The Density Spectrum of Extensive Air Showers of Cosmic Rays

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(Received December 6, 1948)

The density spectrum of extensive showers has been measured both at 260 and at 3260 m above sea level. It has been found that the spectrum in both places is not a pure exponential.

I. METHODS FOR THE EVALUATION OF γ

It has been established that the frequency $H(\Delta)$ of the extensive air showers in which the particle density (particles per unit surface) is greater than Δ can be satisfactorily expressed by the law:

$$H(\Delta)=K\Delta^{-\gamma}, \tag{1}$$

where K and γ are in first approximation constant in a large density range and vary with the altitude above sea level.

The determination of the value of the exponent γ can be achieved through different methods:

(a) n G-M counters, all with the same effective surface S, are placed at proper distances on a horizontal plane, and coincidences $N_{n-1}(S)$ and $N_n(S)$ are recorded.

From the ratio $N_n(S)/N_{n-1}(S)$ it is possible to deduce γ by using the expression:

$$\frac{N_n(S)}{N_{n-1}(S)}=\frac{\left[\binom{n}{1}-\binom{n}{2}2^{\gamma}+\binom{n}{3}3^{\gamma}-\cdots\right]}{\left[\binom{n-1}{1}-\binom{n-1}{2}2^{\gamma}+\binom{n-1}{3}3^{\gamma}-\cdots\right]}. \tag{2}$$

TABLE I. Experimental evaluations of γ and K.

Author	Method	Elevation (m.a.s.l.)	Density range (m^{-2})	γ	K (Δ in m^{-2}, time in sec.)
Cocconi, etc.[a]	*b*	110	10–1000	1.46	0.124
	b	2200	10–1000	1.55	1.0
Auger, Daudin[b]	*a*	50	~70	1.66	
	a	2060	~70	1.50	
	a	2860	~70	1.46	
Maze, etc.[c]	*a*	0	~100	1.67	
	a	6700	~100	1.41	
Treat, Greisen[d]	*a*	3260–4300	30–200	1.55	
	b	3260–4300	30–200	1.40	
Loverdo, Daudin[e]	*a*	2900	4–800	1.52–1.65	
	b	2900	4–800	1.30	
Williams[f]	*c*	3050	300–2000	1.50–1.90	2.70

[a] G. Cocconi, A. Loverdo, and V. Tongiorgi, Nuovo Cimento **2**, 14 (1944); Phys. Rev. **70**, 841 (1946).
[b] P. Auger and J. Daudin, J. de phys. et rad. **6**, 233 (1945).
[c] R. Maze, A. Fréon, and P. Auger, Phys. Rev. **73**, 418 (1948).
[d] J. E. Treat and K. I. Greisen, Phys. Rev. **74**, 414 (1948).
[e] A. Loverdo and J. Daudin, J. de phys. et rad. **9**, 134 (1948).
[f] R. W. Williams, Phys. Rev. **74**, 1689 (1948).

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In this case it is tacitly assumed that the density of showers is the same at each of the n counters.

By varying the surfaces S, γ can be evaluated for showers of different densities, because the showers recorded by counters of surface S have a mean density $\sim 1/S$. When γ is known, the value of the constant K in the integral density spectrum (1) can be immediately deduced from the expression:

$$K=\frac{N_n(S)}{I_{(n,\gamma)}S^{\gamma}\cdot\gamma}, \tag{3}$$

where

$$I_{(n,\gamma)}=\int_0^\infty x^{-(\gamma+1)}(1-e^{-x})^n dx,$$

which can be written

$$I_{(n,\gamma)}=-\frac{\Gamma_{(2-\gamma)}}{\gamma(\gamma-1)}\left[\binom{n}{1}-\binom{n}{2}2^{\gamma}+\binom{n}{3}3^{\gamma}-\cdots\right]^{**}.$$

(b) When $N_n(S)$ is measured for different values of the surface S of the counters, γ may be deduced from the formula:

$$\gamma=\frac{d\log N_n(S)}{d\log S}. \tag{4}$$

If this is done for different values of S, here too γ may be evaluated for different densities.

Because of the fact that the space distribution function in the showers is very likely the same for all showers, it can easily be shown that this method does not require the assumption of uniform density over all the counters.

(c) With either ionization chambers or cloud chambers the extensive showers can be studied. From the graph of the frequency of the showers, *versus* the number of particles crossing the apparatus, γ can be deduced.

(d) Finally γ can be deduced theoretically; this requires, of course, hypotheses about the origin of the showers and the processes taking place in their

** This expression of the integral $I(n,\gamma)$ was pointed out to us by Mr. Aadne Ore, whom we want to thank. Other authors have used similar expressions, e.g., J. Daudin, J. de phys. et rad. **8**, 301 (1947).

development as well as about the spectrum of the shower-producing radiation.

II. RESULTS OF PREVIOUS EVALUATIONS

The experimental evaluations of γ and K obtained thus far are collected in Table I.

As for theoretical evaluations of the density spectrum, it is well known that the behavior of the extensive showers cannot be satisfactorily described in terms of pure cascade theory, since besides electrons and photons, penetrating particles and neutrons are produced in such showers. Nevertheless, calculations have been carried out by several authors on the basis of the cascade theory, with the assumption of single electrons as primary particles.

Assuming an exponent 1.8 for the exponential spectrum of primary electrons, Cocconi[1] found for showers with densities between 10 and 1000 particles m^{-2},

$$\gamma=1.36 \quad \text{at sea level,}$$
$$\gamma=1.50 \quad \text{at 2300 m,}$$
$$\gamma=1.64 \quad \text{at 4100 m.}$$

We have now extended these calculations in order to evaluate γ over a larger range of densities, and we have found that for densities between 0.3 and 2.10^4 the best values for γ are, expressing Δ in m^{-2} and using natural logarithms,

$$\gamma=1.29+0.025\log\Delta \quad \text{at sea level,}$$
$$\gamma=1.41+0.028\log\Delta \quad \text{at 2300 m,}$$
$$\gamma=1.50+0.034\log\Delta \quad \text{at 4100 m.} \qquad (5)$$

The experimental results of Table I are not in good agreement among each other as well as with the theory, hence they do not give information on the question of the small variations of γ with the density of the showers and the altitude above sea level.

In our opinion the discrepancies arise from inaccuracy in the experimental procedures, from uncertainty in the corrections applied to the observed data, and from unreliability of the comparison of results obtained with different apparatus.

Therefore we thought it worth while to set up experiments in order to evaluate γ as accurately as possible at different altitudes and in a large density range. For the same reason we shall discuss here extensively the experimental details and the corrections to the measurements.

III. EXPERIMENTAL ARRANGEMENT

The experiments were performed both at Ithaca (260 m above sea level) and at Echo Lake (3260 m), and the arrangement was chosen in such a way as to allow evaluation of γ with both methods a and b.

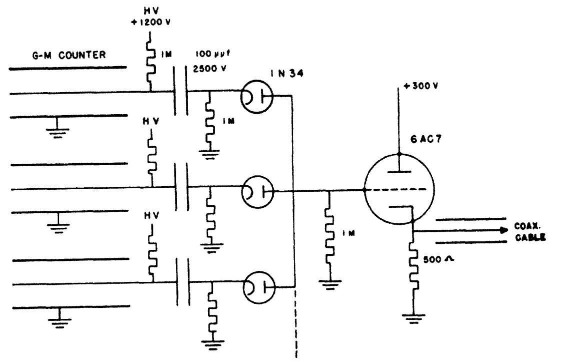

FIG. 1. Diagram of the mixing circuit.

The extensive showers were detected by four groups of counters, A, B, C, and D, placed in a horizontal plane, ABC being at the vertices of an equilateral triangle and D at the center. Threefold coincidences $A+B+C$ and fourfold $A+B+C+D$ were simultaneously recorded.

At both stations the counters were placed in light homosote boxes kept at constant temperature, which constituted the only shield from the open air.

In order to explore a density range as large as possible the following effective surfaces were used:

9 cm^2	(a single counter $\frac{1}{2}''\times 3''$)
29 cm^2	(a single counter $\frac{1}{2}''\times 10''$)
98 cm^2	(a single counter $1''\times 16''$)
500 cm^2	(a single counter $2''\times 40''$)
1500 cm^2	(three counters $2''\times 40''$)
5000 cm^2	(ten counters $2''\times 40''$)

The counters, all-metal type, were filled with an alcohol-argon mixture. The effective length of the central wire was measured and the sensitive surface of the counters was calculated as the product of such a length, times the internal diameter of the brass cylinder.

Inaccuracy in the determination of the effective surfaces may arise from the fact that some showers may fall on the counter trays with large zenith angles. Tests were made in order to evaluate the order of magnitude of such an effect. At Ithaca the frequencies of the extensive showers were recorded, once with the counter trays $ABCD$ consisting each of three counters (each $2''\times 40''$) close together, once with the same counters $2''$ apart. If the number of showers in very slanting directions is significant, one has to expect lower frequencies with the counters arranged in the packed trays. No difference in the counting rates has been found, within the statistical errors (2 percent), therefore the conclusion was reached that the number of extensive showers in very slanting directions recorded by our apparatus was negligible.*** Yet, in order to

[1] G. Cocconi, Phys. Rev. 72, 974 (1947).

*** The same conclusion has been reached by Loverdo and Daudin (see reference e of Table I) and may be independently

TABLE II. Results of measurements.

1 Counter surface S (m²)	2 Time (hr.)	3 Observed coinc. ABC T	4 Observed coinc. $ABCD$ Q	5 Corrected values T/h	6 T/Q	7 S	8 Deduced γ Method a	9 Method b
Ithaca								
0.0029	761.2	102	63	0.134±0.013	1.62±0.12	0.0029	1.58±0.14	1.45±0.10
0.0098	336.2	260	172	0.780±0.05	1.51±0.07	0.0098	1.44±0.10	1.53±0.04
0.0500	271.7	2702	1751	9.350±0.2	1.525±0.02	0.0500	1.46±0.03	1.34±0.02
0.1500	133.2	5388	3584	40±0.6	1.49±0.014	0.1480	1.41±0.03	1.36±0.015
0.5000	24.3	5115	3313	206±3.0	1.47±0.015	0.4940	1.38±0.03	
Echo Lake								
0.0009	372.4	70	44	0.188±0.023	1.59±0.15	0.0009	1.55±0.19	1.55±0.12
0.0029	190.0	215	140	1.150±0.08	1.53±0.075	0.0029	1.47±0.12	1.50±0.06
0.0098	169.2	1187	799	7.120±0.21	1.48±0.03	0.0098	1.40±0.05	1.51±0.021
0.0500	41.65	3109	2019	83±1.4	1.50±0.02	0.0500	1.425±0.03	1.36±0.016
0.1560	30.6	11646	7881	378±3.7	1.46±0.01	0.1520	1.365±0.02	1.33±0.016
0.5000	2.35	4425	3011	1790±30.0	1.40±0.013	0.4880	1.27±0.03	

minimize the effect, if it exists, both at Ithaca and at Echo Lake, all the counter trays of large area were set up with the counters 2″ apart.

The negative pulses of the counters were fed into cathode followers in order to drive the coax cables with low impedance.

Particular precaution was taken when several counters were connected in parallel in order to obtain trays of large area. It is well known that the simple connection in parallel of the anodes of several counters causes an increase in the capacity of each counter and consequently decreases the size of its pulses; furthermore it introduces fluctuations of the voltage applied on each counter, owing to the pulses of the other counters of the tray. In order to avoid these troubles, the "mixing circuit" with crystal diodes drawn in Fig. 1 has always been used between the counters and the cathode followers. Mixing circuits and cathode followers were placed very close to the counters in the same boxes that contained the counters. The recording circuit consisted of channels of amplifiers and blocking oscillators, the outputs of which were put in coincidence through crystal diodes. The resolving time, measured by chance coincidences between two counters with very high rates, was 4.1 μsec. The outputs of the counters, as well as the circuits, were carefully shielded from electrical noise.

The side of the equilateral triangle at the vertices of which counters ABC were placed, was chosen as 4 m at Ithaca and 6 m at Echo Lake, in order to take into account the increase of the average spread of the showers, due to the lower atmospheric pressure at Echo Lake. In recording extensive showers, the choice of a proper distance between the counters is important because if it is too small, either showers very poor in particles, generated by knocks-on electrons of the mesons, or locally generated penetrating showers can be recorded in large percentage besides extensive showers; while if it is too large, on the other hand, some of the smaller extensive showers can be missed. Test measurements have been performed at Ithaca with the counters ($S=1500$ cm²) arranged in a triangle once of 4-m side, once of 8-m side. While the absolute rate of the coincidences shows a decrease in the second arrangement of ~10 percent, the ratios $A+B+C+D/A+B+C$ remains practically constant within the statistical errors (2 percent), which proves that in our arrangement the ratios of recorded rates do not depend critically on the distance among the counters. Of course, if the distance between the counters is increased to many meters, one has to expect a variation not only of the rate of the showers but also of the density spectrum, the smaller showers not being recorded: this has been observed by Loverdo and Daudin.†

reached from the measurement of the zenith angle distribution of extensive showers (see e.g., M. Deutschmann, Zeits. f. Natfor. 2, 61 (1947)) which shows a very pronounced maximum in the vertical direction.

IV. RESULTS AND CORRECTIONS

The results of the experiments at Ithaca and at Echo Lake are collected in columns 1, 2, 3, and 4 of Table II. The following corrections were then taken into account:

(*1*) *Chance coincidences:* Threefold and fourfold chance coincidences have been calculated in the usual way, the higher contribution to them arising from chance coincidence between a shower striking $n-1$ counters (and not n), and an incoherent particle striking the remaining counter.

(*2*) *Barometric effect:* All the data have been normalized to the mean barometric pressure at Ithaca and at Echo Lake by assuming a barometric coefficient of 10 percent per cm Hg, as deduced from the cascade theory of the extensive showers.[1]

† See reference e of Table I.

The experimental determinations of such a coefficient, though rather uncertain, are in agreement with the theoretical value.

(*3*) *Inefficiency of the counters:* It is known that the inefficiency ϵ of G-M counters is mainly due to the dead time following a discharge. We measured the dead time of our counters and calculated their efficiencies. The efficiency $(1-\epsilon)$ of the four kinds of counters we used ($\frac{1}{2}''\times3''$, $\frac{1}{2}''\times10''$, $1''\times16''$, $2''\times40''$) were, respectively:

at Ithaca 99.9%; 99.9%; 99.7%; 98.8%
at Echo Lake 99.9%; 99.8%; 99.5%; 97.5%.

The correction for inefficiency, if applied to the coincidences registered with trays of counters in parallel, requires the knowledge of the degree of coherence of the particles striking the trays; in fact if a tray is struck by two coherent particles, very likely the two particles hit two different counters, so that the efficiency of the tray is $(1-\epsilon^2)$ rather than $(1-\epsilon)$. Such considerations can be avoided by correcting, instead of the recorded coincidences, the effective surface of the counters' tray, i.e., by assuming as effective surface the real one, multiplied by the efficiency $(1-\epsilon)$.

(*4*) *Counter walls:* Another correction, at least in principle, must be introduced in order to take into account the effect of the walls of the counters on the particles in the showers. All our counters had brass walls 0.5–0.6 mm thick and were protected from the open air by a wall of homosote $\frac{1}{2}''$ thick, in total about 1.5 g cm^{-2}. Yet unpublished results obtained by E. Palmatier**** this summer at Echo Lake show that the rate of the extensive showers remains practically constant when the thickness of the walls of the counters varies from 0.4 mm Al to 0.4 mm Al plus 1.6 mm brass. Probably this result is due to multiplication of photons which compensates the absorption of the electrons. In any case we think that no correction must be introduced in our data for the walls of the counters.

Corrections 1, 2, 3 have been applied to the numbers of columns 3 and 4 of Table II, and the corrected rates and areas are listed in columns 5, 6, and 7. The rates of column 5, *versus* surface of the counters (column 7), are plotted on a log-log scale in Fig. 2. The values of γ deduced from the ratios fourfold/threefold (column 6) with formula (2) are written in column 8 of the table. In column 9 are reported instead the values of γ deduced with formula (4) from the slope of the curve of Fig. 2 taken at various S. All the values of γ deduced with the two methods are plotted in Fig. 3 as a function of the logarithm of $1/S$, which is proportional to the mean density of the showers. A check of the validity of the measurements is the fact that the agreement between the results deduced with the two methods is quite satisfactory both at Ithaca and at Echo Lake. From the results it is possible to reach the following conclusions:

(1) γ is observed to increase with the density of the showers. The lines drawn in Fig. 3 are expressed by the equations:

$$\begin{array}{ll}\text{Ithaca (260 m)} & \gamma=1.31+0.038\log 1/S \\ \text{Echo Lake (3260 m)} & \gamma=1.26+0.043\log 1/S \end{array} \quad (6)$$
(S in m^2 and natural logarithms).

Remembering what has been said at the beginning, in first approximation it is legitimate to substitute $\bar{\Delta}$ for $1/S$ in expressions (6).

(2) The variation of γ with Δ implies a dependence of the constant K in formula (1) on Δ. This means that the law (1) cannot be considered true for ranges of densities in which $\Delta\max/\Delta\min>10^2$.

(3) If we confine ourselves within the densities between 10 and 1000 particles per square meter, the value of K and γ which best fit our experimental points are (with Δ measured in m^{-2} and time in sec.).

$$\begin{array}{lll}\text{at Ithaca} & K=0.200 & \gamma=1.48, \\ \text{at Echo Lake} & K=1.68 & \gamma=1.46. \end{array} \quad (7)$$

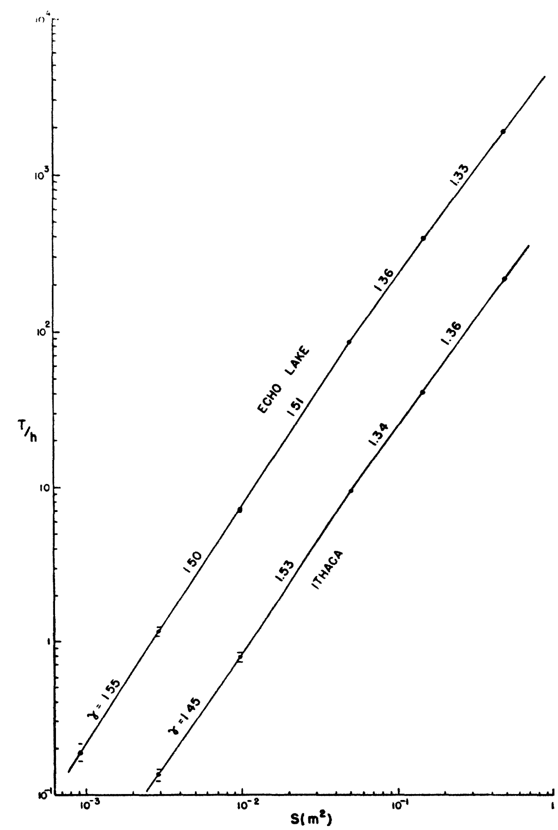

FIG. 2. Rate of threefold coincidences *versus* surface of the counters at Ithaca and Echo Lake.

**** We are very grateful to Mr. E. Palmatier for making his results available to us.

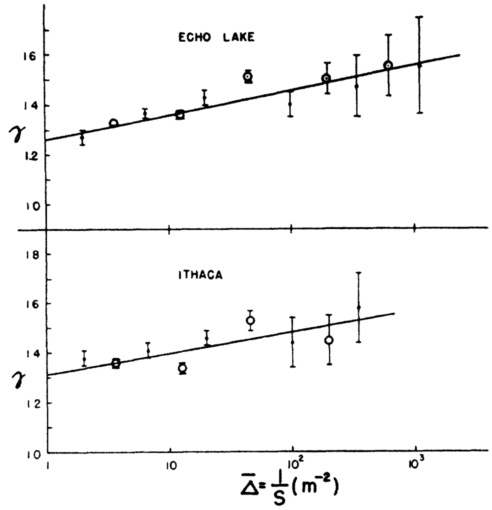

FIG. 3. γ *versus* mean density of the showers at Ithaca and Echo Lake.

Outside these limits, (7) are not applicable. For other ranges of densities, K varies, slightly increasing with Δ.

(4) The controversial point of the variation of γ with altitude is more complex than was thought before; while γ decreases with altitude for showers of density smaller than 10^3-10^4 m^{-2}, yet, if one can extrapolate our lines in Fig. 3, γ is increasing with height for bigger densities.

For showers recorded with counters of normal size it seems that γ decreases very slightly with altitude, qualitatively confirming the measurements of Auger and Daudin[2] and Maze and co-workers,[3] though the decrease observed by us is much smaller than the one they measured and the absolute values of K quite different. As for our previous results,[4] which showed an increase of γ with height, the value of γ obtained at sea level agrees very well with (7), but we probably overestimated the influence of the roof over the counters in the higher station; in fact the value of γ deduced there from the uncorrected measurements (1.45) is in excellent agreement with the present one. Our old values of K are instead not well in agreement with the present ones, even correcting them for the altitude differences. We consider the present figures more reliable. As for the showers observed by Williams[5] with ionization chambers, one has to expect γ's larger than ours, because the densities of the showers he recorded are generally bigger than ours, but even in the range of density where our measurements and his measurements overlap ($300<\Delta<1000$ m^{-2}), and the geometrical disposition of the chambers and of the counters is the same, his γ is larger. This may be in part due to the statistical errors, which are rather large in both measurements, but we think that something else contributes to the discrepancies. One has to remember that all data on the extensive showers taken in the past with ionization chambers were meaningless, because other phenomena (stars, local showers) strongly affected the measurements; all these data led to γ much bigger than the present ones. Maybe Williams did not eliminate completely the spurious phenomena which affected the previous measurements. Even the value of K found by Williams is bigger than ours, but this may be due, at least in part, to the increase of K with γ, as pointed out before.

(5) The comparison with the theoretical evaluations confirms what has been said before: the agreement is only qualitative, and we think this is due more than to defects in the cascade theory, to the inapplicability of such a scheme to the extensive showers, in which other processes, beside the multiplication of electrons and photons, take place.

The authors wish to thank Prof. K. Greisen for many helpful suggestions and discussions. They are grateful to the Research Corporation for a grant which covered part of the expenses of performing the experiments described above. The facilities of the Inter-University High Altitude Laboratory Association, and the help of Professors Čohn and Iona of Denver University were an invaluable aid in performing the experiment.

[2] P. Auger and J. Daudin, J. de phys. et rad. 6, 233 (1945).
[3] R. Maze, A. Fréon, and P. Auger, Phys. Rev. **73**, 418 (1948).
[4] G. Cocconi, A. Loverdo, and V. Tongiorgi, Nuovo Cimento 2, 14 (1944); Phys. Rev. **70**, 841 (1946).
[5] R. W. Williams, Phys. Rev. **74**, 1689 (1948).