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On the Inelastic Scattering of Fast Neutrons*

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The available data on the inelastic scattering of fast neutrons are interpreted in terms of: (1) a detailed theory, developed in this paper, for the excitation of individual levels of the target nucleus, applicable when only a few levels of the target nucleus are involved (i.e., incident neutron energy only a few times the low lying level spacing of the target nucleus); or (2) the statistical theory of Weisskopf, applicable when the incident neutron energy is very much greater than the level spacing of the target nucleus (very many levels involved). The data of the Los Alamos group on the scattering of 1.5- and 3-Mev neutrons by Fe, W, and Pb indicate that Fe requires the first type of interpretation, W the second; Pb seems to be more similar to Fe than to W. The level system proposed by Elliott and Deutsch on the basis of β-decay measurements is sufficient to explain the Fe data. The average spacing of the lowest W levels is derived to be approximately 80 kev. The large apparent level spacing of Pb is consistent with the stability of nuclei containing the "magic number" of 82 protons or 126 neutrons.

I. INTRODUCTION

THE interaction of neutrons with atomic nuclei has been extensively investigated over a wide range of neutron energies. The accumulated data, in terms of cross sections for the various possible reactions, have recently been reviewed by Goldsmith, Ibser, and Feld.[1] No inelastic scattering data were included in that compilation since "inelastic scattering measurements are in general not adaptable to graphical representation of cross section *vs.* neutron energy . . . because of the complex nature of the inelastic scattering process and the strong dependence of the measured cross section on the experimental arrangement (especially on the energy sensitivity of the detector)." [2]

The difficulty of interpreting experiments on inelastic scattering arises from the fact that for a given (single) incident neutron energy the emergent scattered neutrons have a heterogeneous energy distribution. If there are only a few energy levels of the target nucleus between the ground level and the incident neutron energy, the emitted neutrons will have a line spectrum, each line corresponding to neutron emission (scattering) in which the product nucleus is left in one of its excited states; a detector with good energy resolution will then detect separate neutron groups.

However, the line spectrum will be "washed-out" if either the energy levels of the target nucleus are not distinct, but overlap, or the resolution of the detector is not good enough to distinguish the separate neutron groups. In this case, the energy distribution of the scattered neutrons will appear as a continuous spectrum. Obviously, the observed spectrum of inelastically scattered neutrons depends critically on the energy sensitivity of the neutron detector.

Furthermore, the energy distribution of the scattered neutrons is sensitively dependent on the incident neutron energy. Hence, if the neutron source has a heterogeneous energy distribution, the distribution of scattered neutrons will—even with a detector of perfect energy resolution—be very difficult to interpret in terms of the level structure of the target nucleus.

Most measurements of inelastic scattering have

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[1] H. H. Goldsmith, H. W. Ibser, and B. T. Feld, Rev. Mod. Phys. 19, 259 (1947).

[2] Reference 1, p. 289.

involved either heteorgeneous neutron sources, or detectors with poor energy resolution, or both.[3] In most of these experiments the detectors were of the "threshold" type—i.e., sensitive only to neutrons above a certain (usually not very sharply defined) energy. In interpreting such experiments, the term inelastic scattering is used, somewhat loosely, to mean any scattering process which degrades the neutron energy by an amount sufficient to render it incapable of producing the "threshold" reaction. Since such experiments will not detect scatterings in which the source neutrons, while losing some energy, still remain above the threshold, and since the energy dependence of the threshold reaction is usually not well known above the threshold and seldom taken into account, and since the presence of elastic scattering seriously complicates the interpretation of such experiments,[3] it is usually impossible to derive from them information concerning details of the level structure of the target nucleus.

There have been considerably fewer experiments in which the neutron source was monoenergetic and the energy distribution of the inelastically scattered neutrons was observed with a detector of reasonably good resolution. Dunlap and Little,[4] using a $D-D$ source of $\sim$2.5-Mev neutrons, observed the distribution of proton recoils in a hydrogen-filled cloud chamber irradiated by the neutrons emerging from a lead scatterer. More recently, Barshall *et al.*,[5] have investigated the inelastically scattered neutrons from a variety of elements for a number of incident neutron energies; as a detector, they employed proton recoils in a proportional counter.

It is the purpose of this paper to indicate how such experiments may be used to obtain detailed information on the level structure of nuclei, and to derive such information from the existing experimental data.

II. THEORY

The general features of the energy dependence of the cross sections for absorption and scattering of neutrons have been discussed by Feshbach, Peaslee, and Weisskopf.[6] While for incident neutrons in the slow neutron energy range, the cross sections are characterized by sharp resonances corresponding to distinct energy levels of the compund nucleus, at higher neutron energies the cross-sectional behavior is smooth, decreasing monotonically toward the value $2\pi a^2$ for neutrons of wave-length small compared to a, the nuclear radius.[7] Roughly half of the total cross section for fast neutrons corresponds to neutron capture with the formation of a compound nucleus; the remainder is associated with the process of "shadow scattering," analogous to the diffraction of a plane wave by a spherical obstacle. Shadow or diffraction scattering is elastic, and is confined to within small angles, of the order λ/a (λ is the neutron's Dirac wave-length), with the incident direction.[8]

Because the process of shadow scattering does not involve capture of the incident neutron into a compound nucleus, it must be distinguished from that part of the elastic scattering which does. Experimentally, the two processes can be separated, at high incident neutron energies, because of the difference in the angular distributions of the scattered neutrons resulting from the two types of elastic scattering. Actually, the two scattering processes are not completely independent, for they are coherent and, therefore, result in interference effects. However, for convenience, the interference effects will be included in the shadow scattering term of the cross section. That portion of the elastic scattering cross section which involves the formation of a compound nucleus, and which would be present even if there were no shadow scattering, will henceforth be called *capture elastic scattering.* Elastic scattering, used without a preceding modifier, includes both capture and shadow scattering.

The cross section for the capture of the incident neutron, πa^2 at high neutron energies, is shared among all the reactions which are energetically possible; these may include (n, p), (n, α), (n, γ), $(n$, fission$)$, $(n, 2n)$, inelastic scattering, capture elastic scattering, etc. In the range of neutron energies between 0.1 and 5 Mev, for all but the lightest and the heaviest nuclei, only the scattering processes are of appreciable importance.

Fundamentally, inelastic and capture elastic scattering are two aspects of the same process, differing only in the state in which the product nucleus is left after the neutron emission. In the latter case, the product nucleus is left in the ground state and the neutron is emitted with its incident energy (minus that given up as recoil energy to the nucleus in order to conserve momentum); in the case of inelastic scattering, the product nucleus is left in an excited state, and the emitted neutron has an energy equal to the incident energy minus the excitation energy of the level of the product nucleus involved (less the recoil energy of the nucleus).

The product nucleus, left in an excited state as a

[3] See, for instance, L. Szilard, S. Bernstein, B. Feld, and J. Ashkin, Phys. Rev. **73**, 1307 (1948).

[4] H. F. Dunlap and R. N. Little, Phys. Rev. **60**, 693 (1941).

[5] H. H. Barschall, M. E. Battat, W. C. Bright, E. R. Graves, T. Jorgensen, and J. H. Manley, Phys. Rev. **72**, 881 (1947); H. H. Barschall, J. H. Manley, and V. F. Weisskopf, Phys. Rev. **72**, 875 (1947).

[6] H. Feshbach, D. C. Peaslee, and V. F. Weisskopf, Phys. Rev. **71**, 145 (1947) henceforth referred to as FPW.

[7] See reference 6, p. 157, Fig. 5.

[8] E. Amaldi, D. Bocciarelli, B. N. Cacciapuoti, and G. C. Trabacchi, *Report of an International Conference on Fundamental Particles and Low Temperatures* (The Physical Society, London, 1947), Vol. 1, p. 97.

result of inelastic scattering, usually emits the excitation energy as one or more gamma-rays. From the energies and numbers of these gamma-rays, it should be possible to derive considerable information concerning the level structure of the target (product) nucleus and the relative cross sections for the excitation of the various levels. Gamma-rays resulting from inelastic scattering have been observed; this investigation will, however, confine itself to the interpretation of measurements on the inelastically scattered neutrons.[9]

For incident neutrons of energy less than the first excited level of the target nucleus, only elastic scattering is possible. As soon as the incident energy exceeds the excitation energy of the first level, inelastic scattering competes with capture elastic scattering.

It is important to note that not all of the levels of the target nucleus will be excited necessarily as a result of inelastic scattering. For a level to be excited it is necessary that its angular momentum and parity properties be such as to permit the required transitions.

The process of inelastic scattering is a two-step process: First, the incident neutron is captured into one of the levels of the compound nucleus. The excited compound nucleus then decays, by neutron emission, to a level of the target (product) nucleus. Since inelastic scattering occurs only for fast neutrons, the levels of the compound nucleus at the excitation energy (incident neutron energy plus the neutron binding energy) will be closely spaced (even overlapping) compared to the energy spread of the incident neutrons, so that many levels will be available for excitation. Furthermore, a beam of incident neutrons contains states of orbital angular momentum $0 \leqslant l \leqslant a/\bar{\lambda}$, so that it is possible to induce transitions from the ground state of the target nucleus to many levels of the compound nucleus. Finally, since the emitted neutrons also can carry away orbital angular momentum, if they have sufficient energy, transitions from the excited compound nucleus to a given level of the product nucleus will almost always be possible. (See Appendix.)

The foregoing discussion indicates that a study of the angular distributions of the various groups of inelastically scattered neutrons would be difficult to interpret unambiguously in terms of the angular momentum properties of the corresponding levels of the target nucleus. There is, as yet, no experimental information on this aspect of inelastic scattering.

1. Excitation of Few Levels

The values of the total cross sections σ_t of a variety of elements have been measured in the fast neutron region.[1] A theoretical expression for σ_t as a function of the incident neutron energy has been derived by FPW, Eq. (50), and Fig. 5.

Of particular interest to this discussion is that fraction of the total cross section which involves the formation of a compound nucleus. A theoretical expression for this cross section, henceforth denoted by σ_c, may be obtained from the expressions immediately preceding Eq. (49) in FPW, by omitting the diffraction scattering term, p_{sc}. For our purposes, it is sufficient to note that

$$\sigma_c \cong \tfrac{1}{2}\sigma_t = (1+\epsilon)\pi a^2, \tag{1}$$

where ϵ is a small factor which decreases monotonically with increasing neutron energy, approaching 0 for $\bar{\lambda} \ll a$.

When the incident neutron energy is sufficient to excite a number of levels of the target nucleus, the cross section for the formation of the compound nucleus can be written

$$\sigma_c = \sum\nolimits_i \sigma_i, \tag{2}$$

where the subscript refers to a scattering in which the product nucleus is left in the ith state.[10]

In general, it is necessary to consider separately the components of the cross section corresponding

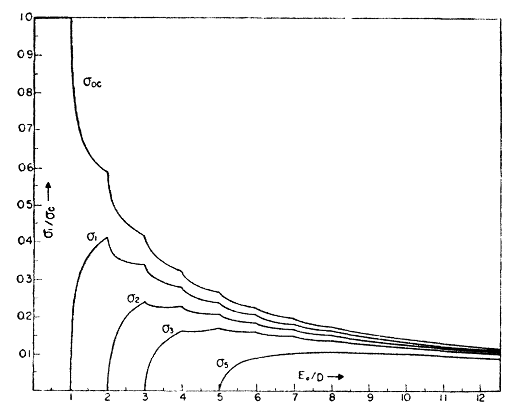

FIG. 1. Cross sections, σ_i, for the excitation of the low-lying levels of the target nucleus as a function the incident neutron energy, E_0. σ_c is the cross section for the formation of a compound nucleus (neutron capture). The distance between levels is assumed constant and equal to D. Further assumptions are:
(1) S-scattering ($l=0$) only;
(2) $\Gamma_i = C(E_0 - E_i)^{\frac{1}{2}}$ with C constant and equal for all levels.

[9] The available date on the gamma-rays associated with inelastic scattering are not sufficiently accurate or detailed to permit interpretation in terms of the level structure of the target nucleus. See, for instance, D. E. Lea, Proc. Roy. Soc. **A150**, 637 (1935); H. Aoki, Proc. Phys. Math. Soc. Jap. **19**, 369 (1935); G. T. Seaborg, G. E. Gibson, and D. C. Grahame, Phys. Rev. **52**, 408 (1937).

[10] The first term in Eq. (2), σ_0, refers only to capture elastic scattering.

TABLE I. Cross sections for the scattering of neutrons of initial energy E_0 to below the (threshold) energy E_i; data of the Los Alamos group (see reference 5).

Element	E_0(Mev)	E_i(Mev)	σ_{in} (barns)	πa^2 (barns)
Fe	1.5	0.40	0.0	1.0
		0.95	0.6	
	3.0	0.75	0.3	
		1.50	0.7	
		2.25	1.1	
W	1.5	0.40	0.9	2.3
		0.90	2.1	
	3.0	0.75	1.4	
		1.50	2.4	
		2.25	2.8	
Pb	1.5	0.40	0.0	2.5
		0.90	0.4	
	3.0	0.75	0.7	
		1.50	1.2	
		2.25	1.6	

to the emission of neutrons in different states of orbital angular momentum, l. In the following discussion, we shall, for simplicity, assume emission in the S-state, $l=0$, only. The extension to take into account states of $l>0$ is discussed in the Appendix.

The cross section for the excitation of a given state (i) of the target nucleus may be written

$$\sigma_i=\sigma_c\Gamma_i/\Gamma, \tag{3}$$

where Γ is the total width for neutron emission by the compound nucleus ($\hbar$ divided by the mean life of the compound nucleus)

$$\Gamma=\Gamma_0+\Gamma_1+\cdots=\Sigma_i\Gamma_i, \tag{4}$$

and Γ_i is the partial width for neutron emission resulting in a product nucleus in the ith state.

By the conservation of energy,

$$\Gamma_i=0, \text{ for } E_0<E_i. \tag{5}$$

(E_0 is the incident neuteron energy, E_i the energy of the ith level.) At energies greater than the ith excitation energy, the theory of FPW [Eq. (46)] gives[11]

$$\Gamma_i=C_i(E_0-E_i)^{\frac{1}{2}}. \tag{6}$$

The variation of C_i from level to level is not very great as long as the transition to the ith level is quantum-mechanically allowed. The magnitude of C_i decreases slowly with increasing E_0, since it is proportional to the mean level spacing of the compound nucleus at the excitation energy resulting from the capture of the incident neutron.

The dependence of σ_i on the incident neutron energy follows from Eqs. (3) through (6). Starting from zero at the threshold, $E_0=E_i$, the cross section rises rapidly, proportional to $(E_0-E_i)^{\frac{1}{2}}$ since, for $E_0-E_i\ll E_0$, Γ increases only slowly with E_0. As Γ_i starts to contribute significantly to Γ, $(E_0-E_i\sim E_0)$, the increase of σ_i, becomes less rapid. Finally, as higher levels contribute successively to Γ, σ_i starts to fall off. At large values of E_0, where many levels are excited, the value of each σ_i becomes quite small, while $\Sigma_i\sigma_i\cong\pi a^2$.

Figure 1 shows the energy dependence of σ_i for small i, based on the assumptions of uniform level spacing, D, in the product nucleus, constancy of C_i, and S-scattering only.

In applying the above considerations to the case where the incident neutron energy is only sufficient to excite a few levels of the product nucleus, it is, of course, necessary to take into account the exact positions of the levels and the variation in C_i from level to level. While this has not been done in deriving Fig. 1, such details can easily be taken into account, if known, in Eq. (6).

On the other hand, the possibility of emission of neutrons in states of higher angular momentum has been neglected in the computation of the cross sections shown in Fig. 1. As discussed in the Appendix, it is possible to take these into account, although the theory then becomes more involved and the computations correspondingly laborious. In particular, as the wave-length of the emitted neutrons becomes much smaller than the nuclear radius, many values of $l>0$ become important, and an accurate theory must take them all into account.

Thus, for incident neutrons of energy great enough to excite many levels of the target nucleus, the analysis becomes rather complicated. Furthermore, it now becomes important to take into account the variation (decrease) of level spacing, both in the product and the compound nucleus, and the resulting decrease of C_i, with increasing neutron energy.

2. Statistical Theory for Many Level Excitation

When many levels can be excited, a measurement of the energy distribution of the emitted neutrons no longer will display separate groups but will, rather, result in a smooth distribution, ranging from zero energy up to the energy of the incident neutron. The shape of the distribution in energy of the inelastically scattered neutrons can be qualitatively deduced as follows: Because the level spacing is expected to decrease exponentially with increasing excitation energy, there are comparatively more levels available for the emission of low energy neutrons. On the other hand, Γ_i is greater for the low lying levels, because of its dependence on the energy of the emitted neutron, resulting in a greater probability, per level, for the emission of faster neutrons. The net effect of these two conflicting

[11] On our assumption of $l=0$ scattering only, $|v_l|^2=1$.

tendencies is that the spectrum of inelastically scattered neutrons will be peaked at some energy intermediate between zero and the incident energy.

Weisskopf[12] has developed a theory applicable to the inelastic scattering of neutrons of energy, large, compared to the level spacing of the target nucleus, using quantum-statistical-mechanical considerations to derive the dependence of level spacing on the excitation energy. According to Weisskopf's *statistical theory*, the distribution of inelastically scattered neutrons is given by:

$$d\sigma(E, E_0) = \sigma_c(E/T^2)e^{-E/T}dE, \tag{7}$$

where $d\sigma(E, E_0)$ is the cross section for the scattering of incident neutrons of energy E_0 into the energy between E and $E+dE$, and T is a parameter, analogous to a temperature, of the excited target nucleus,

$$T \cong (DE_0)^{\frac{1}{2}}. \tag{8}$$

In Eq. (8), D is the average level spacing of the lowest levels of the target nucleus. Eqs. (7) and (8) apply when $T \ll E_0$.

Thus, for incident neutrons of energy $E_0 \gg D$, the inelastically scattered neutrons have a Maxwell-type energy distribution, which can be predicted if D is known. Conversely, a measurement of the energy distribution of the scattered neutrons can be used to infer D, since the average energy of the Maxwell distribution is

$$\bar{E} = 2T \cong 2(DE_0)^{\frac{1}{2}}. \tag{9}$$

Recently, Bradt and Tendam[13] have compared the cross sections for the (α, n) and $(\alpha, 2n)$ reactions in Rh^{103} and Ag^{109}. The results of their experiments are in excellent agreement with the predictions of the statistical theory.

The evidence, to be discussed, on the inelastic scattering of fast neutrons by tungsten, provides further confirmation of the applicability of the statistical theory to the emission of neutrons from highly excited nuclei.

III. EXPERIMENTAL EVIDENCE

The most extensive series of inelastic scattering measurements, using monoenergetic neutron sources, is due to the Los Alamos group.[5] They detected the neutrons scattered from various substances by observing proton recoils in a proportional counter. By varying the "bias" of the detector so that only proton recoils of energy greater than a predetermined threshold value were counted, they obtained a rough measure of the energy distribution of the inelastically scattered neutrons.

[12] V. Weisskopf, Phys. Rev. **52**, 295 (1937).
[13] H. L. Bradt and D. J. Tendam, Phys. Rev. **72**, 1117 (1947).

Their results, for the three elements for which they consider them most reliable, are summarized in Table I. The values of E_t, given in the third column, correspond to the proportional counter bias settings at which the measurements were made. From the response curves for the proportional counter used,[14] it is evident that the effective threshold of the detector is somewhat greater than the bias energy—the more so the greater the bias energy. For the purposes of the following discussion, it will be assumed that the detector thresholds are sharp and equal to the values of E_t in Table I.

The values of σ_{in} given in the table were obtained as differences between the total scattering cross section and the cross section as measured by the detector with the bias energy E_t. In computing these differences, the observed angular dependence of the elastically scattered neutrons was taken into account. The inelastically scattered neutrons were assumed to have a spherically symmetrical angular distribution. The values in the table are uncertain by $\sim$0.1 barn.

In the last column of Table I, values of πa^2, computed on the assumption

$$a = 1.5A^{\frac{1}{3}} \times 10^{-13} \text{ cm},$$

have been included.

These data are used to obtain a rough energy distribution for the inelastically scattered neutrons, as follows: For a given incident energy E_0 and threshold energy E_t, σ_{in} (as given in Table I) is,

$$\sigma_{in}(E_t, E_0) = \int_0^{E_t} d\sigma(E, E_0) = \int_0^{E_t} N(E, E_0)dE, \tag{10}$$

where $N(E, E_0) = \sigma_c(E/T^2)e^{-E/T}$, from Eq. (7). The average value of $N(E, E_0)$ over an interval between

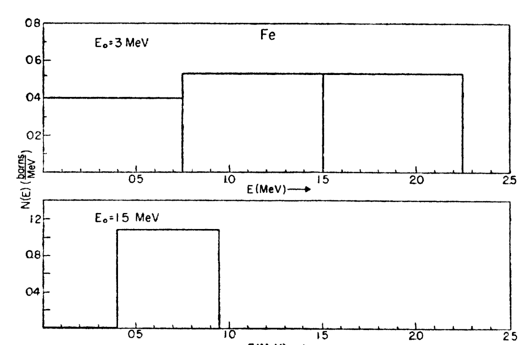

FIG. 2. Observed energy distribution of inelastically scattered neutrons from iron. The experimental results are due to the Los Alamos group.[5] The distributions have been plotted as constant over the energy range between two adjacent thresholds.

[14] Reference 5, Fig. 4.

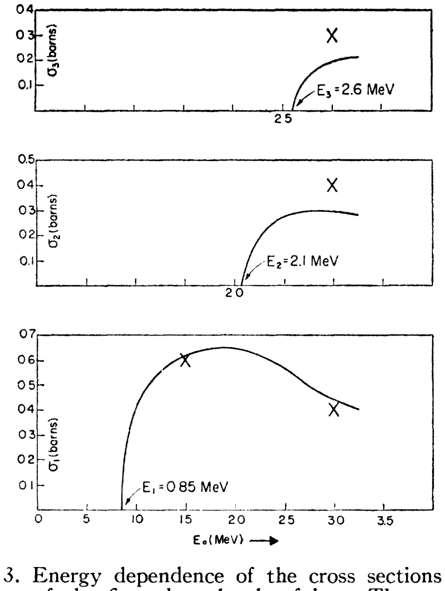

FIG. 3. Energy dependence of the cross sections for the excitation of the first three levels of iron. The crosses are deduced from Table I, assuming only one level in each bias interval. The solid curves are computed on the basis of the simple theory, assuming S-scattering only and equal C for all three levels. The level energies are assumed to be those deduced by Elliott and Deutsch from β-decay experiments.

two threshold values $E_t'>E_t$, is given by

$$\bar{N}=\frac{\int_{E_t}^{E_t'}N(E,E_0)dE}{\int_{E_t}^{E_t'}dE}=\frac{\sigma_{in}(E_t',E_0)-\sigma_{in}(E_t,E_0)}{E_t'-E_t}. \quad (11)$$

1. Interpretation of the Fe Data

The energy distributions corresponding to the data of Table I are shown in Fig. 2. It is immediately evident that these data could not be fitted by the statistical theory, for either of the two incident neutron energies, without assuming a rather large value of T. The data for $E_0=3$ Mev, for instance, would require a $T\sim1.5$ Mev. This value would lead to a level spacing, assuming a Maxwell distribution (Eq. (8)), of $D\sim0.75$ Mev. Such a large level spacing means that only a few levels are available at the incident energies involved; in this case, the statistical theory is not applicable.

It is, therefore, necessary to consider the effects of individual energy levels. While the resolution of the experimental data is certainly not good enough to yield definite energy values for the levels involved, a certain amount of useful information can be inferred, as follows: The $E_0=1.5$ Mev data indicate that there is at least one level in the energy range 0.55–1.1 Mev, and that there are no levels in the range 1.1–1.5 Mev. Assuming this to indicate a level spacing >0.4 Mev, it is reasonable to infer that the entire inelastic scattering for $E_0=1.5$ Mev is probably due to a single level, $E_1\sim0.55$–1.1 Mev, with a cross section $\sigma_1(E_0=1.5)=0.6$ barn. The data at $E_0=3$ Mev indicate one or more levels in the following ranges: 0.75–1.5 Mev, 1.5–2.25 Mev, 2.25–3.0 Mev. Combining the information at the two incident energies, the position of the first level becomes $E_1\sim0.75$–1.1 Mev. Assuming roughly uniform spacing of the low lying levels, it becomes reasonable to conclude that the intervals 1.5–2.25 Mev and 2.25–3.0 Mev contain just one level each. We are thus led to the following level assignments:

$$E_1\simeq0.75\text{–}1.1 \text{ Mev},$$
$$E_2\simeq1.5\text{–}2.25 \text{ Mev},$$
$$E_3\simeq2.25\text{–}3.0 \text{ Mev}.$$

The above conclusions are the best that can reasonably be drawn from the inelastic scattering data. There are, fortunately, other data available on the levels of the Fe nuclei. From a study of the disintegration schemes of Mn^{56} and Co^{56}, Elliott and Deutsch[15] have deduced the following level structure for Fe^{56} (91.6 percent abundant)**:

$$E_1=0.85 \text{ Mev},$$
$$E_2=2.1 \text{ Mev},$$
$$E_3=2.6 \text{ Mev},$$
$$E_4=3.0 \text{ Mev}.$$

Although it is by no means certain that the levels observed in beta-disintegration are in every instance those involved in inelastic scattering, the close correspondence between the results of the two methods makes it at least plausible to assume that they are the same. In the following analysis of the inelastic scattering data, the level scheme of Elliott and Deutsch has been adopted.

In Fig. 3, the measured cross sections for the excitation of these levels (plotted as crosses), as deduced from the data of Table I, are compared with the predictions of the simple theory described in the preceding section. The value of σ_c has been assumed constant and equal to 1.5 barns; this is approximately half of the observed total cross section at these energies.[1] The discontinuities in the slopes of the theoretical cross section curves have been arbitrarily smoothed out in the drawing.

Since the theory assumed (1) S-wave scattering only, (2) constancy of the value of C_i from level to level, and (3) independence of C_i on E_0, and since the experimentally deduced values of σ_i are uncer-

[15] L. G. Elliott and M. Deutsch, Phys. Rev. **64**, 321 (1943).
** They also find a level in Fe^{58} (0.3 percent) at 0.81 Mev; Phys. Rev. **65**, 211 (1944).

tain by $\sim$0.1 barn, the agreement is certainly as good as could be anticipated.

2. Interpretation of the W Data

Figure 4 shows the energy distribution of the inelastically scattered neutrons as deduced from the data in Table I. Comparison with the data on Fe (Fig. 2) indicates the presence of many more low energy neutrons for W a characteristic of the predictions of the statistical theory.

The evidence from β-decay indicates that the lowest levels of the W nuclei are rather closely spaced ($\sim$100 kev).[16] Further evidence is provided by the experiments of Greisen *et al.*,[17] which were designed to observe the inelastic scattering due to the lowest levels. These experiments indicated the presence of a number of low lying levels, the lowest at $\sim$55 kev.

It, therefore, appears to be reasonable to attempt an analysis of the data, plotted as histograms in Fig. 4, on the basis of the statistical theory. The smooth curves of Fig. 4 are obtained, from Eq. (7), by the following procedure: (a) the average energy, $\bar{E}$, for the observed $E_0=3$-Mev distribution, is $\sim$0.85 Mev. To take into account the (unmeasured) interval 2.25–3.0 Mev, the dotted section has been added in Fig. 4a; including this interval, $\bar{E}\cong 1$ Mev. (b) from Eq. (9), $T=\frac{1}{2}\bar{E}\cong 0.5$ Mev, at $E_0=3$ Mev. (c) from Table I, $\sigma_{in}(3\text{ Mev})=2.8$ barns. Increasing this value by the area under the dotted portion of the curve (0.2 barn) and assuming negligible capture elastic scattering, in accordance with the statistical theory, $\sigma_c(3\text{ Mev})=3.0$ barns. This value and the above value of $T=0.5$ Mev have been used to compute the smooth curve in Fig. 4a from Eq. (7). (d) Applying Eq. (8) to the above, $D=83$ kev, in reasonable agreement with expectation. Using this value of D, Eq. (8) gives $T=0.35$ Mev for $E_0=1.5$ Mev. (e) at $E_0=1.5$ Mev, the energy range 0.9–1.5 Mev probably includes a greater fraction of σ_{in} than the unmeasured interval in the previous case. To obtain $\sigma_c(1.5\text{ Mev})$ we make the reasonable assumption that σ_c/σ_t is constant between 1.5 and 3 Mev. Since $\sigma_t(1.5\text{ Mev})=6.3$ barns, $\sigma_t(3\text{ Mev})=5.7$ barns,[1] and $\sigma_c(3\text{ Mev})=3.0$ barns, this gives $\sigma_c(1.5\text{ Mev})=3.3$ barns. The measured cross section for scattering to below 0.9 Mev is 2.1 barns. The unobserved portion (1.2 barns) is plotted as the broken part of the histogram in Fig. 4b. The above σ_c and $T=0.35$ Mev were used in computing the smooth curve of Fig. 4b from Eq. (7).

At both incident energies, the agreement between the statistical theory and the observed energy distribution is reasonably good.

[16] See, for instance, C. L. Peacock and R. G. Wilkinson, Phys. Rev. **74**, 297 (1948).
[17] K. I. Greisen, MDDC-1545, U. S. AEC publication.

3. Interpretation of the Pb Data

The energy distributions, from the data of Table I, are plotted in Fig. 5. Comparison of Fig. 5 with Figs. 2 (Fe) and 4 (W) reveals some similarity to both. While the $E_0=3$ Mev data show the same general trend—decrease of $N(E)$ with increasing E—as the data for W, the effect is considerably less pronounced, and the mean energy (>1 Mev) is intermediate between the Fe and W values. For the $E_0=1.5$ Mev data, on the other hand, the absence of neutrons scattered in the region 0–0.4 Mev (energy loss 1.1–1.5 Mev) is, similar to Fe, a very strong indication that we are dealing with scattering involving only a few energy levels rather than with the type of distribution resulting from many levels and described by the statistical theory.

The value of the measured inelastic scattering cross section adds additional plausibility to the conclusion that only a few levels of Pb are involved in the scattering of neutrons of energies up to 3 Mev. Even allowing a liberal addition for the (unobserved) 2.25–3 Mev range, σ_{in} for $E_0=3$ Mev could hardly be greater than $\sim$2 barns; the observed total cross section at this energy is $\sim$5.5 barns,[1] of which at least $\pi a^2=2.5$ barns should belong to σ_c. Thus, a significant fraction of σ_c must correspond to capture elastic scattering. Appreciable capture elastic scattering is possible only when few levels of the target nucleus are involved.

Proceeding by the same method that was used in the interpretation of the Fe data, we obtain the results summarized in Table II.

Unfortunately, there is no other experimental information available, on the low lying levels of the Pb nuclei, to guide the interpretation beyond this point. The picture is further complicated by the fact that normal Pb consists mainly of three iso-

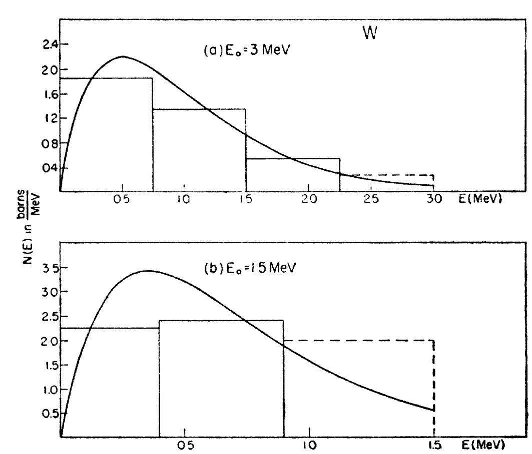

FIG. 4. Energy distribution of inelastically scattered neutrons from tungsten. The experimental curves, from Table I, are plotted as histograms. The smooth curves are derived from the statistical theory.

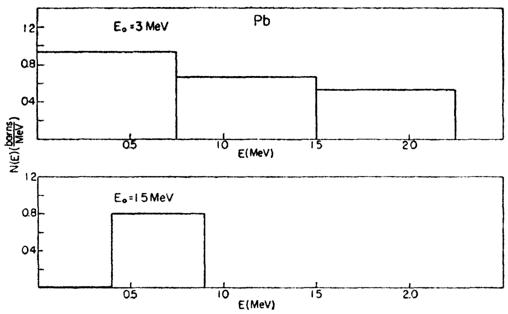

FIG. 5. Energy distribution of inelastically scattered neutrons from lead, from the data in Table I.

topes, roughly half at $A=208$ (82 protons, 126 neutrons), the other half approximately equally divided between $A=207$ and $A=206$. Thus, while the level spacing of each isotope may be large (say, similar to Fe), the observed effect is a result of the superposition of the effects of the three isotopes; it is, therefore, easily possible that more than one level (isotope) is involved in some of the energy regions shown in Table II.

The data of Dunlap and Little[4] are in essential agreement with the above. They measured (with rather poor resolution) the energy distribution of $E_0 \cong 2.5$ Mev neutrons scattered by Pb. While their distribution curve could not be fitted by the statistical theory (as expected if the level spacing is large), it showed a definite peak at $E \sim 1.7$ Mev (a level at ~ 0.8 Mev, see Table II) and a rather flat distribution below this peak, a natural consequence of poor resolution and a number of levels. Their measured cross section, $\sigma_{in}(2.5 \text{ Mev}) = 1.3 \pm 0.5$ barns, is in excellent agreement with the Los Alamos results.

IV. SUMMARY AND DISCUSSION

In the preceding section the experiments of the Los Alamos group have been interpreted in terms of the theory described in Section II. Two extremes have been observed. In the first, exemplified in the inelastic scattering of iron, only a few levels of the target nucleus are involved at the incident neutron energies studied; the energy distribution of the inelastically scattered neutrons should then show a line spectrum, each line corresponding to the excitation of a level of the target nucleus. The experimental evidence is consistent with such a description and with the level scheme suggested by Elliott and Deutsch, as well as with the predictions of the theory concerning the cross sections for the excitation of the various levels.

The second extreme is illustrated by the data on the inelastic scattering by tungsten. In this case, very many levels of the target nucleus are involved in the inelastic scattering of neutrons at the energies investigated. The statistical theory of Weisskopf has been used in the interpretation of these data. An average spacing of the lowest energy levels of tungsten has been deduced (~ 80 kev) which is in agreement with other data.

In the third element considered, lead, the interpretation is less unambiguous. However, the evidence seems to support the conclusion that the behavior of lead, with respect to inelastic scattering, is more similar to that of iron than tungsten. The difficulty of interpretation in terms of a specific level scheme is, at least in part, associated with the fact that there are four lead isotopes, three of which have appreciable and comparable abundance.

It has been pointed out[18] that nuclei containing 20, 50, 82, or 126 protons or neutrons are abnormally stable, compared to neighboring nuclei. This stability has been interpreted as evidence that these combinations of nucleons form tightly bound closed shells. With regard to their interaction with neutrons, such "magic number" nuclei may be expected to behave anomalously in two respects: (1) A nucleus containing a closed shell of neutrons should have an abnormally small binding energy corresponding to the absorption of an additional neutron. (2) A nucleus containing a "magic number" of neutrons, or of protons, or of both should exhibit a comparatively larger level spacing than neighboring nuclei, since the tightly bound nucleons in the closed shells are not expected to participate freely in the general sharing of excitation energy among the nucleons.

These consequences of the closed shell hypothesis are capable of explaining the abnormally small neutron absorption cross sections and the absence of slow neutron resonances observed for Sn($Z=50$), Pb($Z=82$), and Bi($Z=83$, $A-Z=126$).[19] Small neutron absorption and the absence of resonances are both a consequence of a *large level spacing of the compound nucleus* at the excitation energy resulting from the capture of a neutron. Since the Sn isotopes, of which there are ten, are not in the region of a closed shell of neutrons, the large level spacing in this case is likely a consequence of the closed shell of protons in the compound nucleus. The same

TABLE II. Interpretation of the Pb data in terms of individual levels.

Level energy (Mev)	σ_i(barns) $E_0=1.5$ Mev	$E_0=3$ Mev
0.00–0.75	—	—
0.75–1.1	0.4	0.4
1.1 –1.5	0.0	(0)
1.5 –2.25		0.5
2.25–3.0		0.7

[18] W. Elsasser, J. de phys. et rad. **5**, 389 and 625 (1934); M. G. Mayer, Phys. Rev. **74**, 235 (1948).
[19] Reference 1, p. 280.

is probably true of the Pb isotopes, especially since one of the abundant isotopes ($A=207$) lacks one neutron for a closed shell, and therefore should have a rather large binding energy corresponding to the absorption of a neutron. Bi^{209}, on the other hand, already contains a closed shell of neutrons; the binding energy of an additional neutron should therefore be abnormally small. Hence, the excitation energy of the compound nucleus, resulting from the absorption of a neutron, will be small, and the level spacing of the compound nucleus, at this excitation energy, correspondingly large.

The interpretation of inelastic scattering data, on the other hand, involves the *low lying level spacing of the target nucleus*. The data discussed in this paper, as well as other data[3] on the cross sections for the inelastic scattering of fast neutrons by Pb and Bi, provide evidence for anomalously large level spacings of the stable nuclei of these elements. Thus, these data add further confirmation to the hypothesis that 82 protons or 126 neutrons form particularly stable configurations (shells) in nuclei.

The author would like to express his appreciation to many of his colleagues, especially to Professor V. F. Weisskopf and Dr. I. Halpern, for stimulating discussions and criticism of this manuscript.

APPENDIX

In the section on theory, the cross section for the excitation of a given level of the target nucleus has been computed on the assumption that neutrons are *emitted* from the compound nucleus in the S-state ($l=0$) only. The possibility of *absorption* of neutrons in higher angular momentum states was taken into account, since the cross section for the formation of the compound nucleus, σ_c, is assumed to include all possible modes of formation. For a complete description of the inelastic scattering process it is, however, also necessary to take into account the possibility that the emitted neutron can carry away angular momentum.

At the excitation energies (E_0 plus the neutron binding energy) of interest in this discussion, the energy levels of the compound nucleus are very close, or overlapping. Furthermore, the fast neutron sources available have a rather large energy spread compared to the level spacings involved. Thus, on the average, the absorption of an incident fast neutron will result in the excitation of many levels of the compound nucleus.

The excited levels will also have a spread of angular momenta, since incident neutrons of wave-length $\lambda\!\!\!^-$ will appreciably excite all the available levels (of the compound nucleus) whose angular momenta do not differ from that of the ground state (of the target nucleus) by more than $\sim a/\lambda\!\!\!^-$ units.

The probability (width) for the decay of the compound nucleus, to the ith state of the product nucleus, by the emission of a neutron with l units of orbital angular momentum is obtained from FPW, Eq. (46):

$$\Gamma_{il}=C_{il}(E_0-E_i)^{\frac{1}{2}}(1/|v_{il}|^2), \tag{A1}$$

where

$$v_{il}=v_l(x_i), \tag{A2}$$

$$x_i=a/\lambda\!\!\!^-_i. \tag{A3}$$

In the above, $\lambda\!\!\!^-_i$ is the Dirac wave-length of the *emitted* neutron. The values of C_{il} are proportional to the average distance between those excited levels of the compound nucleus which can decay to the ith state by the emission of a neutron carrying l units of orbital angular momentum. Expressions for v_l are given in FPW, Eqs. (44) and (45a). The cross sections, σ_i, are now obtained from Eqs. (3) and (4) of the text, and the relationship

$$\Gamma_i=\Sigma_l\,(2l+1)\Gamma_{il}. \tag{A4}$$

For levels of the target nucleus differing in angular momentum from the ground state by an amount small compared to $a/\lambda\!\!\!^-$ (in units of $\hbar$), the values of C_{il} will not differ much from level to level. Furthermore, since the wave-length of the emitted neutrons is never less than that of the incident neutrons, it follows that if the emission of neutrons of orbital angular momentum l is energetically possible ($|v_{il}|\sim 1$) there will be no lack of appropriate excited levels of the compound nucleus from which such emission can take place. For such levels, the inelastic scattering may be termed "allowed."

For levels of the target nucleus differing in angular momentum from the ground level by an amount large compared to $a/\lambda\!\!\!^-$, the values of Γ_{il} will be small as compared to the previous case. This conclusion follows from the fact that the angular momenta of the excited states of the compound nucleus differ considerably from that of the level under consideration; hence, the decay must involve the emission of neutrons of high orbital angular momentum, which is improbable for small $a/\lambda\!\!\!^-_i(a/\lambda\!\!\!^-_i<a/\lambda\!\!\!^-$, always). For such levels, the inelastic scattering is "forbidden."

From the above arguments, it is clear that there is no sharp transition between allowed and forbidden levels in inelastic scattering. Those levels for which the angular momentum difference from the ground level is $\sim a/\lambda\!\!\!^-$ will comprise an intermediate group, neither fully allowed nor fully forbidden. Furthermore, a level which is forbidden at low incident neutron energies may be allowed at high energies, because of the increase of $a/\lambda\!\!\!^-$. For such levels, the inelastic scattering cross section behavior is quite complicated.

For the allowed levels, the more exact theory differs from that developed in the text because of the energy dependence of the factors v_{il}. The $|v_{il}|^2$ are smoothly varying functions of x_i, decreasing from the value ∞ for $x_i=0$ to 1 for very large x_i (except for $|v_{i0}|\equiv 1$). Thus, for $x_i\gg 1$ it is necessary to take into account many values of l in computing σ_i, or in deriving the energy distribution of the inelastically scattered neutrons by statistical methods.

At the other extreme, $x_i\ll 1$, $|v_{il}|^2\sim x_i^{-2l}$. In this case, $\Gamma_{il}\rightarrow 0$ for all $l>0$, and it is only necessary to take into account emission in the S-state ($l=0$). Thus, the simple theory discussed in the text should adequately describe the shape of the energy dependence of σ_i close to the threshold, $E_0\gtrsim E_i$.