metallic crystal having a cell shape and symmetry departing widely from that of a sphere.

The writer has essentially improved and extended the method by using and generalizing Von der Lage and Bethe's Kubic Harmonics.* The points where the present method differs from the previous ones are as follows: (1) The electron-potentials have been assumed to have the crystal symmetry within a lattice cell. While the Schroedinger equation is then not separable, the eigenfunction can still be expanded in terms which are products of spherical solid harmonics and solutions to a set of simultaneous radial equations.

As in Von der Lage and Bethe's treatment, eigenfunctions belonging to special wave vectors are constructed by suitable linear combinations of spherical harmonics so as to satisfy symmetry requirements. (2) Boundary conditions have been satisfied more exactly than in their treatment by using a different average fitting and, in consequence, the arbitrariness in choosing the fitting points on the cell surface is removed. (3) Essentially different features of the present treatment from preceding ones, such as Shockley's, Ewing and Seitz's and Tibbs', result from the electron-potential adopted. Starting from the picture of point charges, the classical Madelung's field in ionic crystals belonging to the cubic system has been obtained in terms of Kubic Harmonics. This has further been modified by superposing the Hartree fields of nearest neighboring ions. The resulting potential is naturally continuous throughout the crystal except at the lattice points and has the full cubic symmetry. (4) Lastly, reasonable joining of the wave functions at the cell boundaries has been achieved in virtue of this potential. Thus, the only assumptions made in our article are that in the ionic crystal the inner shells of ions are not altered from their form in the free ions and that the contribution of nearest neighboring ions to the potential is given by superposition of the Hartree fields arising from them. The method is being applied to NaCl and CsCl to find the ground state of the valence-electron in both crystals and of the additional electron in rock salt. The results will be published shortly.

* F. C. Von der Lage and H. A. Bethe, Phys. Rev. 71, 612 (1947).

Approximate Densities of Liquid He³ between 1.27° and 2.79°K*

E. R. GRILLY, E. F. HAMMEL, AND S. G. SYDORIAK
Los Alamos Scientific Laboratory, Los Alamos, New Mexico
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THE pure He³ used in these experiments was the same as used in vapor pressure measurements.[1] The principal apparatus consisted of a gas buret-manometer combination connected to a narrow soft glass tube (hereafter called "the capillary") in which the He³ was liquefied. Around the capillary was a bath of liquid He⁴, then liquid air, both Dewars having unsilvered strips. The capillary dimensions were: 1.656-mm I.D., 2.95-mm O.D., 90 mm long. It was fused to another glass capillary (0.687-mm I.D., 4.83-mm O.D.) which connected to the buret. Although this thick-walled section introduced an undesirable temperature gradient, it was necessary in order to conserve volume and yet provide sufficient rigidity. The liquid levels were observed by means of a cathetometer, suitable illumination being obtained from a 2-watt neon lamp on the opposite side.

Liquefied He³ was observed in the capillary to be clear and colorless. The meniscus was sharp, and moved markedly when the capillary was tapped. At one temperature, 1.7°, sudden reduction of He³ pressure caused a violent boiling[2] similar to that in He⁴ (I).

During measurements the capillary immersion in the bath was kept constant by raising the Dewars. The procedure used follows: He³ was liquefied in the capillary to a height of

several mm, then most of this was evaporated by loweringthe mercury in the buret. Since the evaporated liquid was replaced by vapor at the same temperature, the data gave the difference between liquid and vapor densities:

$$\rho_L - \rho_v = \frac{3.02 \times 273 P \Delta V_{\mathrm{bur}}}{22400 \times 760 T_{\mathrm{bur}} \Delta V_L},$$

where ΔV_{bur} = buret volume change at P(mm Hg) and T_{bur}°K. And ΔV_L = liquid volume change due to change in liquid height. The results are given in Table I along with

TABLE I. Differences between liquid and vapor densities of He³.

T	1.27	1.73	2.26	2.52	2.79°K
$\rho_L - \rho_v$	0.0783	0.0781	0.0698	0.0661	0.0542 g/cc
Uncertainty	±2	±3	±2.5	±2.5	±5%

estimated uncertainties in ΔV_L, by far the most significant. After completion of the measurements, an examination of the Dewar windows showed that refractive errors varied considerably with the height of observation.

In the absence of saturated vapor density data, absolute values of liquid densities were estimated, using the law of rectilinear diameters. For both He³ and He⁴, critical densities ρ_c were calculated from critical pressures and temperatures[3] by means of the Dieterici and van der Waals equations of state. For He⁴ ρ_c from the law of rectilinear diameters, using vapor and liquid densities measured at Leiden,[4,5] is about 0.4 of the way between Dieterici and van der Waals values of ρ_c (see Table II and Fig. 1). Assuming a corresponding situa-

TABLE II. Critical densities of He³ and He⁴ by various methods.

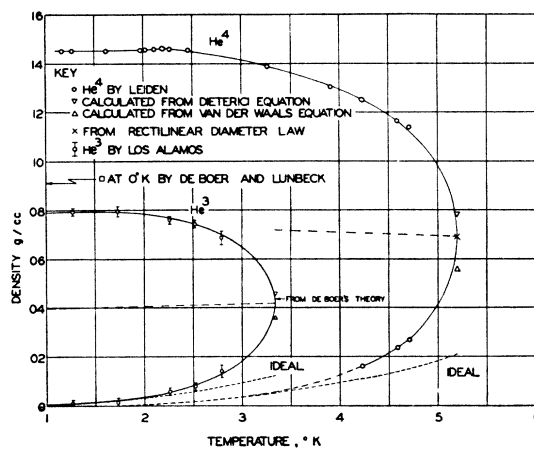|	Dieterici	van der Waals	Rectilinear diameters	De Boer
He³	0.0460	0.0360		0.041–0.046
He⁴	0.0780	0.0564	0.0693	

FIG. 1. Density of helium vs. temperature.

tion for He³, $\rho_c = 0.0420$ was taken to fix one point of the mean density line. The other point used was taken from our data at 1.27°, where the vapor density is small and close to ideal:

$$\tfrac{1}{2}(\rho_L - \rho_v) + (MP/RT) = 0.0401.$$

Our data were then plotted so that $\rho_L - \rho_v$ was divided equally above and below the mean density. The resulting vapor and

TABLE III. Smoothed densities of liquid and vapor He³.

T	1.0	1.5	2.0	2.5	2.8	3.1	3.34°K
Liquid	0.0792	0.0792	0.0784	0.0744	0.0700	0.0618	0.0420 g/cc
Vapor	0.0005	0.0016	0.0036	0.0084	0.0132	0.0220	0.0420 g/cc

liquid density curves are shown in Fig. 1 and the smoothed values in Table III. The density for condensed He^3 at $0°K$ was calculated by De Boer and Lunbeck[6] to be 0.091; and ρ_c was calculated by us, from De Boer's theory,[7] to be in the range 0.041–0.046 g/cc.

Using $\Delta H_{vap} = T\Delta V(dP/dT)$ with our density and vapor pressure[1] data, heats of vaporization as a function of temperature were calculated and are presented in Fig. 2. The reliability of the results is insufficient for establishment of anything but
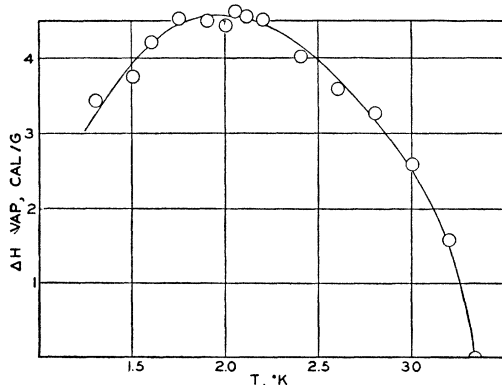

FIG. 2. Heat of vaporization for He^3 vs. temperature.

a smooth curve which has a maximum of 4.5 cal./g between 1.7° and 2.2°.

It is recognized that greater precision is imperative for establishing a lambda-transition such as exists for He^4. To improve density and vapor pressure measurements[8] and to reach lower temperatures, we are making major design changes in our apparatus.

We acknowledge helpful discussions with Dr. F. G. Brickwedde and Dr. J. E. Kilpatrick.

* This document is based on work performed at the Los Alamos Scientific Laboratory of the University of California under government contract W-7405-Eng-36.
[1] S. G. Sydoriak, E. R. Grilly, and E. F. Hammel, Phys. Rev. **75**, 303 (1949).
[2] Such observations at various temperatures might be useful for detecting a phase II in He^3.
[3] For He^4, $P_c = 1718$ mm Hg and $T_c = 5.20°K$ were taken from W. H. Keesom, *Helium* (Elsevier Publishing Company, Inc., New York, 1942), p. 191. For He^3, $P_c = 875$ mm Hg and $T_c = 3.34°K$ were taken from reference 1.
[4] H. Kamerlingh Onnes and J. D. A. Boks, Commun. Leiden, No. 170b (1924).
[5] Mathias, Crommelin, Kamerlingh Onnes, and Swallow, Commun. Leiden, No. 172b (1925).
[6] De Boer and R. J. Lunbeck, Physica **XIV**, No. 7 (September, 1948).
[7] J. De Boer, Physica **XIV**, 139–148 (1948).
[8] In reference 1, we noted the vapor pressure excess in the stainless steel capillary when it contained He^4. We have obtained similar results using the present glass capillary. This excess was independent of capillary immersion depth and was the same whether the surrounding Dewars were wholly silvered or strip-silvered.

Space-Charge Wave Amplification Effects

V. A. BAILEY

School of Physics, University of Sydney, Sydney, Australia
February 21, 1949

IN a letter under the title given above,[1] which has just come to my notice, A. V. Haeff reports on "an entirely new method (of amplification of microwave energy) based on space-charge amplification effects occurring as a result of interaction between streams of charged particles" and states that "this mechanism of interaction between particles and the associated space-charge waves plays an important part in many natural phenomena" such as bursts of solar noise associated with sunspot activity and the excess noise in electron-beam tubes.

The theoretical and experimental results obtained by him are clearly of wide interest, and it is, therefore, of some importance to consider how they are related to certain other work on these topics.

Haeff considers first-order perturbations of the form

$$V = V_0 \exp(\Gamma z + j\omega t),$$

where Γ is the propagation constant corresponding to a frequency ω, in a medium composed of k streams of particles, each of charge e and mass m, of which the typical ith component stream has a particle density ρ_i and velocity v_i (presumably directed along the z axis). He states that Γ is determined by the following equation:

$$\Sigma_1^k \omega_i^2 / [(\omega + j\Gamma v_i)^2] = 1, \tag{1}$$

where

$$\omega_i^2 = e/m \cdot \rho_i/\epsilon.$$

In this way, for the special case $k = 2$, and $\omega_1 = \omega_2$, he finds that spatial amplification occurs over a limited range of values of the factor $\omega(v_1 - v_2)/\omega_1(v_1 + v_2)$. Apparently, the corresponding field wave has no magnetic component, no transverse electric component, and no associated Poynting flux.

This mechanism may be compared with two other mechanisms of plane wave-amplification which were published about seven months previously by J. R. Pierce[2] and V. A. Bailey,[3] respectively.

In Pierce's theory of fluctuations in a stream of electrons, he considers the perturbations which can arise through the presence of a second stream of charged particles, namely, positive ions, and obtains an equation for determining Γ which is formally very similar to Eq. (1). This theory yielded plane wave-amplification even when the stream velocity of the positive ions was neglected.

Bailey's theory of spontaneous waves in discharge tubes and in the solar and other atmospheres is based on the general investigation of plane waves in an ionized gas which is pervaded by static electric and magnetic fields $\mathbf{E}_0$ and $\mathbf{H}_0$, respectively. It was found that, in general, over several ranges of frequency, the ionized medium acts as an amplifier for one or more of the eight possible types of wave, of a given (real) frequency, which can travel through it, and that through this mode of wave-amplification almost any initial random fluctuation can grow into electric noise. The presence of a static electric field was found to be essential and that of a static magnetic field to be favorable to such amplification. The fundamental equations of this theory (when the vibrations of the positive ions are neglected) were given in a paper read to the Royal Society of New South Wales on June 2nd, 1948, and published in its Journal.[4] It was there shown that the (complex) wave number* l is related to ω by a dispersion equation equivalent to $X(Y^2 - \Omega_1^2 Z^2) + U_T^2 l^2 [Y(R - i\nu) - \Omega_1^2 Z]$

$$- RY\Omega_T^2 Z - 2lRZ\Omega_1 \mathbf{\Omega}_T \cdot \mathbf{U}_T = 0 \tag{2}$$

where

$X = R^2 - \tau l^2 - 1 - i\nu R, \quad Y = RZ + R - i\nu Z,$
$Z = l^2 - \omega^2, \quad R = \omega - U_1 l,$
$\mathbf{U}_0 =$ the mean drift velocity of the electrons,
$\mathbf{\Omega}_0 = -\mathbf{H}_0 e/mc,$ (using Gaussian units),
$\nu =$ the collision-frequency of an electron with molecules,
$\tau =$ one-third of the mean square velocity of agitation of the electrons,

the subscripts 1 and T, respectively, denote components along, and transverse to, $0z$, the direction of wave-propagation, and the units of velocity $\mathbf{U}_0$ and frequency ω, are taken as equal, respectively, to c, the velocity of light and p, the electron plasma frequency.

It can be deduced from Eq. (2) that, when ν is not too large, wave-amplification exists in certain frequency-bands under any one of the following three simple conditions:

1. $\mathbf{U}_0$ and $\mathbf{H}_0$ are parallel to $0z$ and $\tau = 0$.
2. $\mathbf{U}_0$ is oblique to $0z$, $\mathbf{H}_0 = 0$ and $\tau = 0$.
3. $\mathbf{U}_0$ is parallel to $0z$, $\tau > U_0^2$ (approx.) and $\mathbf{H}_0 = 0$.