and we do not believe that the discrepancy is serious.

In concluding, it should be pointed out that the experiments were essentially qualitative, designed to test the hypothesis that we were observing the Barkhausen effect, and were not intended as careful studies. In making careful studies, somewhat different techniques would be desirable. We believe that the experiments prove the existence of the ferro-electric Barkhausen effect fairly conclusively, and that they further suggest useful applications of the effect in studying ferro-electricity, particularly in barium titanate. We believe also that the three important measurements for one crystal: size of Barkhausen region 10^{-9} cm^3, total volume which changes orientation discontinuously ≈ 0.4 percent of crystal (at 2000 volts/cm), and velocity of propagation $\approx 10^3$ cm/sec., are correct as to order of magnitude.

We do not believe that the photographs in Fig. 1 of de Bretteville's letter[6] support the conclusion that he was observing Barkhausen jumps. They seem to us more consistent with the hypothesis that his crystal contained regions of different coercive forces. Further, our data do not support his remark that ferro-electric domains are larger than ferro-magnetic ones. In the crystal which we used for most of our tests, the volume of a domain was certainly less than 10^{-3} cm^3, while ferromagnetic domains are frequently as large as 10^{-3} cm^3 or more.

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A Calculation of the Changes in the Conductivity of Metals Produced by Cold-Work*

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The increase in the electrical resistance of severely cold-worked metals has been calculated by assuming that the important change which occurs during cold-work is the introduction of large numbers of Taylor dislocations. The results obtained are as follows: The calculated increase in the electrical resistance of polycrystalline copper is in good agreement with the measured value. In case single crystals are considered, the calculations show that there is a decided dependence of the dislocation resistance on the orientation of the electric field relative to the crystallographic axes. In copper the ratio of the largest dislocation resistance to the smallest is 8. This ratio is large for materials having a small Poisson's ratio. It is found that if the dislocations are not too closely clustered no interference effects will occur. The detailed calculations for copper assume that clustering is unimportant. Measurements on single crystals are in progress.

I. INTRODUCTION

IT is found experimentally that if a metal is severely cold-worked its electrical resistance increases by several percent. The available data are summarized in Table I.[1] In the table $\Delta\rho$ is the change in the electrical resistance produced by cold-work, and ρ is the electrical resistance of the annealed metal at room temperature. It should be noted that except for the data on tungsten the specimens used were polycrystals of undetermined purity.

It is the aim of this paper to calculate theoretically the magnitude of this effect and to calculate the dependence of $\Delta\rho$ upon the orientation of the electric field with respect to the various crystallographic axes in a single crystal. The calculation will be made by assuming that the important change which occurs during cold-work is the introduction of a large number of dislocations[2] of the type shown in Fig. 1. It will

* This research was supported by Contract N6ori-47 with the Office of Naval Research. Some of the work was done while the author was a Westinghouse Research Fellow in 1941.

[1] E. Schmid and W. Boas, *Kristallplastizität* (Verlag Julius Springer, Berlin, 1935), p. 214.

[2] F. Seitz, *The Physics of Metals* (McGraw-Hill Book Company, Inc., 1943), p. 88, references to original work are given.

also be assumed that the standard quantum-mechanical methods of calculating resistivities are applicable to this problem.

An experimental investigation of the increase in the electrical resistance of pure metallic single crystals resulting from cold-work is in progress in the Physics Department at the Carnegie Institute of Technology. The measurement swill be made at low temperatures where the dislocation scattering will produce an appreciable contribution to the total resistance.

II. INTRODUCTION—THE RELAXATION TIME AND THE RESISTIVITY

Consider a metal containing N electrons per unit volume. Suppose that each electron can move freely for a mean time $2t$, after which it suffers a collision and its momentum is destroyed, t is called the "relaxation time." The resistivity is then given by:[3]

$$\rho = m/(Ne^2t), \tag{1}$$

where m is the mass of the electron and e is its charge. It can be shown[4] that this equation is still valid if quantum mechanics is used; the classical and quantum treatments differ in their calculation of t.

It will also be assumed that Matthiesson's rule is valid so that

$$\rho = \rho_{\text{thermal}} + \rho_{\text{dislocation}}, \tag{2}$$

where each kind of resistance is related to its characteristic relaxation time by an equation of type (1). These assumptions lead to the result that

$$\Delta\rho/\rho = \rho_{\text{dislocation}}/\rho_{\text{thermal}} = t_{\text{thermal}}/t_{\text{dislocation}}. \tag{3}$$

Since t_{thermal} has been calculated theoretically,[5] a comparison with experiment can be made when $t_{\text{dislocation}}$ has been calculated. The calculation of $t_{\text{dislocation}}$ is the principal aim of this paper.

A short discussion of the assumptions will be given before proceeding. According to present theoretical ideas,[6] the procedure adopted above should be valid in two temperature regions: First in the range where the temperature is large compared with the Debye Θ of the material, and second in the range where the temperature is small compared with the Debye Θ. Deviations from Matthieson's rule, if they occur at all, should appear at temperatures in the vicinity of the Debye Θ. The experimental data available[7] indicates that Matthiesson's rule is valid over a wide range of temperatures, including the range containing the Debye Θ of the material in question. Deviations are sometimes found at very low temperatures. The experimental tests include cases where the temperature independent part of the resistance was primarily produced by impurities and other cases where it was caused by plastic strains. In Table II some results of experimental tests of the rule have been assembled.[7] In this table Matthiesson's rule has been used to calculate ρ_T(obs.), the "observed" thermal portion of the resistivity, and this thermal resistivity has been adjusted so that it takes the value one at zero degrees centigrade. The calculated values of the thermal resistivity ρ_T(calc.) were obtained by using Grüneisen's semi-emperical expression. Note that the "observed" data has been made to fit the calculated values at zero degrees absolute and at zero degrees centigrade. The proof of the Matthiesson rule stems from the fact that not just one set of results gives an "observed" thermal resistance in agreement with "Grüneisen's relation, but that several sets of data all give thermal resistivities in agreement with the relation. The values of ρ_D give the ratio of the temperature independent part of the resistance to the total resistance at zero degrees centigrade for the particular specimen in question. Similar data has been obtained on other materials.[7–9] The data given in Table II also show that the thermal

TABLE I.*

	Cu	Ni	W	Ag	Pt	Mo
$\Delta\rho/\rho$	0.02	0.08	0.50	0.03	0.06	0.18

* See reference 1.

[3] N. F. Mott and H. Jones, *The Theory of the Properties of Metals and Alloys* (Oxford University Press, London, 1936), p. 241.

[4] See reference 3, p. 247 and following pages.

[5] F. Seitz, *Modern Theory of Solids* (McGraw-Hill Book Company, Inc., 1940), p. 517, and following pages.

[6] A. Sommerfeld and H. Bethe, *Handbuch der Physik* (Verlag Julius Springer, Berlin, 1933), Vol. 24, p. 554.

[7] E. Gruneisen, *Handbuch der Physik* (Verlag Julius Springer, Berlin, 1928), Vol. 13, p. 21.

[8] W. J. DeHaas and G. J. VanDenBerg, Physica **4**, 683 (1937).

[9] W. J. DeHaas and J. DeBoer, Physica **1**, 609 (1934).

resistance can be calculated with good accuracy using Grüneisen's expression.

III. THE INTEGRAL EQUATION GIVING THE RELAXATION TIME

Consider an electron in the crystal lattice in a state specified by the wave number $\mathbf{k}$, where $|k|=2\pi/\lambda$ and λ is the wave-length of the electron. This electron may be scattered by any deviation of the potential from a periodic potential. In such a scattering process the wave number changes from $\mathbf{k}$ to some new value $\mathbf{k}'$.

It is assumed that the Fermi distribution function $f_0(\mathbf{k})$ describes the way in which the conduction electrons are spread over the various allowed energy states in the absence of an external electric field. The Fermi function is:

$$f_0(\mathbf{k})=1/\exp(E-\zeta/KT)+1, \tag{4}$$

where E is the energy of the stationary state associated with wave number $\mathbf{k}$, ζ is the energy of the most energetic electrons when the absolute temperature T is zero, and K is Boltzmann's constant. It is assumed that the electron in state $\mathbf{k}$ has an energy E given by:

$$E=E_0+\alpha k^2. \tag{5}$$

The Fermi function gives the fraction of states having energy E which are occupied; thus the number of electrons which occupy the region $dk_x dk_y dk_z$ in k space is:[10]

$$(2/8\pi^3)f_0(\mathbf{k})dk_x dk_y dk_z. \tag{6}$$

If an external electric field $\boldsymbol{\varepsilon}$ acts, then it can be shown that the $\mathbf{k}$ vector of each electron changes; the time rate of change is[10]

$$d\mathbf{k}/dt=-e\boldsymbol{\varepsilon}/\hbar, \tag{7}$$

where $-e$ is the charge on the electron and $\hbar$ is Planck's constant over 2π. The entire Fermi function therefore drifts in $\mathbf{k}$ space, and the time rate of change of the Fermi function produced by the field $\boldsymbol{\varepsilon}$ is

$$(df/dt)_{\text{field}}=f_0[\mathbf{k}-(e\boldsymbol{\varepsilon}/\hbar)dt]-f_0(\mathbf{k})/dt = -\partial f_0/\partial k[e(\boldsymbol{\varepsilon}\cdot\mathbf{k})/\hbar k], \tag{8}$$

where k represents the magnitude of $\mathbf{k}$. Note that $f_0(\mathbf{k})$ depends only on the magnitude of k and not on its direction. The second expression for $(df/dt)_{\text{field}}$ is valid only because $f_0(\mathbf{k})$ is a spherically symmetric function in $\mathbf{k}$ space.

In Section II it was shown that it is admissible to calculate the thermal and the dislocation contributions to the resistance separately. We therefore shall attempt to calculate the dislocation contribution by supposing that in a specimen at temperature T a large number of dislocations exist (where it is supposed that the temperature influences the Fermi distribution but that no thermal oscillation of the lattice occurs). In the presence of the external field $\boldsymbol{\varepsilon}$ the Fermi distribution will drift in k space until a steady state is reached in which the increase in the momentum of the electrons per second produced by the field is equal to the loss per second produced by collisions with the distorted lattice in the vicinity of the dislocations. This balance is described by

$$(df/dt)_{\text{field}}+(df/dt)_{\text{collisions}}=0. \tag{9}$$

The first term in this equation has already been calculated; an expression for the second term will now be obtained.

The decrease in $f(\mathbf{k})$ per second produced by collisions which change $\mathbf{k}$ to some other value $\mathbf{k}'$ is given by the product of:

(a) The fraction of states k originally filled, i.e., $f(\mathbf{k})$.

(b) The sum of the transition probabilities $P(\mathbf{k}\mathbf{k}')dS'$ to all other states of the same energy multiplied by the probability that the state in question is unoccupied, i.e., by $\{1-f(\mathbf{k}')\}$.

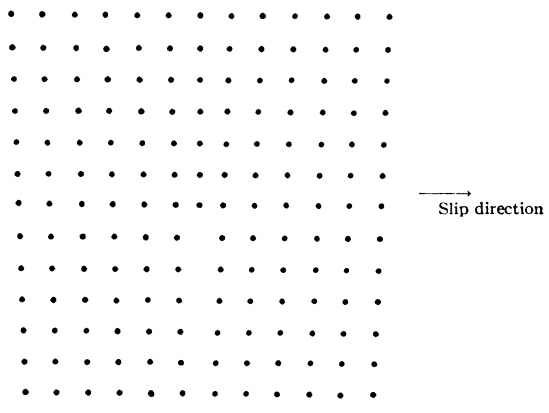

FIG. 1. A schematic diagram of a positive Taylor line dislocation. Atomic planes above and below the plane of the figure are identical with the plane shown. A negative dislocation would be present if the extra atomic plane would appear in the lower portion of the figure rather than in the upper part as shown.

[10] See reference 3, p. 259.

The increase in $f(\mathbf{k})$ per second produced by collisions which change $\mathbf{k}'$ to $\mathbf{k}$ is given by the product of:

(a) The fraction of unoccupied states $\mathbf{k}$, i.e., $\{1-f(\mathbf{k})\}$.
(b) The sum of the transition probabilities $P(\mathbf{k}'\mathbf{k})dS'$ from all other states of the same energy multiplied by the probability that they are filled, i.e., by $f(\mathbf{k}')$.

In the above dS' is the element of area on a sphere in $\mathbf{k}$ space which is associated with the state $\mathbf{k}'$. The sphere gives the end points of all $\mathbf{k}'$ vectors associated with states which have the same energy as the state $\mathbf{k}$. The probability per unit time $P(\mathbf{k}\mathbf{k}')dS'$ that an electron in state k makes a transition to a region dS' having the same energy is given by Mott[11] as

$$P(kk')dS'=\frac{V|\,V(kk')\,|^2dS'}{4\pi^2\hbar(dE'/dk')}.$$

Thus the net change in the Fermi function per second as a result of collisions is

$$(df/dt)_{\text{collisions}}=[1-f(k)]\int f(k')P(k'k)dS'$$

$$-f(k)\int[1-f(k')]P(kk')dS',$$

where the first term gives the change produced by transitions into $\mathbf{k}$ and the second gives the change resulting from transitions out of $\mathbf{k}$. Note that since the dislocations constitute a static perturbation, the collisions are elastic and energy is conserved by the electron in a collision. Note also that the Fermi functions in the last equation are the distorted functions appropriate for the distribution which exists when the balance is achieved. Since the collisions are elastic, one finds that

$$|\,V(kk')\,|^2=|\,V(k'k)\,|^2. \tag{10}$$

The expression for the rate of change in the Fermi function produced by collisions can therefore be written as

$$(df/dt)_{\text{collisions}}=V/4\pi^2\hbar(dE/dk)$$

$$\times\int\{f(k')-f(k)\}\,|\,V(kk')\,|^2dS'. \tag{11}$$

[11] See reference 3, p. 251.

TABLE II. Experimental data on Mattiesson's rule.

	Gold, $\Theta=190°$K					Platinum, $\Theta=230°$K						
T degrees Kelvin	Onnes* $\rho_D=0.00210$ ρr(obs.)	Meissner* $\rho_D=0.00039$ ρr(obs.)	Northrup* $\rho_D=0.04$ ρr(obs.)	DeHaas** $\rho_D=0.01575$ ρr(obs.)	ρr(calc.)	T degrees Kelvin	Henning* $\rho_D=0.0016$ ρr(obs.)	Henning* $\rho_D=0.0033$ ρr(obs.)	Meissner* $\rho_D=0.0016$ ρr(obs.)	Onnes* $\rho_D=0.0098$ ρr(obs.)	$\rho_D=0.000375$ ρr(obs.)	ρr(calc.)
4.2		0.00001			0.000015	1.63					0.0000040	0.0000002
6.78				0.000068	0.000087	2.39					0.0000085	0.0000008
7.57				0.000098	0.000135	3.12					0.0000148	0.0000022
11.18	0.00044			0.000545	0.00064	4.23			0.00004		0.0000282	0.0000074
12.1	0.00061				0.00088	5.47					0.0000520	0.0000207
14.3	0.00150			0.001729	0.00172	7.60					0.0001214	0.0000770
18.08	0.0036			0.004590	0.00439	9.60					0.0002626	0.0001963
20.4	0.0060	0.0060		0.007384	0.00692	14.20				0.00093		0.000940
57.8	0.1315				0.131	15.17					0.001369	0.00122
68.1	0.177				0.174	17.9				0.0027		0.0024
75.2	0.207				0.204	18.08					0.002700	0.0025
80.0		0.2249			0.225	20.44	0.0044		0.0044	0.0044	0.004322	0.0039
81.8		0.2317			0.232	56.5				0.1015		0.1020
90.1	0.270				0.268	68.4				0.1515		0.1513
91.5		0.275			0.273	90.1	0.2441					0.2448
169.3	0.592				0.589	90.3			0.2432	0.2454		0.2457
373.7		1.402			1.405	197.1	0.6935					0.6934
573.1			2.244		2.25	373.1		1.3927				1.3941
773.1			3.164		3.18	373.8			1.3966			1.3968
1073.1			4.77		4.79	473.1		1.7738				1.7739
1273.1			6.03		6.03	573.1		2.1431				2.3261

* See reference 7.

** See reference 8.

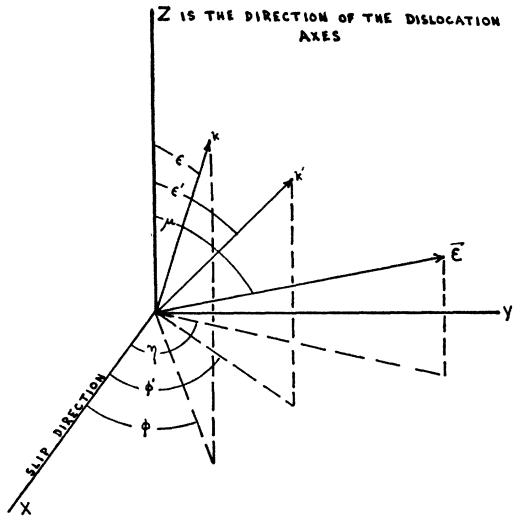

FIG. 2. Diagram showing the scattering geometry. The xy plane is the slip plane; the dislocation axes are parallel to z; k and k' are the wave number vectors of an electron before and after scattering; $\mathbf{\varepsilon}$ is the electric field.

This expression can be written in terms of the Fermi function for the case where no field is present by introducing the notion of the relaxation time. Consider a metal single crystal containing a large number of dislocations which have their axes parallel. If a field $\mathbf{\varepsilon}$ is suddenly applied, the Fermi function will drift in k space until collisions establish the balance mentioned. The resultant distortion produced in the Fermi function will depend on the orientation of the field relative to the dislocations and on the direction of motion of the electron under consideration relative to the dislocations. Figure 2 shows the important directions involved in the problem. The xz plane is the slip plane. Using the angles just introduced, Eq. (8) becomes

$$(df/dt)_{\text{field}} = -(e\varepsilon/\hbar)(\partial f_0/\partial k) \times \{\sin\epsilon\cos\phi\sin\mu\cos\eta + \sin\epsilon\sin\phi\sin\mu\sin\eta + \cos\epsilon\cos\mu\}. \quad (12)$$

In addition, using the fact that $f_0(\mathbf{k})$ is a spherically symmetric function in $\mathbf{k}$ space,

$$f(\mathbf{k}) = f_0[\mathbf{k} - t(\mathbf{k},\varepsilon)e\varepsilon/\hbar] = f_0(k) - (e(\varepsilon\cdot\mathbf{k})/\hbar k)(\partial f_0/\partial k)t(\mathbf{k},\varepsilon), \quad (13)$$

where $t(\mathbf{k},\varepsilon)$ is the relaxation time associated with dislocation scattering. t depends on $\mathbf{k}$ and on the direction of ε. A similar expression can be written for $f(\mathbf{k}')$. Inserting the angles, one finds

$$f(\mathbf{k}) = f_0(\mathbf{k}) - (e\epsilon/\hbar)(\partial f_0/\partial k)t(\mathbf{k},\varepsilon) \times \{\sin\epsilon\cos\phi\sin\mu\cos\eta + \sin\epsilon\sin\phi \times\sin\mu\sin\eta + \cos\epsilon\cos\mu\}. \quad (14)$$

The integral equation obtained by inserting (11), (12), (14), and a similar equation for $f(\mathbf{k}')$ into Eq. (9) is

$$\sin\epsilon\cos\phi\sin\mu\cos\eta + \sin\epsilon\sin\phi\sin\mu\sin\eta + \cos\epsilon\cos\mu = V/4\pi^2\hbar(dE/dk)\int |V(kk')|^2 dS' \cdot[t(\mathbf{k},\varepsilon)\{\sin\epsilon\cos\phi\sin\mu\cos\eta + \sin\epsilon\sin\phi\sin\mu\sin\eta + \cos\epsilon\cos\mu\} - t(\mathbf{k}',\epsilon)\{\sin\epsilon'\cos\phi'\sin\mu\cos\eta + \sin\epsilon'\sin\phi'\sin\mu\sin\eta + \cos\epsilon'\cos\mu\}]. \quad (15)$$

Note that since energy is conserved in a collision $k=k'$. The integration $\mathbf{k}'$ will therefore be over the surface of a sphere in $\mathbf{k}$ space.

The integral Eq. (15) will eventually be used to determine the relaxation time for dislocation scattering $t(\mathbf{k},\varepsilon)$.

IV. THE EVALUATION OF THE MATRIX ELEMENT GIVING DISLOCATION SCATTERING

The matrix element will first be evaluated for a dislocation pair which have their axes parallel to the z axis. The positive dislocation is located at $x=0$, $y=+R/2$; the negative dislocation is at $x=0$, $y=-R/2$. The displacements produced by the pair at a point x,y are[12]

$$\begin{aligned} u_x &= A\{\tan^{-1}((y-R/2)/x) - \tan^{-1}((y+R/2)/x)\} \\ &\quad + 2Bx\{(y-R/2)/x^2 + (y-R/2)^2 \\ &\quad - (y+R/2)/x^2 + (y+R/2)^2\}, \\ y_y &= -C\log(x^2+(y-R/2)^2/x^2+(y+R/2)^2)^{\frac{1}{2}} \\ &\quad - 2Bx^2\{1/x^2+(y-R/2)^2 \\ &\quad - 1/x^2+(y+R/2)^2\}, \end{aligned} \quad (16)$$

where

$$A = \lambda/2\pi, \quad B = (\lambda/8\pi)(m/(m-1)), \quad C = (\lambda/4\pi)((m-2)/(m-1));$$

λ is the unit crystallographic slip distance, i.e., it is the smallest interatomic distance encountered

[12] J. S. Koehler, Phys. Rev. 60, 398 (1941).

considering atoms lying along the slip direction. m is the reciprocal of Poisson's ratio.

Let $\mathbf{a}_j$ be a vector from the origin of our coordinates to the lattice position of the jth positive ion. The potential in which the valence electrons move in a perfect crystal is taken to be

$$V(\mathbf{r})=\sum_j V_j(\mathbf{r}-\mathbf{a}_j). \quad (17)$$

The potential of a single ion, $V_j(\mathbf{r}-\mathbf{a}_j)$ is taken to be screened Coulomb field, i.e.,

$$V_j(\mathbf{r}-\mathbf{a}_j)=-Ze^2\exp(-q|\mathbf{r}-\mathbf{a}_j|)/|\mathbf{r}-\mathbf{a}_j|, \quad (18)$$

where Ze is the charge of the ion and q is the screening constant. According to Mott[13] the value of q for copper lies between 1.8×10^8 and 3.3×10^8 cm^{-1}.

If all of the ions experience displacements, then the deviation of the resulting potential from that which occurs in a perfect crystal is approximately:

$$\Delta V(\mathbf{r})=-\sum_j \mathbf{u}_j\cdot\operatorname{grad} V_j(\mathbf{r}-\mathbf{a}_j), \quad (19)$$

where $\mathbf{u}_j$ is the displacement of the jth ion. Using this as the perturbation, the matrix element associated with the scattering produced by dislocations is

$$V(kk')=-1/V\int \exp(-ik'\cdot r)\times(\textstyle\sum_j u_j\cdot\operatorname{grad} V_j(\mathbf{r}-\mathbf{a}_j))\exp(i\mathbf{k}\cdot\mathbf{r})dv, \quad (20)$$

where V is the volume of the crystal and where the electronic wave functions associated with the states k and k' have been assumed to be free electron wave functions. The integration is over the entire crystal. Exchanging summation and integration and using as variable in the resulting integral $(\mathbf{r}-\mathbf{a}_j)$, one obtains

$$V(kk')=-1/V\sum_j\exp(-i(\mathbf{k}'-\mathbf{k})\cdot a_j)\int\exp(-i\mathbf{k}'\cdot(\mathbf{r}-\mathbf{a}_j))\mathbf{u}_j\cdot\operatorname{grad}V_j\times\exp(ik\cdot(\mathbf{r}-\mathbf{a}_j))dv. \quad (21)$$

Since all of the dislocations considered have their axes parallel to z, and since each dislocation therefore produces displacements which are perpendicular to z, the resulting u_{iz} is zero. Equation (21) can therefore be written:

$$V(kk')=\textstyle\sum_j\exp(-i\mathbf{K}\cdot\mathbf{a}_j)[u_{xj}P+u_{yj}Q] \quad (22)$$

where $\mathbf{K}=(\mathbf{k}'-\mathbf{k})$ and

$$P=1/V\int\exp(-i\mathbf{k}'\cdot\mathbf{r})(\partial/\partial X)\times\{Ze^2\exp(-qr)/r\}\exp(i\mathbf{k}\cdot\mathbf{r})dv=(4\pi iZe^2/V)K_x/(q^2+K^2). \quad (23)$$

Also,

$$Q=(4\pi iZe^2/V)(K_y/(q^2+K^2)). \quad (24)$$

The sum over j can now be converted into an integral. The deflection at each successive atom is assumed to vary slowly with j. Then

$$V(kk')=1/V_0\int\exp(-i\mathbf{K}\cdot\mathbf{a})[u_xP+u_yQ]da, \quad (25)$$

where V_0 is the volume associated with one atom, and the integral extends over the entire crystal.

Let us calculate in detail the value of the matrix element $v(kk')$ for the case where the displacement u is that produced by a single pair of dislocations. This matrix element is given by (25) if the displacements (16) are used. The integrations can be done using Bierens De Haan, and one finds

$$|v(kk')|^2=\frac{(16\pi^2)^2Z^2e^4|\delta(K_z)|^2\sin^2(K_yR/2)}{V^2V_0^2(q^2+K^2)^2}\times\left[\frac{A^2K_x^2}{K^4}+\frac{C^2K_y^2}{K^4}-\frac{16BCK_x^2K_y^2}{K^6}\right], \quad (26)$$

where the delta-function $\delta(K_z)$ is given by

$$\delta(K_z)=\int_0^{V^{\frac{1}{3}}}\exp(i\mathbf{K}\cdot\mathbf{z})dz=\frac{\{\exp(iV^{\frac{1}{3}}K_z)-1\}}{iK_z}. \quad (27)$$

The appearance of this delta-function means that only those transitions occur in which k_z equals k_z'. The possibility that interference might occur between the wave scattered from the positive dislocation and the wave scattered by the negative dislocation is manifest in Eq. (26) by the presence of the factor $\sin^2(K_yR/2)$. Assuming a

[13] See reference 3, pp. 88 and 294.

uniform distribution throughout the solid, the dislocations in a severely cold-worked metal are separated by distances of the order of 10^{-6} cm.[14] In the case of copper, $K_y/2$ can take values from zero to 1.36×10^{8} cm^{-1}. Thus as the direction and magnitude of K are varied the argument of the sine goes through many periods while the rest of the matrix element is changing only slightly. It is therefore appropriate to put the average value of $\sin^2(K_yR/2)$ into the expression when dealing with metals. It is interesting to note that these diffraction effects could be of importance if one were discussing the dislocation resistance of a semi-conductor in which the wave-length of the current carriers was about 10^{-6} cm. Experiments on such materials might yield information on the way in which dislocations are distributed in the solid. Diffraction effects might also conceivably occur in a metal if the dislocations were clustered along the slip bands, but this seems rather improbable.[15] If the average value of the sine square is used, then the matrix element appropriate for a large number of dislocation pairs is

$$|V(kk')|^2=n\langle|v(kk')|^2\rangle_{\mathrm{Av}}, \tag{28}$$

where the averaged value of the matrix element has been used. n is the total number of dislocation pairs in the specimen.

V. THE SOLUTION OF THE INTEGRAL EQUATION

Inserting (28) and (26) into the integral equation as given by (15), one obtains

$$\sin\epsilon\cos\phi\sin\mu\cos\eta+\sin\epsilon\sin\phi\sin\mu\sin\eta+\cos\epsilon\cos\mu=32\pi^2Z^2e^4n/VV_0^2(dE/dk)\hbar$$

$$\int dS'|\delta(K_z)|^2/(q^2+K^2)^2\cdot(A^2K_x^2/K^4+C^2K_y^2/K^4-16BCK_x^2K_y^2/K^6)$$

$$\times[t(\mathbf{k}\boldsymbol{\varepsilon})\{\sin\epsilon\cos\phi\sin\mu\cos\eta+\sin\epsilon\sin\phi\sin\mu\sin\eta+\cos\epsilon\cos\eta\}$$

$$-t(\mathbf{k}'\boldsymbol{\varepsilon})\{\sin\epsilon'\cos\phi'\sin\mu\cos\eta+\sin\epsilon'\cos\phi'\sin\mu\sin\eta+\cos\epsilon'\cos\eta\}]. \tag{29}$$

The integrations can be carried out as follows. In the first place the presence of the delta-function enables one to reduce the integration from a two-dimensional integration to a one-dimensional integration. Since all quantities except the delta-function vary slowly with k_z', the value of the rest of the integrand at the position of the maxima of the delta-function can be used with the integrated value of $|\delta(K_z)|^2$. The result of integrating ϵ' from 0 to π is

$$\int dS'|\delta(K_z)|^2=k^2\int\sin\epsilon' d\epsilon' d\phi'|\delta(K_z)|^2$$

$$=4\pi kV^{\frac{1}{3}}d\phi'. \tag{30}$$

It is next assumed that the relaxation time can be written in the following form:

$$t(\mathbf{k},\boldsymbol{\varepsilon})=a_0+a_2\cos2\phi+a_4\cos4\phi+a_6\cos6\phi+\cdots+b_2\sin2\phi+b_4\sin4\phi+b_6\sin6\phi+\cdots. \tag{31}$$

The symmetry of the problem leads one to believe that sine terms and cosines containing odd multiple of ϕ will not be needed. It should be noted that the coefficients a_{2i} depend on ϵ, on k, and on the field $\boldsymbol{\varepsilon}$. The integration of Eq. (29) can be facilitated by introducing the angle u defined as follows:

$$u=\tfrac{1}{2}(\phi'-\phi). \tag{32}$$

When this is done and when the results of Eqs. (30) and (31) are used, the integral equation becomes

[14] See reference 12, p. 410.

[15] Diffraction effects would probably only be observable if $KyR/2$ varied from zero to about 15. This would produce about 10 oscillations in the scattering probability as the electric field was shifted from the plane $\eta=0$ to the plane $\eta=90°$. The value of R for this limiting case is only 11×10^{-8} cm. Since this is only four times the smallest interatomic distance in copper, it seems unlikely that such pairs will be formed. This can be seen as follows: A dislocation pair having the separation mentioned in copper can only be separated if an external stress of 260 kg/mm^2 is applied. The tensile strength of polycrystalline copper is 22.6 kg/mm^2. If most of the dislocations in a cold-worked copper specimen had the separation distance mentioned, it would be impossible to account for the extensive plastic deformation which occurs prior to fracture.

$$(1)\cos\phi+(2)\sin\phi+(3)=\frac{\mathbf{A}}{\pi\sin^6\epsilon}\int_0^\pi \frac{du[A^2\sin^2(u+\phi)+C^2\cos^2(u+\phi)-16BC\sin^2(u+\phi)\cos^2(u+\phi)]}{\sin^2u[s^2+\sin^2u]^2}$$

$$\times[+2a_0(1)\sin u\sin(u+\phi)-2a_0(2)\sin u\cos(u+\phi)+a_2(1)\{\sin u\sin(u+\phi)+\sin 3u\sin 3(u+\phi)\}$$
$$+a_2(2)\{\sin u\cos(u+\phi)-\sin 3u\cos 3(u+\phi)\}+2a_2(3)\sin 2u\sin 2(u+\phi)$$
$$+a_1(1)\{\sin 3u\sin 3(u+\phi)+\sin 5u\sin 5(u+\phi)\}+a_4(2)\{\sin 3u\cos 3(u+\phi)-\sin 5u\cos 5(u+\phi)\}$$
$$+2a_4(3)\sin 4u\sin 4(u+\phi)+\cdots+b_2(2)\{\sin u\sin(u+\phi)-\sin 3u\sin 3(u+\phi)\}$$
$$-b_2(1)\{\sin u\cos(u+\phi)+\sin 3u\cos 3(u+\phi)\}-2b_2(3)\sin 2u\cos 2(u+\phi)+b_4(2)\{\sin 3u\sin 3(u+\phi)$$
$$-\sin 5u\sin 5(u+\phi)\}-b_4(1)\{\sin 3u\cos 3(u+\phi)+\sin 5u\cos 5(u+\phi)\}-2b_4(3)\sin 4u\cos 4(u+\phi)+\cdots], \tag{33}$$

where

$\mathbf{A}=2\pi^4Z^2e^4n/V_0^2\alpha k^6\hbar$, $s^2=q^2/4k^2\sin^2\epsilon$, $(1)=\sin\epsilon\sin\mu\cos\eta$, $(2)=\sin\epsilon\sin\mu\sin\eta$, $(3)=\cos\epsilon\cos\mu$.

Since (33) is an identity in ϕ the coefficients of $\cos n\phi$ must be equal.

Coefficient of $\cos\phi$:

$$4(1)\sin^6\epsilon/\mathbf{A}\pi=2a_0(1)d_1\{3A^2+C^2-8BC\}+a_2(1)\{d_1(3A^2+C^2-8BC)-d_3(A^2-C^2+4BC)\}$$
$$+a_4(1)\{-d_3(A^2-C^2+4BC)+d_5 4BC\}+a_6(1)d_5 4BC+b_2(2)\{d_1(3A^2+C^2-8BC)$$
$$+d_3(A^2-C^2+4BC)\}+b_4(2)\{-d_3(A^2-C^2+4BC)-d_5 4BC\}+b_6(2)d_5 4BC.$$

Coefficient of $\cos 3\phi$:

$$0=-2a_0(1)d_3\{A^2-C^2+4BC\}+a_2(1)\{-d_3(A^2-C^2+4BC)+2(d_1+d_3+d_5)(A^2+C^2-4BC)\}$$
$$+a_4(1)\{2(d_1+d_3+d_5)(A^2+C^2-4BC)-(d_3+d_5+d_7)(A^2-C^2)\}+a_6(1)\{-(d_3+d_5+d_7)$$
$$\times(A^2-C^2)+(d_5+d_7+d_9)4BC\}+\cdots+b_2(2)\{-d_3(A^2-C^2+4BC)-2(d_1+d_3+d_5)$$
$$\times(A^2+C^2-4BC)\}+b_4(2)\{2(d_1+d_3+d_5)(A^2+C^2-4BC)+(d_3+d_5+d_7)(A^2-C^2)\}$$
$$+b_6(2)\{-(d_3+d_5+d_7)(A^2-C^2)-(d_5+d_7+d_9)4BC\}+\cdots.$$

Coefficient of $\cos 5\phi$:

$$0=+2a_0(1)d_5 4BC+a_2(1)\{+d_5 4BC-(d_3+d_5+d_7)(A^2-C^2)\}+a_4(1)\{-(d_3+d_5+d_7)(A^2-C^2)$$
$$+2(d_1+\cdots+d_9)(A^2+C^2-4BC)\}+a_6(1)\{2(d_1+\cdots+d_9)(A^2+C^2-4BC)$$
$$-(d_3+\cdots+d_{11})(A^2-C^2)\}+b_2(2)\{+d_5 4BC+(d_3+d_5+d_7)(A^2-C^2)\}$$
$$+b_4(2)\{-(d_3+d_5+d_7)(A^2-C^2)-2(d_1+\cdots+d_9)(A^2+C^2-4BC)\}$$
$$+b_6(2)\{2(d_1+\cdots+d_9)(A^2+C^2-4BC)+(d_3+\cdots+d_{11})(A^2-C^2)\}+\cdots.$$

Coefficient of $\cos 7\phi$:

$$0=+a_2(1)\{(d_5+d_7+d_9)4BC\}+a_4(1)\{+(d_5+d_7+d_9)4BC-(d_3+\cdots+d_{11})(A^2-C^2)\}$$
$$+a_6(1)\{-(d_3+\cdots+d_{11})(A^2-C^2)+2(d_1+\cdots+d_{13})(A^2+C^2-4BC)\}+\cdots$$
$$-b_2(2)\{(d_5+d_7+d_9)4BC\}+b_4(2)\{+(d_5+d_7+d_9)4BC+(d_3+\cdots+d_{11})(A^2-C^2)\}$$
$$+b_6(2)\{-(d_3+\cdots+d_{11})(A^2-C^2)-2(d_1+\cdots+d_{13})(A^2+C^2-4BC)\}+\cdots. \tag{34}$$

Coefficient of $\sin\phi$:

$$4(2)\ \sin^6\epsilon/\mathbf{A}\pi = 2a_0(2)d_1\{A^2+3C^2-8BC\}+a_2(2)\{-d_1(A^2+3C^2-8BC)-d_3(A^2-C^2-4BC)\}$$
$$+a_4(2)\{+d_3(A^2-C^2-4BC)+d_5 4BC\}-a_6(2)d_5 4BC+b_2(1)\{+d_1(A^2+3C^2-8BC$$
$$-d_3(A^2-C^2-4BC)\}+b_4(1)\{-d_3(A^2-C^2-4BC)+d_5 4BC\}+b_6(1)d_5 4BC.$$

Coefficient of $\sin 3\phi$:

$$0=-2a_0(2)d_3(A^2-C^2-4BC)+a_2(2)\{d_3(A^2-C^2-4BC)+2(d_1+d_3+d_5)(A^2+C^2-4BC)\}$$
$$+a_4(2)\{-2(d_1+d_3+d_5)(A^2+C^2-4BC)-(d_3+d_5+d_7)(A^2-C^2)\}+a_6(2)\{(d_3+d_5+d_7)(A^2-C^2)$$
$$+(d_5+d_7+d_9)4BC\}+\cdots+b_2(1)\{-d_3(A^2-C^2-4BC)+2(d_1+d_3+d_5)(A^2+C^2-4BC)\}$$
$$+b_4(1)\{+2(d_1+d_3+d_5)(A^2+C^2-4BC)-(d_3+d_5+d_7)(A^2-C^2)\}$$
$$+b_6(1)\{-(d_3+d_5+d_7)(A^2-C^2)+(d_5+d_7+d_9)4BC\}+\cdots.$$

Coefficient of $\sin 5\phi$:

$$0=+2a_0(2)d_5 4BC+a_2(2)\{-d_5 4BC-(d_3+d_5+d_7)(A^2-C^2)\}+a_4(2)\{+(d_3+d_5+d_7)(A^2-C^2)$$
$$+2(d_1+\cdots+d_9)(A^2+C^2-4BC)\}+a_6(2)\{-2(d_1+\cdots+d_9)(A^2+C^2-4BC)$$
$$-(d_3+\cdots+d_{11})(A^2-C^2)\}+\cdots+b_2(1)\{+d_5 4BC-(d_3+d_5+d_7)(A^2-C^2)\}$$
$$+b_4(1)\{-(d_3+d_5+d_7)(A^2-C^2)+2(d_1+\cdots+d_9)(A^2+C^2-4BC)\}$$
$$+b_6(1)\{+2(d_1+\cdots+d_9)(A^2+C^2-4BC)-(d_3+\cdots+d_{11})(A^2-C^2)\}+\cdots.$$

Coefficient of $\sin 7\phi$:

$$0=+a_2(2)\{(d_5+d_7+d_9)4BC\}+a_4(2)\{-(d_5+d_7+d_9)4BC-(d_3+\cdots+d_{11})(A^2-C^2)\}$$
$$+a_6(2)\{+(d_3+\cdots+d_{11})(A^2-C^2)+2(d_1+\cdots+d_{13})(A^2+C^2-4BC)\}+\cdots$$
$$+b_2(1)\{(d_5+d_7+d_9)4BC\}+b_4(1)\{+(d_5+d_7+d_9)4BC-(d_3+\cdots+d_{11})(A^2-C^2)\}$$
$$+b_6(1)\{-(d_3+\cdots+d_{11})(A^2-C^2)+2(d_1+\cdots+d_{13})(A^2+C^2-4BC)\}+\cdots, \quad (35)$$
$$\vdots$$

where the d_i are the following functions of s:

$$d_1=\tfrac{1}{2}(1+2s^2/s^3\{1+s^2\}^{\frac{1}{2}}),$$
$$d_3=-2/s\{1+s^2\}^{\frac{1}{2}}+\tfrac{3}{2}(1+2s^2/s^3\{1+s^2\}^{\frac{1}{2}}),$$
$$d_5=16-\tfrac{1}{2}(20+32s^2/s\{1+s^2\}^{\frac{1}{2}})+\tfrac{1}{2}(\{5-16s^4\}\{1+2s^2\}/s^3\{1+s^2\}^{\frac{1}{2}}),$$
$$d_7=(112+128s^2)-32+\tfrac{1}{2}(56+224s^2+192s^4/s\{1+s^2\}^{\frac{1}{2}})+\tfrac{1}{2}(\{7-112s^4-128s^6\}\{1+2s^2\}/s^3\{1+s^2\}^{\frac{1}{2}}),$$
$$d_9=(432+1152s^2+768s^4)-(288+256s^2)+96-\tfrac{1}{2}(120+864s^2+1728s^4+1024s^6/s\{1+s^2\}^{\frac{1}{2}})$$
$$+\tfrac{1}{2}(9-432s^4-1152s^6-768s^8\{1+2s^2\}/s^3\{1+s^2\}^{\frac{1}{2}}). \quad (36)$$

In general,

$$d_{2i+1}=1/\pi\int_0^\pi du\ \sin(2i+1)u/\sin u\{s^2+\sin^2 u\}^2. \quad (37)$$

If s is much less than one, then

$$d_{2i+1}=2i+1/2s^3. \tag{38}$$

If s is much greater than one,

$$d_{2i+1}=\tfrac{1}{2}/s^4-(i+2)/2^{2i+1}s^{2i+6}+0/s^{2i+8}+\cdots. \tag{39}$$

Equations (34) constitute the requirement that the drift of the Fermi function produced by the x component of the field by balanced by collisions. Equations (35), if satisfied, guarantee that the drift produced by the y component of the field will be balanced by collisions. Sines and cosines of even multiples of ϕ also appear in the integral equation. The equations obtained by considering these terms govern the drift produced by the z component of the field. We have supposed that all dislocation pairs have their axes parallel to z. In this case if thermal scattering is neglected there will be a steady drift of the Fermi function in the k_z direction, i.e., the equation which governs the drift produced by the z component of the field cannot be satisfied.

Equations (34) and (35) are solved for the coefficients a_{2i} and b_{2i}, and the resulting relaxation time is averaged over all directions of $\mathbf{k}$, since electrons traveling in all directions are present. The relaxation time $\bar{t}$ which results will depend only on the orientation of the field direction relative to the array of dislocations. It can be seen from (31) that only a_0 will contribute to $\bar{t}$, since the other contributions average out. The $\bar{t}$ then gives the dislocation resistance when used in Eq. (1). The method used in practice to carry through these calculations will be described in the next section where the averaged relaxation time is calculated numerically for copper.

VI. THE NUMERICAL EVALUATION OF t FOR COPPER

It is assumed that copper has one valence electron per atom. The effective mass of this valence electron is taken to be just the mass of a free electron.[16] The width in energy of the filled portion of the Fermi band is[17] 7.04 ev. This, together with the values just quoted, gives $k=1.36\times10^{+8}$ cm^{-1} and $\alpha=6.095\times10^{-28}$ erg cm^2. The other numerical constants used are given in Table III. The values of the first three constants are taken from a paper by Koehler.[18] The density of dislocation pairs is chosen to give the correct value for the energy stored in copper during work hardening. The pairs are assumed to be uniformly distributed. The value of V_0 is that given by Mott and Jones.[19] The thermal portion of the electrical resistivity is the value quoted in the *Handbook of Chemistry and Physics* (Chemical Rubber Publishing Company, Cleveland, Ohio) (30th edition, 1947).

There are two cases in which the value of a_0 can be calculated simply. If the electric field vector lies in the xz plane, η equals zero, and since $(2)=\sin\epsilon\sin\mu\sin\eta$ is also zero, the coefficients of the b_{2i} terms in (34) vanish; similarly, the coefficients of the a_{2i} terms in (35) vanish. Equations (35) are then a set of linear homogeneous equations in the b_{2i}s. Since the determinant of the coefficients is not, in general, zero, the coefficients b_{2i} must be zero in this case. Equations (34) can then be solved for the a_{2i}s—in particular for a_0. The fact that the b_{2i} should all vanish can also be seen from the symmetry conditions which exist in this case. The symmetry of t is determined by that of the matrix element and by the orientation of the field with respect to the dislocations. In the case under discussion it is easily seen that t should be an even function of ϕ, and hence all the sine terms must be zero. Substituting the numerical values into (34) one obtains the following equations for the case where η is zero:

$$\begin{aligned}
T&=+1.048090a_0+0.094359a_2\\
&\quad -0.302463a_4+0.127223a_6,\\
0&=-0.859372a_0+0.815058a_2\\
&\quad +0.278139a_4-0.581710a_6,\\
0&=+0.254447a_0-0.839382a_2\\
&\quad +1.247998a_4+0.590756a_6,\\
0&=+0.384895a_2-1.238952a_4+1.556807a_6.
\end{aligned} \tag{40}$$

The terms in a_8 and in higher coefficients are assumed to be unimportant for the determination of a_0. Solving this, one finds that

$$a_0=+0.974020T, \text{ when } \eta=0, \tag{41}$$

where

$$T=16\pi\sin^6\epsilon/\mathbf{A}\lambda^2.$$

[16] See reference 5, p. 153.
[17] See reference 5, p. 146.
[18] See reference 12, pp. 409 and 410.
[19] See reference 3, p. 318.

TABLE III. Constants used in calculations on copper.

m = reciprocal of Poisson's ratio = 2.941
λ = unit slip distance = 2.552×10^{-8} cm
n = density of dislocation pairs (when highly cold-worked) = $1/2(\pi R^2) = 2.906\times10^{11}$ cm^{-2}
V_0 = volume occupied by one atom = $1/N = 1.1708\times10^{-23}$ cm^3
q = the screening constant for the atomic potential in which the electron moves = 2.5×10^8 cm^{-1} (Mott* gives $1.8\times10^8 \leqslant q \leqslant 3.3\times10^8$ cm^{-1})
$s^2 = \dfrac{q^2}{4k^2\langle\sin^2\epsilon\rangle_{Av}} = q^2/2k^2 = 1.692$
$d_1 = +0.22549$
$d_3 = +0.32836$
$d_5 = +0.34629$
$d_7 = +0.35235$
$d_9 = d_{11} = d_{13} = d_{15} = d_{17} = 1/s^4 = 0.349301$
ρ_{thermal}(observed 20°C) = 1.692×10^{-6} ohm cm = 1.889×10^{-18} e.s.u. (sec. cm^{-3})

* See reference 13.

Similarly, if the electric field lies in the yz plane η is 90° and $(1)=\sin\epsilon\sin\mu\cos\eta$ is zero. In this case the coefficients of the b_{2i} terms in (35) vanish and Eqs. (34) become a set of linear homogeneous equations in the b_{2i} which are again zero. In this case, as before, t should be an even function of ϕ, and again symmetry conditions enable us to understand the vanishing of the coefficients b_{2i}. Substituting numerical values into (35), the following equations are obtained for the case where η is 90°:

$$\begin{aligned}
T &= +0.199172a_0 - 0.288000a_2 \\
&\quad + 0.315637a_4 - 0.127223a_6, \\
0 &= -0.376828a_0 + 1.433158a_2 \\
&\quad - 2.211349a_4 + 1.351500a_6, \\
0 &= +0.254447a_0 - 1.093828a_2 \\
&\quad + 3.181208a_4 - 3.838450a_6, \\
0 &= +0.384895a_2 - 2.008742a_4 + 4.804501a_6, \qquad (42)
\end{aligned}$$

where T has the value previously given. The value of a_0 in this case is

$$a_0 = +8.037938T, \text{ when } \eta = 90°. \qquad (43)$$

It is more difficult to solve the equations for other values of η. Because of the symmetry of the problem, the complete dependence of $\dot{t}$ on η is known if its behavior from 0° to 90° has been obtained. We have also solved the equations for the case where $\eta = 45°$. Some simplification then occurs because (1) equals (2). The equations which result are:

$$\begin{aligned}
T &= +1.048090a_0 + 0.094359a_2 + 0.953731b_2 \\
&\quad - 0.302463a_4 - 0.556909b_4 \\
&\quad + 0.127223a_6 + 0.127223b_6, \\
T &= +0.199172a_0 - 0.288000a_2 - 0.088828b_2 \\
&\quad + 0.315637a_4 - 0.061191b_4 \\
&\quad - 0.127223a_6 + 0.127223b_6, \\
0 &= -0.859372a_0 + 0.815058a_2 - 1.674430b_2 \\
&\quad + 0.278139a_4 + 2.211349b_4 \\
&\quad - 0.581710a_6 - 1.351500b_6, \\
0 &= -0.376828a_0 + 1.433158a_2 + 1.056330b_2 \\
&\quad - 2.211349a_4 + 0.278139b_4 \\
&\quad + 1.351500a_6 - 0.581710b_6, \\
0 &= +0.254447a_0 - 0.839382a_2 + 1.093828b_2 \\
&\quad + 1.247998a_4 - 3.181208b_4 \\
&\quad + 0.590756a_6 + 3.838450b_6, \\
0 &= +0.254447a_0 - 1.093828a_2 - 0.839382b_2 \\
&\quad + 3.181208a_4 + 1.247998b_4 \\
&\quad - 3.838450a_6 + 0.590756b_6, \\
0 &= +0.384895a_2 - 0.384895b_2 - 1.238952a_4 \\
&\quad + 2.008742b_4 \\
&\quad + 1.556807a_6 - 4.804501b_6, \qquad (44)
\end{aligned}$$

where the first, the third, the fifth, and the seventh equations are from (34). The others are from (35). Solving these equations one finds that

$$a_0 = +2.518534T, \text{ when } \eta = 45°. \qquad (45)$$

The angular dependence of the average relaxation time can now be determined. The symmetry of the problem is such that $\partial\dot{t}/\partial\eta$ is zero at $\eta=0$ and at $\eta=90°$. $\dot{t}$ can therefore be written in the following form:

$$\dot{t} = a_0(\mu\eta) = T\{\alpha_0 + \beta_0\cos2\eta + \gamma_0\cos4\eta + \delta_0\cos6\eta + \cdots\}. \qquad (46)$$

Since the matrix element contains terms in $\cos4\eta$ but does not contain more rapidly oscillating terms, one would expect that the first three coefficients in (46) would be large in comparison with higher coefficients. It is assumed here that δ_0 and higher coefficients are negligible. The values of a_0 given in (45), (43), and (41) then suffice to calculate the coefficients; they are:

$$\begin{aligned}
\alpha_0 &= +3.498157, \\
\beta_0 &= -3.531951, \qquad (47) \\
\gamma_0 &= +1.007815.
\end{aligned}$$

The value of the dislocation resistance can now be calculated. The value of **A** is found using

the expression given below (33). The result is used to obtain T which is given by the relation below (41). The average value of $\sin^6\epsilon$ is inserted in the expression for T (i.e., 15/48). The dislocation resistance is then obtained by substituting from (46) and (47) into (1). The result for copper is

$$\Delta\rho = 1.491\times10^{-20}/(1.000-1.010\cos2\eta + 0.288\cos4\eta) \text{ e.s.u.} \quad (48)$$

It should be noted that $\Delta\rho$ does not depend on μ, the angle between the field direction and the dislocation axes. This can be qualitatively understood as follows: As μ decreases from 90° towards zero the magnitude of the matrix element and hence the scattering probability decrease and become zero when μ is zero. On the other hand, the time spent in the perturbed regions around the dislocations by the electrons which contribute the largest part of the dislocation resistance (i.e., the electrons moving along the field direction) increases as μ decreases; this times goes to infinity when μ is zero. Thus the two effects tend to cancel out.

The dependence on η is most interesting. The values of the dislocation resistance in the slip direction, at $\eta=45°$, and perpendicular to the slip direction in copper are

$$\Delta\rho_{\eta=0°} = 5.37\times10^{-20} \text{ e.s.u.},$$
$$\Delta\rho_{\eta=45°} = 2.09\times10^{-20} \text{ e.s.u.},$$
$$\Delta\rho_{\eta=90°} = 0.648\times10^{-20} \text{ e.s.u.}$$

The ratio of the extreme values is

$$\Delta\rho_{\eta=0°}/\Delta\rho_{\eta=90°} = 8.30.$$

This means that the dislocation resistance depends sensitively on the direction of the electric field. The ratio of the extreme values depends on Poisson's ratio for the material and is large for materials having a low value of Poisson's ratio.

In a polycrystalline material all values of η are equally likely. If the relaxation time is averaged over η and is then used in (1), the result obtained is appropriate for comparison with the experimental results on polycrystals mentioned in the introduction. In the case of copper the result is

$$\langle\Delta\rho\rangle_{Av} = 1.491\times10^{-20} \text{ e.s.u.}$$

The averaged dislocation resistance in highly cold-worked copper is therefore calculated to be slightly less than 1 percent of the thermal portion of the electrical resistance at 20°C. Experimentally, (see ρ_{thermal} in Table III) the dislocation resistance amounts to 2 percent of the thermal resistance at room temperature. Considering the complexity of the calculation, the agreement is remarkable.

The treatment given here is incomplete in two respects. First, similar calculations should be made assuming that the electrons are scattered by screw type dislocations.[20] Second, studies should be made to determine just what kind of three-dimensional dislocation arrays are present in the various types of crystals when they undergo certain simple kinds of plastic strain, such as simple glide in one slip direction on one slip plane.

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[20] J. M. Burgers, Proc. K. Akad. Amst. **42**, 263, 378 (1939).