electrons in the air showers, changes strongly with distance from the shower core. Hence the experimental numbers of high-energy cascade particles are strongly weighted averages and should be considered only approximate in absolute value.

The authors are grateful to the Research Corporation for a grant which covered the expense of performing the experiment described above. The cost of constructing the apparatus was provided through a Navy contract. The facilities of the Inter-University High Altitude Laboratories, and the help of Professors Cohn and Iona of Denver University, were an invaluable aid in performing the experiment. The authors thank Mr. G. Branch for assistance in the measurements made in Ithaca.

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Range of Cascade Showers in Lead

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A calculation has been made of the probability of a cascade particle being detected under large thicknesses of lead, taking into account the effect of the low energy photons produced in the lead. The low energy photons increase the probability of detection very greatly, and make the concept of "range" of a cascade shower in lead very indeterminate. The results have been applied to the case of large air showers, where general agreement is found between the calculated and experimental counting rates under large thicknesses.

In this paper, an attempt is made to calculate the expected counting rate in a G-M counter under a large thickness of lead, when a known spectrum of photons and electrons is incident on the lead. The solution is of importance in the interpretation of many cosmic-ray experiments, and may be useful in work with high energy synchrotron beams.

If we express the incident spectrum $f(W)$ as the number of particles per logarithmic interval of energy W, the counting rate C is given by

$$C(T)=\int_0^\infty f(W)P(W,T)d(\log W) \tag{1}$$

where $P(W, T)$ is the probability of obtaining a count under T radiation lengths of lead when a particle of energy W strikes the lead. Thus the problem is reduced to a computation of $P(W, T)$.

In this paper, we consider only large values of T, greater than about 20 radiation lengths (4 inches of Pb); and the probabilities P have been evaluated only for the case of photons striking the lead. Indeed, for energies W large compared with the critical energy of lead, the result does not depend strongly on whether the incident particle is a positron, electron or photon: while if W is less than the critical energy, the probabilities P are large only for incident photons. Also, the number of low energy photons is large compared with the number of low energy electrons, both in cosmic rays and in a beam emerging from a synchrotron.

Solutions have already been computed, according to the cascade theory, for the function $\pi(W, O, T)$, which is the average number of electrons above zero energy at a depth T in a cascade initiated by a photon of energy W. In first approximation, then, our solution for P may be written

$$P_0=1-e^{-\pi}. \tag{2}$$

This formula assumes a Poisson form for the fluctuations in the number of shower particles, which is admittedly an underestimate of the fluctuations in the case $\pi\gg 1$. But in this case P is large anyway, and the error is not serious. For $\pi<1$, the physical explanation for the long tail of the shower curve is that one of the photons may survive beyond the depth where the rest of the shower is practically exhausted, and release an electron in the neighborhood of T. This accounts for the shape of the tail, which is approximately $\pi\sim e^{-\sigma T}$, σ being the nearly constant absorption coefficient of high energy photons. In this case, which is the one of importance in the present calculations, Eq. (2) represents the fluctuations correctly.

Our problem would be completely solved by Eqs. (1) and (2) if the quantity π had been completely and correctly evaluated. This has not been done, however, for an arbitrary depth T in the shower, but only for the thickness corresponding to the maximum, or integrated over the shower length. The solutions which have been given for arbitrary T have taken the collision loss of electrons into account, but have ignored the variation of the absorption coefficient of the photons with their energy. This simplification is not serious for materials of low atomic number, in which the absorption coefficient is never much less than its asymptotic value. It is very bad, however, at large depths

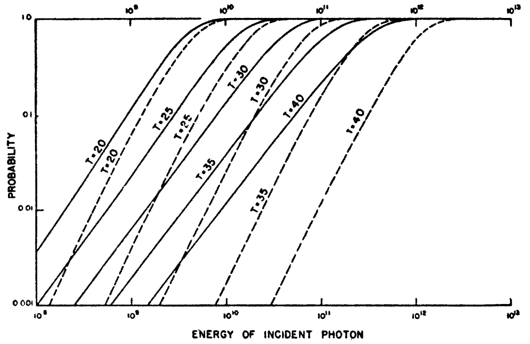

FIG. 1. Probability of detecting an incident photon by a G-M counter under various thicknesses of lead, measured in radiation units (5 rad. units = 1 inch). Dashed curves are calculated by ignoring the low energy photons produced. Solid curves include effect of low energy photons.

in materials of high atomic number like lead, in which low energy photons have a very long mean free path. But this is just the material which is most frequently used in shielding against electronic radiation.

For lead, we take Eq. (2) for P_0 as representing the contribution to P from all components of the shower except the low energy photons. The function π, calculated by Snyder and Serber, is quoted from the survey article by Rossi and Greisen,[1] whose terminology we follow. π is expressed by two equations, with a parameter s:

$$\pi(W, O, T)=\frac{1}{(2\pi s)^{\frac{1}{2}}}\frac{M\cdot K}{[\lambda_1''T+(1/2s^2)]^{\frac{1}{2}}}\left(\frac{W}{\epsilon}\right)^s e^{\lambda_1 t}$$
$$\log(W/\epsilon)=(1/2s)-\lambda_1'T. \tag{3}$$

The probabilities P_0 calculated with this function are shown by the dashed curves in Fig. 1.

As low energy photons in lead, we consider the group between 1.2 and 7 Mev, which have an approximately constant absorption coefficient of about 0.23 per radiation length (see Fig. 13a, reference 1). The precise width of the group is not important, since it will enter only logarithmically in the result. Photons of energy below 1.2 Mev have a rapidly increasing absorption coefficient, and a decreasing probability of detection even if they reach the counters, so the low energy cut-off is rather well determined. Photons of energy above 7 Mev have a rapidly increasing absorption coefficient also, and moreover, their effects are already at least partially included in the probability P_0. Between 1.2 and 7 Mev, the absorption coefficient is nearly constant; the number of photons is distributed as dW/W and the efficiency of detection is proportional to W; hence all parts of the interval are about equally effective. For this group, we approximate the absorption coefficient as being exactly constant (0.23), and the efficiency of detection as constant also at 0.020. The latter figure assumes copper or brass counter walls, since the low energy photons are detected mainly by producing Compton electrons in the walls of the counters. The value of the efficiency is taken from the work of G-A. Renard,[2] and is actually the efficiency for photons of 3 Mev.

In good approximation, the absorption of a low energy gamma-ray simply removes it from the shower, and does not lead to further production of photons in the same energy interval because the electrons produced are below the critical energy and are quickly stopped by ionization. In fair approximation, the probability of production of a low energy photon of energy W to $W+dW$ is dW/W per radiation length per electron of energy above 7 Mev (the critical energy in Pb), and zero for electrons of lower energy. Hence, the photons between 1.2 and 7 Mev, produced in dt at depth t of a cascade, that survive to depth T are

$$dN_\gamma=\log(7/1.2)e^{-.23(T-t)}\pi(W, \epsilon, t)dt,$$

and the total number of low energy photons that reach T is

$$N_\gamma(W, T)=1.8e^{-.23T}\int_0^T e^{.23t}\pi(W, \epsilon, t)dt.$$

We consider only values of T far beyond the maximum of the cascade shower, therefore the upper limit of the integral can be taken as infinite, and the integral is then the Laplace transform of π with parameter $\lambda=-0.23$,

$$N_\gamma(W, T)=1.8e^{-.23T}\mathfrak{L}_\pi(W, \epsilon, -0.23). \tag{4}$$

The Laplace transform has also been given in reference 1, calculated with consideration of the ionization loss, under approximations that are good for energies above ϵ. The parametric equations are

$$\mathfrak{L}_\pi(W, \epsilon, -.23)=-\frac{1}{s}\frac{B}{(\lambda_1-\lambda_2)\lambda_1'}\left(\frac{W}{\rho_1\epsilon}\right)^s.$$
$$\lambda_1(s)=-.23 \tag{5}$$

Insertion of the numerical values of the constants yields

$$N_\gamma(W, T)=1.0e^{-.23T}(W/12)^{1.29} \tag{6}$$

with W expressed in Mev.

The probability of a counter being discharged by one of the N_γ photons is

$$P_\gamma=1-e^{-.02N_\gamma}, \tag{7}$$

and the total probability that a counter be discharged when a photon of energy W strikes the lead is

$$P=P_0+(1-P_0)P_\gamma. \tag{8}$$

[1] B. Rossi and K. Greisen, Rev. Mod. Phys. **13**, 240 (1941).

[2] G-A. Renard, J. de phys. et rad. 9, 212 (1948).

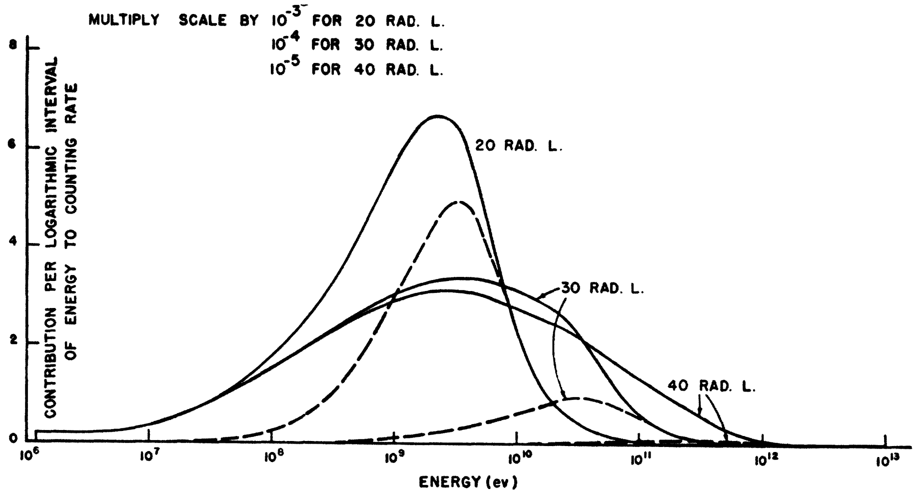

FIG. 2. Contribution to counting rate under various thicknesses of lead made by incident photons and electrons of various energies in an extensive air shower of primary energy 5×10^{13} ev, at a depth of 17 radiation lengths. Dashed curves ignore effect of low energy photons generated in the lead. Solid curves include effect of low energy photons.

In Fig. 1, P has been plotted as a function of W for $T=20$, 25, 30, 35 and 40 radiation lengths (4, 5, 6, 7 and 8 inches Pb). The values of P may be contrasted with the values of P_0, which appear as dashed curves on the same graph. The difference is not strong for a lead thickness of 4 inches, but increases with thickness and is very great at 8 inches. The probability of recording an incident photon of moderate energy (say 2.10^9 ev) is observed to decrease much more slowly with T than would be predicted if the low energy photons were ignored.

These results have been applied in the case of the extensive showers observed at 3260 meters elevation with three or four unshielded counters of 1500 cm^2 area in coincidence. Most of the showers recorded are slightly beyond their maximum development, considering *all* the electrons in the shower; i.e., $s\approx1.2$ in the function $\pi(E_0, O, t)$ describing the shower in the air. Most of the showers recorded probably originate about 17 radiation lengths (610 g/cm^2) above the apparatus. By making use of the formulas in reference 1, we find therefore that a typical initiating energy for the recorded air showers is 5.10^{13} ev. For the spectrum of the air showers we have, therefore, computed the spectrum of photons and electrons at 17 radiation lengths depth, in a shower of 5.10^{13} ev primary energy.* Down to energies of about 2.10^8, the formulas in reference 1 have been used for deducing the spectrum. Below 2.10^8, the spectrum calculated by Richards and Nordheim[3] has been used. This should be accurate, because the shape of the low energy end of the shower spectrum does not change greatly with depth near the maximum of the shower.

The spectrum has been expressed as the number of particles (photons and electrons) per logarithmic interval of energy, relative to the total number of electrons in the shower at the same depth. The total number of electrons, for the typical shower assumed above, is 3.5×10^4. When the spectrum is expressed in this way, it is rather insensitive to the precise value assumed for the typical primary energy.

If the spectrum of the shower particles is multiplied by the probability curves of Fig. 1, graphs like those shown in Fig. 2 result.** Following Eq.

TABLE I. Comparison of experimental and calculated values of the part of R due to soft component in extensive showers. $R(T)$ is the number of particles detected under thickness T of lead, relative to the total number of electrons in the showers.

Lead thickness (inches)	Experimental value of R	Calculated value considering low energy photons	Calculated value ignoring low energy photons
	$10^{-3}\times$	$10^{-3}\times$	$10^{-3}\times$
4	31–33	25.0	14.0
5	9–13	6.1	2.2
6	3.0–4.5	2.1	0.30
7	0.5–2.0	0.65	0.036
8	0.0–0.5	0.21	0.0036

* Occasionally, the "normal spectrum" or spectrum at the maximum of a shower has been treated. It should be noted that no shower ever has the normal spectrum at any fixed depth, because the depth of the maximum is different for different energies of the secondary particles. The "normal spectrum" of a shower is of the form dW/W^{s+1} with constant s. The actual spectrum of a shower at a fixed depth is of the same form, except that s is not constant; instead, s increases logarithmically with W.

[3] J. A. Richards, Jr. and L. W. Nordheim, Phys. Rev. **74**, 1106 (1948).

** Below 10^8 ev, the Laplace transforms used to compute P were not calculable. For energies from 1.2 to 7 Mev, we ignored the electrons striking the lead and considered that the photons were equivalent to photons of the same energy generated in the lead at depth $t=0$. Only the exponential absorption, $\exp(-0.23T)$, and the efficiency of detection by the counters had to be considered for these photons. Between 7 Mev and 100 Mev, we joined the two parts of the curves smoothly. The area under the curves below 10^8 ev was small compared with the total area, hence slight errors in the smooth joining of the curves have a negligible effect. The contribution below 1.2 Mev goes very rapidly to zero because of the increase in absorption coefficient of the gamma-rays.

(1), the area under such a graph represents the number of shower particles detected under thickness T of absorber, relative to the total number of electrons in the showers. This quantity corresponds to the quantity $R(T)$ measured at 3260 meters elevation by G. Cocconi, V. T. Cocconi, and the author (see accompanying paper). We list for comparison in Table I, (a) the experimental values of the part of $R(T)$ that is due to electrons and photons, (b) the corresponding values of R given by the areas under the solid curves in Fig. 2 (and other similar graphs), and (c) the values of R which would be deduced if the effect of the low energy photons were not considered; i.e., if the probabilities P_0 were used instead of P.

Considering the approximations made both in the present calculations and in the experimental determination of the part of R due to soft component, the agreement between (a) and (b) in Table I is very good. If the low energy photons were not considered, however, it would seem that some of the shower particles were much too penetrating to be photons and electrons. Taking into account the effect of the low energy photons removes the necessity for imagining the existence of new particles.

It is apparent from Fig. 2 that when one increases the absorber thickness beyond about 5 or 6 inches (25 or 30 radiation lengths), one does not detect incident cascade particles of higher average energy. Indeed, beyond 6 inches of lead, practically the only cascade particles detected are the low energy photons that have a slow exponential absorption (a factor 10 in two inches Pb), and may equally well originate from low energy particles as from high energy particles striking the lead.

Note added in proof: It has been pointed out to us that our neglect of post-Compton photons has led us to use too *large* an effective absorption coefficient for the low-energy gamma-rays. Thus we have *underestimated* the effect of the low-energy gamma-rays under large thicknesses of lead. From the work of Hirschfelder *et al.* [Phys. Rev. **73**, 852 and 863 (1948)], we find that an effective absorption coefficient of 0.19 per radiation length is better than the value 0.23 which we have used. Applying this change, Eq. (6) becomes

$$N_\gamma(W,T)=0.9e^{-.19T}(W/12)^{1.23},$$

from which the corrected values of P_γ and P (Eq. (7) and (8)) may be calculated.

The qualitative conclusions in the above article are not affected by this change. Applying it to the air showers, the numbers in column 3 of Table I become $10^{-3}\times 31.5$, 11.0, 4.2, 1.7 and 0.68 for the calculated values of R taking into account the low-energy photons. The agreement with the experimental values is even improved by the present correction. However, such excellent agreement must be regarded as somewhat fortuitous, because the method of calculation is such that the results are only expected to be accurate within a factor of about 2.

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Some Properties of the Cosmic-Ray Ionizing Particles That Generate Penetrating Showers

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The mean free path of the ionizing particles that produce penetrating showers has been measured in various materials at 260 and 3260 m above sea level. It has been found that in both places the mean free path is much larger in Pb than in C, and that it increases with the thickness of the absorbers. Possible interpretations are discussed.

THE experiments described below have been performed in order to study the total cross section in different materials of the ionizing particles of the cosmic radiation which produce penetrating showers. The information thus far acquired concerning such phenomena seems to indicate that the shower-producing particles are likely protons and that penetrating showers are more frequently produced in materials with low atomic number.[1]

EXPERIMENTAL APPARATUS AND RESULTS

The apparatus used is drawn in Fig. 1. The G-M counters were of all-metal type, filled with alcohol-argon mixture, 1″×16″ effective surface, and brass walls 0.5 mm thick. The large surface A, 16″×20″ in area, was realized with 20 counters connected in parallel through the crystal diode mixing circuit described in another paper in this issue.[2] Counters B, C, and D constitute three other groups of counters, each group consisting of three counters

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[1] G. Cocconi and K. Greisen, Phys. Rev. **74**, 62 (1948); H. A. Meyer, G. Schwachheim, and A. Wataghin, Phys. Rev. **74**, 846 (1948).

[2] G. Cocconi and V. Cocconi Tongiorgi, Phys. Rev. **75**, 1058 (1949).