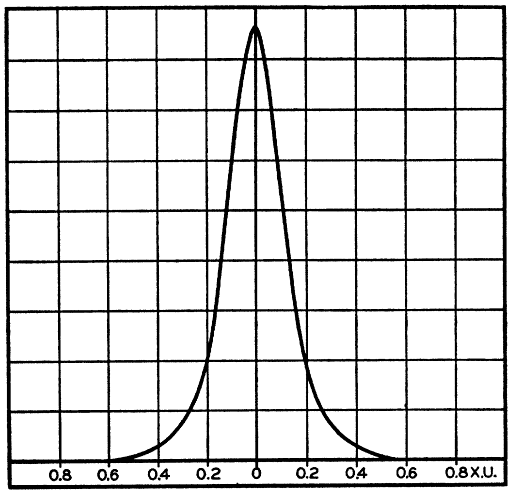

FIG. 10. The gamma-ray spectrometer "window" curve. This is the curve obtained by folding the profiles of Figs. 8 and 9 together. It may be regarded as the "tool" or "window" with which the instrument explores the spectrum.

The spectral profiles of the annihilation line appear to be significantly broader than a careful analysis of all instrumental sources of broadening will alone account for. It seemed worth while, therefore, to try to separate from the observed line structure the influence of the various instrumental causes of broadening so as to get an approximate idea of the "natural" breadth of the line. Since, unfortunately, the excess broadening and the instrumental broadening are of the same order of magnitude and the observational uncertainty of the profile is also far from negligible on this scale, no very great accuracy can be claimed for this process and we, therefore, did not feel warranted in applying very elaborate analytical methods to resolve the natural structure.

In order to obtain as much precision in the shape of the observed line profile as was available, all of the individual line profiles were superposed and

[13] Two independent types of direct precision determination of the Faraday have been made (one with the iodine, the other with the silver voltameter), and it is well known that these disagree by considerably more than the estimated probable error of either one, the silver value being the lower of the two. G. W. Vinal and L. H. Brickwedde of the U. S. National Bureau of Standards in a recent communication have given results on the iodine and silver determinations in which the difference (13 parts in 10^5) is estimated to be three or four times the average deviation of a single observation in either the iodine or silver experiments and is uncomfortably large even when the uncertainties in the atomic weights of iodine and silver are taken into account. Vinal and Brickwedde adopt the arithmetic average of the iodine and silver experiments with equal weights. For a long while R. T. Birge attached considerably greater weight to the silver value of the Faraday, but since his 1941 review of the general physical constants he has adopted a weighted average value which is very close to that of Vinal and Brickwedde. As DuMond and Cohen point out in their (1947) paper (reference 11), all the experimental work on the Faraday was done before the discovery of isotopes and there may be some error introduced (perhaps by selective deposition) in the case of silver which in nature consists of two nearly equally abundant isotopic constituents. Iodine, on the contrary, is isotopically homogeneous. H. Urey and other experts in the field, however, tend to consider this suggested source of error as unlikely to be the correct explanation of the discrepancy. As Vinal and Brickwedde point out, the two types of Faraday determination are subject to quite different kinds of errors and one, therefore, furnishes a most valuable check on the other. When the principal work on the silver voltameter was done, the emphasis was placed on perfecting the voltameter *as a primary standard for the international ampere.* The emphasis was on reproducibility rather than on the absolute significance of the results for the determination of the Faraday. It would seem, therefore, most desirable that new voltameter measurements having for their primary object the redetermination of the Faraday should be made with the better facilities now available both for chemical and electrical measurements.

averaged so as to obtain a composite mean profile. This is shown in Fig. 7. An instrumental "window curve" was then prepared by taking into account all of the known instrumental causes of broadening. These are (i) the geometric slit width as defined by the lead slit jaws, (ii) the effect of absorption in the edges of the lead jaws, and (iii) the effect of imperfect focusing of the curved crystal from geometric aberration. In determining (ii), which is far from negligible, the different obliquities at the slit of the rays defined by each active slot in the lead collimator was taken in account, together with the relative importance of each of these rays as determined by previous measurements using one slot at a time. The combined effects of (i) and (ii) lead to a curve (in which the effect of absorption at the slit edges is clearly visible) which we may call the source function, pictured in Fig. 8. The fold of this curve into the curve descriptive of the imperfect focusing of the crystal (Fig. 9) gives the complete instrumental "window" curve shown in Fig. 10.

The rather irregular looking curve of Fig. 9 was determined by means of earlier measurements[4] made on the 0.41-Mev line from Au^{198} on the assumption that the "natural spectral width" of the latter line is completely negligible for our purposes. Partly from a knowledge of the aberrations of focus and partly by trial and error, the curve of Fig. 9 was given such a shape that when folded into a rectangular distribution 0.13 x.u. wide (chosen to represent the geometry of the Au^{198} source) it would yield the composite line profile actually obtained from the average of all of the Au^{198} measurements. This curve of Fig. 9 is also not inconsistent with a still earlier exploration of the geometrical aberrations of focus of the curved crystal made by using tungsten $K\alpha_1$ x-radiation and making repeated observations of the position and intensity of the $WK\alpha_1$ line utilizing separately each one of six of the collimator slots. In this last case the x-ray tube was placed on the upper beam CR' of Fig. 3 and a very fine slit was substituted for the source and placed exactly on the focal circle inside the spherical lead source bomb. Figure 11 shows the information

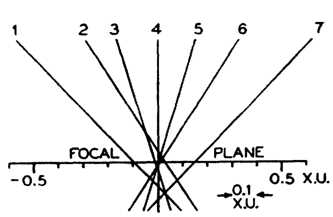

FIG. 11. Ray diagram made with tungsten $K\alpha_1$ radiation to test the aberrations of focus of the curved crystal. Each of the seven numbered rays corresponds to a collimator slot. The obliquities of these rays are 100-fold exaggerated, i.e., the tangents of their angles relative to ray 4 are 100 times natural scale. In the annihilation radiation study, slot 7 was blocked with lead.

regarding the aberration pattern of the crystal as obtained by this x-ray method. In the present work slot number 7 has been blocked off with lead. The remaining pattern is in reasonable agreement with the distribution of Fig. 9. In all these curves (Figs. 7 to 10 inclusive) the abscissa scale has been expressed in nominal scale x.u. of the spectrometer and, to the accuracy of these line structure studies, this can be taken as identical either to x.u. or to $\text{A}\times10^{-3}$.

The desired "natural" line profile of the annihilation radiation is then such a profile that if it is folded into the instrument window curve of Fig. 10 the resultant will be the observed curve of Fig. 7. There are well-known methods of unfolding or "dividing out" one of the members of a fold which depend on finding the Fourier transforms of the curves involved (in this case the curves of Figs. 7 and 10). We felt that the accuracy in the present instance was hardly sufficient to warrant such an elaborate solution and we have contented ourselves with fitting simple Gaussian error functions of form $A_i(2\pi)^{-\frac{1}{2}}\sigma_i^{-1}\exp(-x^2/2\sigma_i^2)$ to each of the curves of Figs. 7 and 10 with width parameters σ_0 and σ_w, respectively, and assuming that the desired "natural" spectral profile is given by a third Gaussian error curve with width parameter σ_n related to the first two by the relation

$$\sigma_n^2=\sigma_0^2-\sigma_w^2. \qquad (2)$$

(It is well known that when Gaussian error functions are folded together the squares of the respective width parameters add to form the square of the width parameter of the folded resultant.) Figures 12 and 13 show how closely the curves of Figs. 7 and 10 can each be approximated by a single Gaussian error curve. In Figs. 12 and 13 the full lines are the original curves and the points indicate the Gaussian approximation. A simple and effective geometrical construction to obtain a good value for a width parameter σ, in fitting a Gaussian error curve to a given curve which nearly approximates it, is to project the tangents to the points of steepest slope on either side of the given curve upward so as to locate their point of mutual intersection. The half-width of the given curve at a height half that of this intersection point may then be taken as giving σ. This method was used in the present case. It will be seen that the Gaussian approximations to the curves of Figs. 7 and 10 are quite satisfactory save for minor departures in the wings.

By this method we arrive at the conclusion that the "natural" structure of the annihilation line from Cu^{64} is approximately represented by a Gaussian error curve of width parameter

$$\sigma_n=0.096 \text{ x.u.}$$

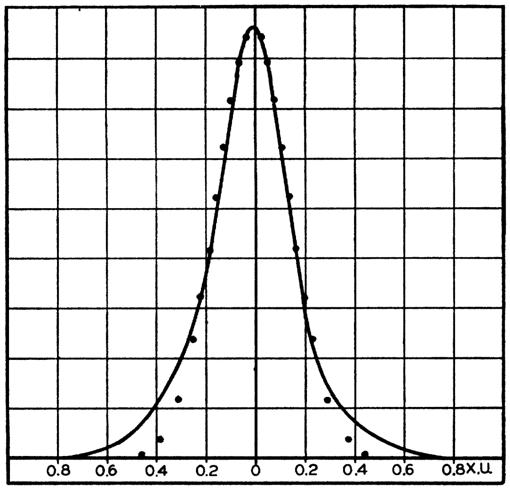

FIG. 12. To show how closely the curve of Fig. 7 can be approximated by a Gaussian error function. The full line is the curve of Fig. 7 (the observed annihilation line profile). The points lie on the Gaussian curve. For this latter the width parameter, σ_0, is 0.15 x.u.

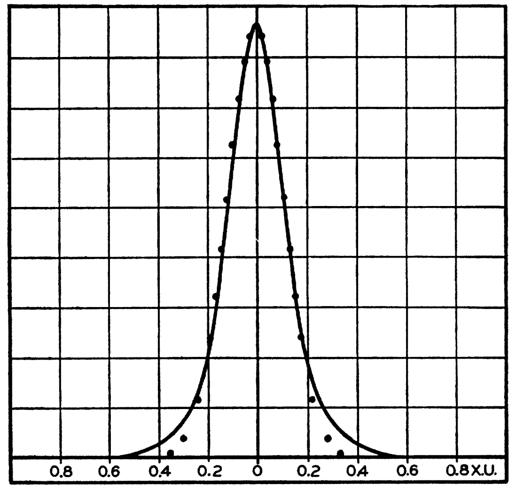

FIG. 13. To show how closely the curve of Fig. 10 can be approximated by a Gaussian error function. The full line is the curve of Fig. 10 (the instrumental window profile). The points lie on the Gaussian curve. For this latter the width parameter, σ_w, is 0.115 x.u.

If we now compute what speeds $\beta = v/c$ of the center of mass of recombining positron-electron pairs would account for such a width as this we arrive at the value $\beta = 0.0040$ corresponding to the speed of an electron having a kinetic energy of only 4.1 volts. If our rough picture of nearly stationary ("thermal velocity") positrons recombining with moving K, L, M, and conduction electrons of copper proposed in our introduction is a correct one, the speeds of the *moving members* of the pairs must be twice as great and the kinetic energy four times as great as the above, hence 16 ev. *The conduction electrons in copper are the only ones with which such a low order of speed can be associated.* Assuming one conduction electron per atom and an effective electronic mass equal to m_0, Seitz[14] calculates that the fastest conduction electrons in copper should have kinetic energies of about 7 ev. Since our line breadth determinations are rather rough, this may be regarded as a reasonable agreement.

The extremely narrow half-width of the "line" ($\Delta\lambda \sim 0.096$ x.u.) gives us great confidence in the reliability of the value of $h/(m_0c)$ which we have computed from its measured wave-length λ since any *shift* $\delta\lambda$ from this ideal value must almost certainly be a much smaller order of magnitude when expressed as a fraction of λ than is the ratio $\Delta\lambda/\lambda$. Applying formula (1) we conclude that the line is not shifted more than 16 parts per million, that is to say, $\delta\lambda/\lambda \cong 16\times 10^{-6}$.

However, the above considerations do not exclude the possibility that the more rapidly moving electrons may also be involved in the annihilation process. The speeds of the K, L, and M electrons in copper are roughly given by $\beta_K = 0.20$; $\beta_L = 0.060$; $\beta_M = 0.017$. The M electrons, of which there are 18 in copper, have such speeds that on the basis of our simple picture one would expect a Doppler broadening of half-breadth about 0.21 x.u. This is not at all inconsistent with the breadth observed near the base of the present line where the wings flare out rather emphatically and it may be that the less energetic M electrons do contribute somewhat to the annihilation process.

The L and K electrons have speeds which would give Doppler broadened distributions, respectively, about 3 and 9 times as broad as the M. Such very broad distributions with our present resolving power could easily be confused with and interpreted as part of the continuous background coming from various causes, chiefly scattering from the partitions of the collimator. We have, therefore, tried to estimate from our previous experiment in measuring the wave-length of the 0.41-Mev line from the isotope Au^{198} what background intensity we ought to expect in the present instance. It turns out that our observed background in the present experiment is some 50 percent higher than this expected value which favors the supposition that part of it could be a broad distribution of annihilation radiation. However, little positive weight can be attached to this fact since the estimate is subject to errors of the same order as the excess. Unfortunately, because of the short half-life of Cu^{64}, our strong source had decayed below the level for further spectral investigation before this interesting question occurred to us. We believe it might be of

[14] F. Seitz, *Modern Theory of Solids* (McGraw-Hill Book Company, Inc., New York, 1940), p. 146.

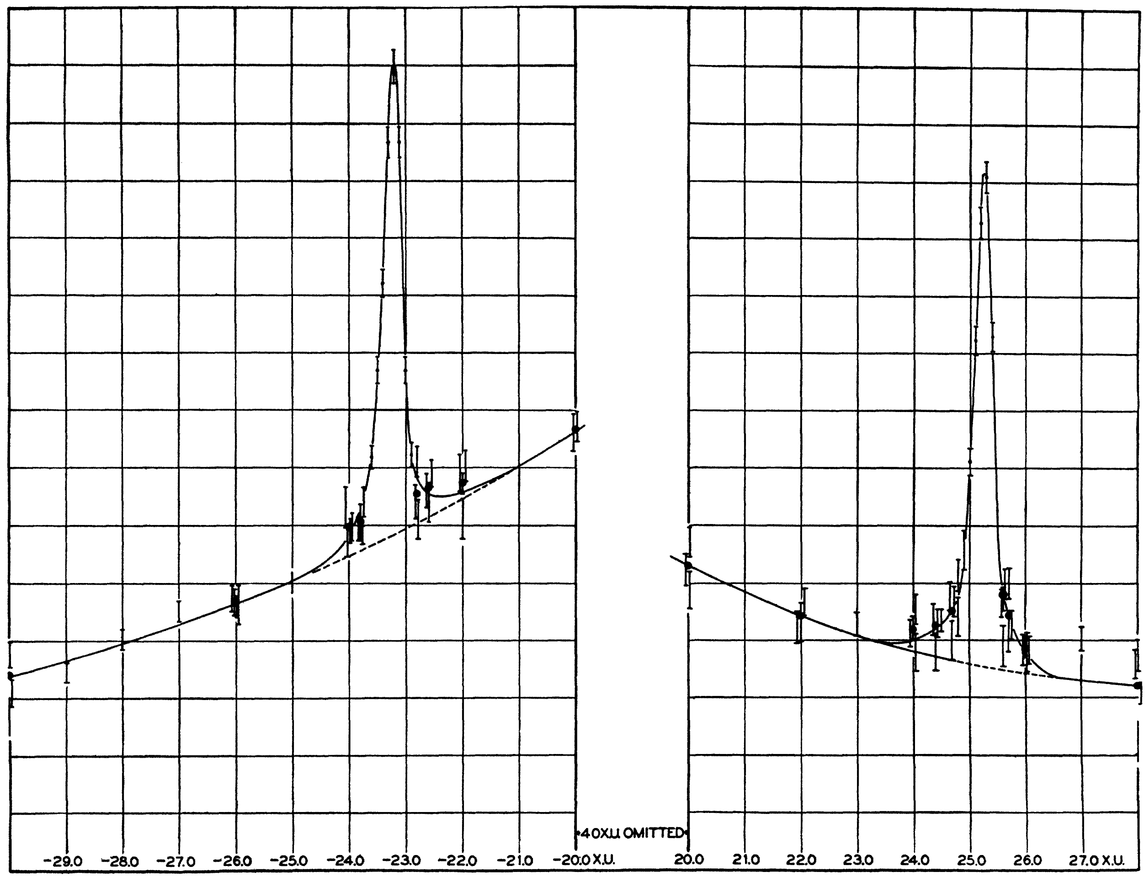

FIG. 14. Composite study of background profile from data taken in runs 1 to 4 inclusive. The individual observations are shown as vertical strokes (indicative of the statistical counting uncertainty) and where more than one such occurs at a given wave-length setting the weighted average is also shown by a black dot. Individual observations belonging to the same wave-length setting are displaced slightly to right and left of that setting to avoid confusion. In the more salient parts of the annihilation line only the observations from run 1 are shown, to avoid confusion, although many more observations were actually taken. Such points are distinguished by a dot in the middle of the vertical stroke. See text for method of normalizing the observations of different runs. There is no positive evidence for a broad inhomogeneous distribution of annihilation radiation. The background is probably entirely from scattering in the collimator.

considerable interest to repeat the experiment with a considerably wider slit and lower resolving power in an effort to see if our present line is actually superposed on one or more much broader structures also ascribable to the annihilation process. We think that participation of the K and L electrons in the annihilation process is unlikely for the following reason. At the low electron velocities in question the product of the velocity by the cross section for annihilation according to Heitler[2] is constant for all electrons and equal to $\pi r_0^2 c$ where r_0 is the classical radius of the electron so that $\pi r_0^2 = 2.5 \times 10^{-25}$ cm^2 and c is the velocity of light. This product multiplied by the positron number density per unit volume gives the time rate of annihilation per electron. If the various classes of electrons could be regarded as having homogeneous density throughout the interior of the copper, one would therefore expect that the area of the spectral distribution of annihilation radiation ascribable to each class of electron would be simply proportional to the number of electrons in that class. This, however, ignores the fact that the density of positrons awaiting annihilation is, because of nuclear repulsion, certainly enormously lower in the vicinity of the K and L electrons. The K, L, and M x-ray absorption discontinuities of copper indicate binding energies for these classes of electrons: $W_K = 9000$ ev, $W_{L\,\text{III}} = 930$ ev, and $W_{M\,\text{III}} = 75$ ev. The work to bring a stationary positron from infinity up to the same general order of proximity to the nucleus as these electrons should be, therefore, about double these values. Clearly, "thermal" positrons with energies of the order kT or 0.024 ev would have practically no density whatever in regions of this order of potential energy. These considerations, though rough, argue strongly against the existence of any annihilation radiation differing greatly in wave-length from that indicated in our observed "line."

In Fig. 14 we show a composite of all the background observations obtained in the first four runs of the present experiment reduced to the same ordinate scale by multiplying the ordinates of different runs by appropriate factors to take care of the decay of the source. In these curves the "natural" background of the counter has been subtracted so that what remains is entirely due to the presence of Cu^{64} in the spectrometer. In order to avoid any possible error or unconscious bias the multiplying factors for the different runs have been determined so as to make the peak of the annihilation "line" the same height in every case, letting the background ordinates fall where they would. Figure 14 shows both weighted *average* background ordinates (black circles) and individual observations (vertical strokes). The heights of these strokes are indicative of the statistical uncertainty. Where more than one individual observation falls at a given wave-length the corresponding vertical strokes are displaced slightly to right and left of the true position, if necessary, to keep them distinct. Figure 14 is plotted primarily to give information on the background profile and, therefore, in the more salient parts of the annihilation line only the points from run 1 have been shown to avoid confusion. Such points are indicated by placing a small dot in the middle of the vertical stroke.

We have purposely plotted the right- and left-hand profiles separately. The difference in background height on the two sides is not a result of source decay since a correction for this has already been made. (This can be seen by the equality in the heights of the lines above the background level.) We ascribe this difference on the two sides in part to the fact that some slight misalignment of the collimator may result in a different angle of scattering of the primary beam on the collimator partitions in the two cases[15] and in part to possible differences in the character of the surfaces of the lead partitions on their two different sides. This difference illustrates well how difficult and uncertain it is to make any prediction of background intensity from previous observations of background at different wave-lengths and angles. At these small angles of incidence of the primary beam on the collimator partitions the background intensity varies very rapidly with the angular setting as can be clearly seen from the background curves themselves. The increase in slope with decreasing wave-length setting is characteristic of all the background profiles we have ever obtained. The chief conclusion we draw from these curves is that they furnish no positive evidence for any broad distribution of annihilation radiation at distances from the core of the line much in excess of ± 1.5 x.u. If we divide the line, after subtracting the sloping background, into horizontal strips, Fig. 14 makes it clear that only a very small fraction of the total line area would be associated with a strip of half-width 1.5 x.u. Such a strip would correspond to a class of electrons of speed $\beta = v/c = 0.12$ and energy 3.6 kev. This, of course, ignores the fact that a large part of the line width at the base comes from the instrumental "window curve" and is not a spectral characteristic of the annihilation radiation.

[15] The kinematics of the instrument (Figs. 2 and 3) are such that a misalignment of the collimator will result in a different mean angle of scattering of the primary beam on the collimator partition but will, nevertheless, give exactly the same obliquity in the collimator for the transmission of the selectively reflected beam in both right- and left-hand settings.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge with gratitude our indebtedness to David Klein, David Muller, James Brown, and E. Richard Cohen for very valuable assistance in the present experiment both as regards the observation and reduction of data and the tedious calibration procedures. Mr. Cohen's assistance in discussions regarding the interpretation of the results has also been of much value.

Note added in proof: After this article was submitted to the editors, Bethe and Longmire,[16] discussing the work of Taub and Kush, have announced that the results of the latter on the ratio of magnetic moments of electron and proton combined with the recent hyperfine structure results of Nafe and Nelson yield a very accurate value of α $\alpha^{-1} = (e^2/hc)^{-1} = 137.041 \pm 0.005$. This result plots on the consistency chart in such a way as to favor the *lower* or *silver value* of the Faraday.

This value of α (combined with the Rydberg constant) yields a value of h/m directly comparable on the consistency chart with a result on λ_c in the present article. The present annihilation radiation measurement yields a value of λ_c slightly *longer* in wave-length than λ_c computed from α, but the discrepancy is not serious relative to the probable error ranges. C. C. Lauritsen has suggested that small angle Compton shifted scattering of the emerging annihilation radiation *within the copper source itself* might account for a slight shift of this order toward longer wave-lengths. A careful analysis reveals, however, that any such shift effect is far too small. These points are more fully discussed in forthcoming letters to the editor.[17]

[16] H. A. Bethe and C. Longmire, Phys. Rev. **75**, 306 (1949).
[17] Jesse W. M. DuMond, Phys. Rev. **75**, 1266 (1949); also, Phys. Rev. **75**, 1267 (1949).