quire a knowledge of the internal dynamics of the participating systems; only their mass energies are required in order to set a rigorous lower limit on the beam energy for single or multiple production of mesons.

The most convenient coordinate frame for the analysis of the threshold is that in which the center of gravity of the colliding systems is at rest. The net momentum then vanishes, and when the kinetic energy of all products also vanishes, the threshold for the reaction leading to these particular products has been reached. Furthermore, the absolute threshold for the production of any particular product, such as a meson, is reached when all other products of the reaction are left in their combined state of lowest energy (often a single system in its ground state). For example, when protons bombard C^{12} the threshold reaction yielding positive mesons is the one simultaneously producing C^{13} in its ground state. Above threshold resonances should be observable in which C^{13} is left in its various excited states, and at any bombarding energy the meson spectrum should show a corresponding structure near the upper energy limit. On the other hand, the threshold production of negative mesons, when neutrons bombard Li^7, yields in addition two alpha-particles of zero kinetic energy.

For a quantitative treatment suppose that in laboratory coordinates a moving system, mass m, of kinetic energy T' bombards a target system, mass M, at rest. The sum of their kinetic energies, T, in the center-of-gravity system is then:

$$(m+M)c^2\{[1+(2MT'/(m+M)^2c^2)]^{\frac{1}{2}}-1\}. \tag{1}$$

For a reaction leading to particular products of mass m_1 and other products of total rest mass M_1 in their ith discrete energy state, we have:

$$(m+M)c^2=(m_1+M_1)c^2+Q_i, \tag{2}$$

as the defining equation for Q_i.

We set $T=-Q_i$ and thus obtain as a general theorem:

$$T_i'=[(m_1+M_1)^2-(m+M)^2]c^2/2M, \tag{3}$$

T_i' being the kinetic energy of the beam in the laboratory system at which the ith resonance occurs.

The absolute thresholds of a number of typical reactions leading to single meson production are given in Table I.

TABLE I. Thresholds for typical reaction leading to single meson production.

Beam	Target	Meson produced	Other products	Threshold Mev
γ	p	Pos.	n	159
γ	C^{12}	Pos.	B^{12}	161
γ	C^{12}	Neg.	N^{12}	165
p	p	Pos.	d	301
p	C^{12}	Pos.	C^{13}	155
p	C^{12}	Neg.	$N^{12}+p$	178
n	p	Neg.	$2p$	301
n	p	Pos.	$2n$	306
n	C^{12}	Neg.	N^{13}	156
n	C^{12}	Pos.	B^{13}	168
d	C^{12}	Pos.	C^{14}	162
d	C^{12}	Neg.	O^{14}	166
α	C^{12}	Neg.	F^{16}	207
α	C^{12}	Pos.	N^{16}	202

Masses of bare nuclei have been used in the calculations, the coupling to electronic systems being quite negligible in the energy range where mesons are produced. The meson mass used was 286 electron masses.

This type of threshold calculation invokes the over-all conservation of momentum and energy. The internal kinetic energies[1] of the reacting systems and the Coulomb barriers will enter only if one undertakes further to estimate cross sections for such reactions. A deficiency of internal kinetic energy, as in the deuteron, may increase the *observable* threshold if mesons can be produced only by the energetic collisions of pairs of nucleons, but lower thresholds than those here calculated cannot be obtained consistent with energy and momentum conservation. However, when one compares the threshold energy (3) with that calculated by the admittedly approximate theory of McMillan and Teller, their result often is considerably *lower*. The discrepancy arises because their formulation is inexact in the following details:

(a) The meson is supposed to be produced by the collision of a beam nucleon with a target nucleon, which at that instant is approaching the beam nucleon. The velocities then compound to raise the available energy. In order for the momentum of the target nucleus to remain zero, however, the remaining nucleons of the target must instaneously have a net momentum in the direction of the beam. The corresponding kinetic energy term, which will usually be small, has been omitted. Furthermore, the velociy of the remainder of the nucleus makes it necessary to calculate the kinetic energy of the product nucleons in the moving reference frame of the assemblage before equating their kinetic energies to the Fermi energy. McMillan and Teller neglected this effect.

(b) Errors are introduced by their use of non-relativistic mechanics.

(c) In the light nuclei, especially, the binding energy[2] of a nucleon to the nucleus may deviate widely from 8 Mev, and as they note the kinetic energy may be poorly approximated by the Fermi energy.

(d) The threshold state is not, as assumed, that in which the meson has zero kinetic energy in the laboratory coordinates, but one in which it has the velocity of the center of gravity of the reacting systems.

The valuable comments of Professor Robert Serber in discussions with the writer on this subject are much appreciated.

[1] W. G. McMillan and E. Teller, Phys. Rev. 72, 1 (1947).
[2] W. H. Barkas, Phys. Rev. 55, 692 (1939), Fig. 1.

Erratum: On the Numerical Calculation of the Internal Conversion in the K-Shell—The Electric Dipole Case

[Phys. Rev. 75, 534 (1949)]

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IN the above letter,

(1) The formula for I_K should read as follows:

$$I_K=\frac{\gamma^{3+2\beta}\{[1+\beta+(1/\theta)]^2-1\}^{-(\frac{3}{2}+\beta)}e^{b\pi}|\Gamma(1+\beta-ib)|^2}{24(137)(2+\beta)\Gamma(3+2\beta)|(-z)^{-(1+\beta-ib)}|^2}\times[2|P|^2+|Q|^2].$$

(2) The explanation of $2I_K$ given in the note below Table I is based on Hulme's simplified model of the system considered. This is now known to be incorrect. Accordingly, the first paragraph of this note should read as follows:

In the above table, $2I_K$ represents the ratio of observed K-electrons to observed photons of energy $h\nu$. No allowance for screening effects has been made.

For further details, see N. F. Mott, and I. N. Sneddon, *Wave Mechanics and Its Applications* (Oxford University Press, London, 1948).

Alpha-Particle Ionization in Argon and in Air and the Range-Energy Curves

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February 17, 1949

WE have recently compared by a method already described[1,2] the relative ionization for individual samarium and polonium alpha-particles in argon and in air. We find the ionization for samarium relative to polonium about 5 percent less in air than in argon. These preliminary measurements would indicate what has long been suspected,[3] that, while in hydrogen and the noble gases the total alpha-ionization is fairly well proportional to the particle energy, air is distinctly anomalous in this respect. Thus, should we assume the above relation fulfilled in each gas, we get for a computed

TABLE I. Derived values of $(n-H^1)$ mass difference.

Range-energy curve	$(n-H^1)$ from $N^{14}(n, p)C^{14}$ (measured range $=0.991$ cm)	$(n-H^1)$ from $He^3(n, p)H^3$ (range $=0.980$ cm)	Mean of $(n-H^1)$ from the two determinations
Uncorrected Bethe-Livingston	0.751 Mev	0.754 Mev	0.753 Mev
Corrected Bethe-Livingston	0.771	0.782	0.777
Parkinson, Herb, Bellamy, Hudson	0.822	0.845	0.833

energy for the Sm alpha (Po = 5.298 Mev) 2.18 Mev from our argon comparison and only 2.07 Mev in air, a distinct discrepancy far above the experimental error.

If air behaves in the anomalous manner which these preliminary measurements would indicate, the effect of such behavior on the alpha-range-energy curves in current use is at once apparent. The energy-range curve derived from the very careful measurements of Holloway and Livingston,[4] for instance, is in reality an ionization in air-range curve. The authors have themselves explicitly stated that the validity of their relative energy values depends upon an assumed proportionality between the alpha-particle energy and the total ionization in air produced by it. If this condition is not fulfilled, as our preliminary measurements would indicate, then corrections to their energy values must be made, particularly in the region of lower alpha-energies.

Such corrections in the region from 1 to 3 Mev we have attempted to make from our measurements above. This results in raising the energy values of the Holloway-Livingston curve in this region by amounts varying from 60 to 130 kev. The maximum correction appears to fall at about 1.5 Mev. This is the region where the present curve is known to give much too low energy values for the measured ranges, as for example in the (n, α) reaction for B^{10}. From a compilation of data by L. H. Gray[3] based on the original ionization measurements of Gurney we have computed corrections to the range-energy curve almost identical with our own. Finally, we have found in the literature a limited number of reactions where alpha-ranges in air have been determined and elsewhere reaction energies independent of range measurements. The plotted points in all such cases are in better agreement with the corrected curve than with the original.

It should be emphasized that our energy values still rest upon the assumption of the proportionality between alpha-energy and alpha-ionization in argon. We have merely assumed that argon is a gas much superior to air in this respect—an assumption we think, well justified by our own experience and that of others.

We have also attempted to correct the proton[5] range-energy curve from the corrected alpha-curve. This would seem logical, since in the region above about 0.3 Mev the proton curve was apparently derived from the alpha-curve by use of the equation $R_H(E)=1.0072R_\alpha(3.971E)-c$ where R and R_H denote, respectively, the corresponding alpha- and proton-ranges and $c=0.20$ cm is an empirical constant introduced by Blackett to account for the difference in behavior of protons and alpha-particles at low energies. Obviously, a change in the alpha-curve necessitates a change in the proton curve. The energies for the corrected Livingston-Bethe proton curve are slightly higher than in the original (about 20 kev at range 1 cm) but still much lower than in the curve derived from the data of Parkinson, Herb, Bellamy, and Hudson.

Perhaps the most interesting aspect at the moment of the corrected proton range-energy curve is the effect it will have upon the energies derived from the proton range determinations by Hughes and Eggler[6] in the (n, p) reactions for N^{14} and He^3 and their bearing upon the now uncertain $(n-H^1)$ mass difference. Table I shows the derived values for $(n-H^1)$ from the three possible range-energy curves. The beta-end points have been taken as 155 kev and 19 kev for C^{14} and H^3, respectively. It will be seen that the use of the corrected curve raises the $(n-H^1)$ value derived from the proton ranges from a value in good agreement with the formerly accepted value of 0.755 Mev to a value 0.777, about halfway between the former value and the new Chalk River[7] value of 0.804 Mev. Probably the greatest uncertainty in the value 0.777 Mev lies in the uncertainty in the empirical Blackett constant c, which uncertainty affects equally the corrected and uncorrected L and B curves. A minor uncertainty may lie in the assumption mentioned above as to the energy-ionization relation in argon for alpha-particles.

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3 L. H. Gray, Proc. Camb. Phil. Soc., **40**, 95 (1944).
4 M. G. Holloway and M. S. Livingston, Phys. Rev. **54**, 18 (1938).
5 M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. **9**, 245 (1937).
6 D. J. Hughes and C. Eggler, Phys. Rev. **73**, 809 (1948).
7 R. E. Bell and L. G. Elliot, Phys. Rev. **74**, 1552 (1948).

Radiative Corrections to the Klein-Nishina-Formula

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IT is well known that the new developments in quantum electrodynamics allow the computation of the radiative corrections to various first-order effects.[1] We have applied these methods to the scattering of light by a charged particle of spin ½.[2] The calculation was first carried through using the older methods of perturbation theory in momentum space. After subtraction of the photon and particle self-energy-operators,[3] one can interpret the remaining divergencies (for high momenta) as a charge renormalization.[4] This charge renormalization must take place in such a fashion that in the extreme non-relativistic limiting case the corrections disappear (compare reference 2). In addition, one obtains integrals which diverge for small photon momenta, and which in a known way are compensated by the scattering cross section for the double Compton effect.[5] The formulas can then be transformed into x space where they are seen to be gauge and Lorentz invariant. Since these formulas are very long, we reserve their publication for a later detailed paper. The evaluation leads to the same uniqueness difficulties as in the case of the Lamb-Retherford shift and the anomalous magnetic moment of the electron.[6] The investigation of these difficulties is now under way. The detailed calculation for the non-relativistic limiting case (that is, limiting our results to terms $\propto k^2 \log k$, but neglecting terms $\propto k^2$ where k is the momentum of the photon in the rest system of the particle) gives for the correction to the differential cross section:[7]

$$d\sigma_6=(r_0^2/137)(d\Omega/4\pi)(k/m)^2\{[1+\cos^2\vartheta][2\log(k/m)+(8/3)\log(\omega/m)]-(8/3)[1+\cos^2\vartheta]\cos\vartheta\log(\omega/m)-(4/3)[1-\cos^2\vartheta]\cos\vartheta\log(k/m)\},$$

where k is the momentum of the photon, m the mass of the electron, r_0 the classical electron radius,[8] ϑ the scattering angle, $d\Omega$ the solid angle element, and ω the cut-off radius of the double Compton effect. Thus $d\sigma_6$ is the cross section for a process in which, in addition to the scattered photon (ϑ), at most one of momentum $\leqq\omega$ is emitted.

I am very grateful to Professor Pauli for his kind interest in this work and also to Dr. Res Jost, who suggested this